Problems: Work Along a Space Curve

1. Find the work done by the force $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ in moving a particle from $(0,0,0)$ to $(2,4,8)$
(a) along a line segment
(b) along the path $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$.

<u>Answer:</u>
(a) We use the parametrization $x = 2t$, $y = 4t$, $z = 8t$, where $0 \le t \le 1$. Other parametrizations should also work.

$$\begin{aligned} W &= \int_C M\,dx + N\,dy + P\,dz \\ &= \int_C -y\,dx + x\,dy + z\,dz \\ &= \int_0^1 -4t\,dt + 2t\,dt + 8t\,dt \\ &= \int_0^1 6t\,dt = 3. \end{aligned}$$

(b) We use the parametrization we were given:

$$\begin{aligned} W &= \int_C -y\,dx + x\,dy + z\,dz \\ &= \int_0^2 (-t^2)dt + t\,(2t\,dt) + t^3(3t^2\,dt) \\ &= \int_0^2 3t^5 + t^2\,dt = \frac{104}{3}. \end{aligned}$$

Note that for this force field, work done is not path independent.

2. Let $\mathbf{F} = \nabla f$, where $f = \dfrac{1}{(x+y+z)^2+1}$. Find the work done by $\mathbf{F}$ in moving a particle from the origin to infinity along a ray.

<u>Answer:</u> The fundamental theorem tells us that $\int_C \mathbf{F}\cdot d\mathbf{r} = f(P_1) - f(0)$ if C goes from 0 to P_1. In this example $f(0) = 1$, and as P_1 goes to infinity $f(P_1)$ approaches 0. Thus the work done is $0 - 1 = -1$.