18.03 Hour Exam II
March 17, 2010

Your Name
Your Recitation Leader's Name
Your Recitation Time

Problem	Points
1	
2	
3	
4	
5	
Total	

Do not open this booklet till told to do so. There are five problems. Use your test-taking skills—be sure you get to all the problems. Do all your work on these pages. No calculators or notes may be used. The point value (out of 100) of each problem is marked in the margin. Solutions will be available on the web after 4:00 today, and at recitation.

[8]   **1. (a)** For what value of k is the system represented by $\ddot{x} + \dot{x} + kx = 0$ critically damped?

[4]   **(b)** For k greater than that value, is the system overdamped or underdamped?

[8]   **(c)** Suppose a solution of $\ddot{x} + \dot{x} + kx = 0$ vanishes at $t = 1$, and then again for $t = 2$ (but not in between). What is k?

[10]   **2. (a)** Find a solution of $\ddot{x} + x = 5te^{2t}$.

[10]   **(b)** Suppose that $y(t)$ is a solution of the same equation, $\ddot{x}+x = 5te^{2t}$, such that $y(0) = 1$ and $\dot{y}(0) = 2$. (This is probably *not* the solution you found in **(a)**.) Use $y(t)$ and other functions to write down a solution $x(t)$ such that $x(0) = 3$ and $\dot{x}(0) = 5$.

[10] **3. (a)** Consider the equation $\ddot{x} + b\dot{x} + kx = \cos(\omega t)$. We will vary the spring constant but keep b fixed. For what value of k is the amplitude of the sinusoidal solution of $\ddot{x} + b\dot{x} + kx = \cos(\omega t)$ maximal? (Your answer will be a function of ω and may depend upon b as well.)

[10] **(b)** (Unrelated to the above.) Find the general solution of $\dfrac{d^3x}{dt^3} - \dfrac{dx}{dt} = 0$.

4. A certain system has input signal y and system response x related by the differential equation $\ddot{x} + \dot{x} + 6x = 6y$. It is subjected to a sinusoidal input signal.

[10]   **(a)** Calculate the complex gain $H(\omega)$.

[5]   **(b)** Compute the gain at $\omega = 2$.

[5]   **(c)** Compute the phase lag at $\omega = 2$.

5. Suppose that $\frac{1}{2}t\sin(2t)$ is a solution to a certain equation $m\ddot{x} + b\dot{x} + kx = 4\cos(2t)$.

[4] **(a)** Write down a solution to $m\ddot{x} + b\dot{x} + kx = 4\cos(2t - 1)$.

[4] **(b)** Write down a solution to $m\ddot{x} + b\dot{x} + kx = 8\cos(2t)$.

[12] **(c)** Determine m, b, and k.

MIT OpenCourseWare

18.03 Differential Equations
Spring 2010