Equation of a Plane

1. Later we will return to the topic of planes in more detail. Here we will content ourself with one example.

Find the equation of the plane containing the three points $P_1 = (1,3,1)$, $P_2 = (1,2,2)$, $P_3 = (2,3,3)$.

<u>**Answer:**</u>

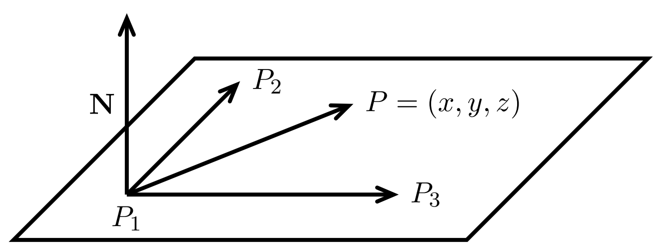
The vectors $\overrightarrow{\mathbf{P_1P_2}}$ and $\overrightarrow{\mathbf{P_1P_3}}$ are in the plane, so

$$\mathbf{N} = \overrightarrow{\mathbf{P_1P_2}} \times \overrightarrow{\mathbf{P_1P_3}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = \mathbf{i}(-2) - \mathbf{j}(-1) + \mathbf{k}(1) = \langle -2, 1, 1 \rangle.$$

is orthogonal to the plane.

Now for any point $P = (x, y, z)$ in the plane, the vector $\overrightarrow{\mathbf{P_1P}}$ is also in the plane and is therefore orthogonal to $\mathbf{N}$. Expressing this with the dot product we get

$$\begin{aligned} & \mathbf{N} \cdot \overrightarrow{\mathbf{P_1P}} = 0 \\ \Leftrightarrow \quad & \langle -2, 1, 1 \rangle \cdot \langle x-1, y-3, z-1 \rangle = 0 \\ \Leftrightarrow \quad & -2(x-1) + (y-3) + (z-1) = 0 \\ \Leftrightarrow \quad & -2x + y + z = 2. \end{aligned}$$



The equation of the plane is $-2x + y + z = 2$. You should check that the three points P_1, P_2, P_3 do, in fact, satisfy this equation.

The standard terminology for the vector $\mathbf{N}$ is to call it a *normal* to the plane.