Matrix multiplication

1. Let $A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 5 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{pmatrix}$, $D = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$, $E = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

For each of the following say whether it makes sense to compute it. If it makes sense then do the computation.

(i) AA (ii) AB (iii) AC (iv) AE (v) DA (vi) CE (vii) $A + B$ (viii) $A + D$.

Answer: i) A 2×2 matrix times a 2×2 matrix is a 2×2 matrix:

$$A \cdot A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 13 & 8 \\ 24 & 37 \end{pmatrix}.$$

ii) (2×2) times (2×3) = (2×3): $A \cdot A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 13 & 16 & 19 \\ 24 & 29 & 34 \end{pmatrix}.$

iii) (2×2) times (3×2) does not make sense.

iv) (2×2) times (2×1) = (2×1): $A \cdot E = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ 35 \end{pmatrix}.$

v) (2×2) times (2×2) = 2×2: $D \cdot A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ 12 & 15 \end{pmatrix}.$

vi) (3×2) times (2×1) = (3×1): $\begin{pmatrix} 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 17 \\ 20 \\ 23 \end{pmatrix}.$

vii) For addition the matrices have to be the same size, so this does not make sense.

viii) This makes sense, the addition is done entrywise:

$$A + D = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 4 & 8 \end{pmatrix}.$$

2. Let $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. Find a column vector B such that $AB = \begin{pmatrix} b \\ e \\ h \end{pmatrix}$ (the middle column of A).

Answer: $B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow AB = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} b \\ e \\ h \end{pmatrix}.$

3. Write the following system in matrix form

$$\begin{array}{rcrcrcl}
2x & + & 3y & + & 5z & = & 2 \\
 & & 2y & + & z & = & 1 \\
x & - & 2y & + & & = & 3.
\end{array}$$

Answer:

$$\begin{pmatrix} 2 & 3 & 5 \\ 0 & 2 & 1 \\ 1 & -2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}.$$

18.02SC Multivariable Calculus
Fall 2010