18.034 ; Feb 4, 2004

Lecture 1

1. Discussed general idea of modeling a real-world problem with a D.E.
   (i) Set-up the model: decide on relevant independent variables, relevant dependent variables or state variables, relevant constants/ parameters, and use real-world lens to find a diff. equations the dependent variables satisfy.
   (ii) Analyze the D.E.: Find a closed form solution, prove existence and uniqueness, determine long-time behavior, determine dependence of solution on the parameters, etc.
   (iii) Interpret the mathematical analysis in terms of the original problem: Does a population become extinct? Does feedback cause a spring to break? Will increasing harvesting of a predator in a predator-prey system increase/ decrease the equil. pop. of the prey.

2. Terminology: PDE, independent variable, dependent variable, solution, ODE, interval, initial time, initial value, initial condition, IVP, order, linear ODE, nonlinear ODE, normal form, autonomous.

3. 2 models. A. Spring: (i) Model using Newton's 2nd law + models
   $mx'' = -kx \rightsquigarrow x(t) = A\cos(\omega t - \phi)$ where $\omega^2 = \frac{k}{m}$.
   (ii) Model using cons. of energy $\frac{1}{2}m(x')^2 + \frac{1}{2}kx^2 = E_0$.
   Exactly solved and observed existence doesn't rerr. hold, and uniqueness doesn't hold.

B. Logistic model with harvesting: $y' = ay - cy^2 - H$.