18.02 Practice Exam 3 A

1. Let $(\bar{x}, \bar{y})$ be the center of mass of the triangle with vertices at $(-2, 0)$, $(0, 1)$, $(2, 0)$ and uniform density $\delta = 1$.

a) (10) Write an integral formula for $\bar{y}$. Do not evaluate the integral(s), but write explicitly the integrand and limits of integration.

b) (5) Find $\bar{x}$.

2. (15) Find the polar moment of inertia of the unit disk with density equal to the distance from the y-axis.

3. Let $\vec{F} = (ax^2y + y^3 + 1)\hat{\mathbf{i}} + (2x^3 + bxy^2 + 2)\hat{\mathbf{j}}$ be a vector field, where a and b are constants.

a) (5) Find the values of a and b for which $\vec{F}$ is conservative.

b) (5) For these values of a and b, find $f(x, y)$ such that $\vec{F} = \nabla f$.

c) (5) Still using the values of a and b from part (a), compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ along the curve C such that $x = e^t \cos t$, $y = e^t \sin t$, $0 \le t \le \pi$.

4. (10) For $\vec{F} = yx^3\hat{\mathbf{i}} + y^2\hat{\mathbf{j}}$, find $\int_C \vec{F} \cdot d\vec{r}$ on the portion of the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.

5. Consider the region R in the first quadrant bounded by the curves $y = x^2$, $y = x^2/5$, $xy = 2$, and $xy = 4$.

a) (10) Compute $dxdy$ in terms of $dudv$ if $u = x^2/y$ and $v = xy$.

b) (10) Find a double integral for the area of R in uv coordinates and evaluate it.

6. a) (5) Let C be a simple closed curve going counterclockwise around a region R. Let $M = M(x, y)$. Express $\displaystyle\oint_C M dx$ as a double integral over R.

b) (5) Find M so that $\displaystyle\oint_C M dx$ is the mass of R with density $\delta(x, y) = (x + y)^2$.

7. Consider the region R enclosed by the x-axis, $x = 1$ and $y = x^3$.

a) (5) Use the normal form of Green's theorem to find the flux of $\vec{F} = (1 + y^2)\hat{\mathbf{j}}$ out of R.

b) (5) Find the flux out of R through the two sides C_1 (the horizontal segment) and C_2 (the vertical segment).

c) (5) Use parts (a) and (b) to find the flux out of the third side C_3.