Problems: Calculating Flux

1. Find the flux of $\mathbf{F} = \langle x, y, z \rangle$ through the surface $x^2 + y^2 + z^2 = 1$, where $z \geq 0$.

Answer: The surface in question is the upper unit half-sphere and $\mathbf{F}$ is identical to the outward unit normal. Therefore, $\mathbf{F} \cdot \mathbf{n} = 1$ and $\displaystyle\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \text{Area} = 2\pi r^2$.

2. Find the flux of $\mathbf{F} = \langle 0, x, 0 \rangle$ through the portion of the plane $x + z = 1$ for which $x > 0$, $0 < y < 1$ and $z > 0$.

Answer: The surface in question is a rectangle in the first octant. It has constant normal $\langle 1, 0, 1 \rangle$ which is everywhere orthogonal to $\mathbf{F}$, so $\mathbf{F} \cdot \mathbf{n} = 0$ over the surface and the flux is 0.

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