Components and Projection

If $\mathbf{A}$ is any vector and $\widehat{\mathbf{u}}$ is a unit vector then the *component* of $\mathbf{A}$ in the direction of $\widehat{\mathbf{u}}$ is

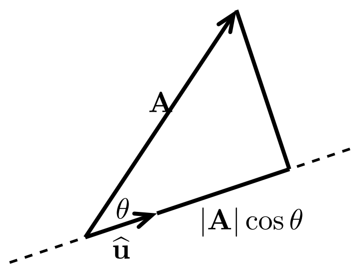
$$\mathbf{A} \cdot \widehat{\mathbf{u}}.$$

(Note: the component is a scalar.)

If θ is the angle between $\mathbf{A}$ and $\widehat{\mathbf{u}}$ then since $|\widehat{\mathbf{u}}| = 1$

$$A \cdot \widehat{\mathbf{u}} = |\mathbf{A}||\widehat{\mathbf{u}}| \cos\theta = |\mathbf{A}| \cos\theta.$$

The figure shows that geometrically this is the length of the leg of the right triangle with hypotenuse $\mathbf{A}$ and one leg parallel to $\widehat{\mathbf{u}}$.



We also call the leg parallel to $\widehat{\mathbf{u}}$ the *orthogonal projection* of $\mathbf{A}$ on $\widehat{\mathbf{u}}$.

For a non-unit vector: the component of $\mathbf{A}$ in the direction of $\mathbf{B}$ is simply the component of $\mathbf{A}$ in the direction of $\widehat{\mathbf{u}} = \frac{\mathbf{B}}{|\mathbf{B}|}$. ($\widehat{\mathbf{u}}$ is the unit vector in the same direction as $\mathbf{B}$.)

Example: Find the component of $\mathbf{A}$ in the direction of $\mathbf{B}$.

i) $|\mathbf{A}| = 2$, $|\mathbf{B}| = 5$, $\theta = \pi/4$.

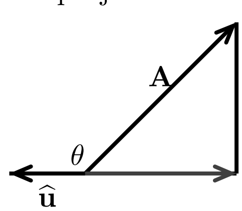
Answer: Referring to the figure above: the component is $|\mathbf{A}| \cos\theta = 2\cos(\pi/4) = \sqrt{2}$. Note, the length of $\mathbf{B}$ given is irrelevant, since we only care about the unit vector parallel to $\mathbf{B}$.

ii) $\mathbf{A} = \mathbf{i} + 2\mathbf{j}$, $\mathbf{B} = 3\mathbf{i} + 4\mathbf{j}$.

Answer: Unit vector in direction of $\mathbf{B}$ is $\frac{\mathbf{B}}{|\mathbf{B}|} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j} \;\Rightarrow\;$ component is $\mathbf{A} \cdot \mathbf{B}/|\mathbf{B}| = 3/5 + 8/5 = 11/5$.

iii) Find the component of $\mathbf{A} = \langle 2, 2 \rangle$ in the direction of $\widehat{\mathbf{u}} = \langle -1, 0 \rangle$

Answer: The vector $\widehat{\mathbf{u}}$ is a unit vector, so the component is $\mathbf{A} \cdot \widehat{\mathbf{u}} = \langle 2, 2 \rangle \cdot \langle -1, 0 \rangle = -2$. The negative component is okay, it says the projection of $\mathbf{A}$ and $\widehat{\mathbf{u}}$ point in opposite directions.



We emphasize one more time that the component of a vector is a *scalar*.