Proofs using vectors

1. The median of a triangle is a vector from a vertex to the midpoint of the opposite side.

Show the sum of the medians of a triangle = $\mathbf{0}$.

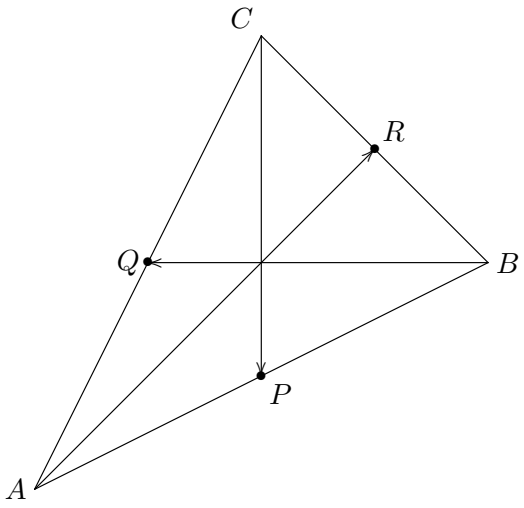
Answer: The median of side AB is the vector from vertex C to the midpoint of AB. Label this midpoint as P. As usual we write $\mathbf{P}$ for the origin vector $\overrightarrow{\mathbf{OP}}$.

The midpoint $\mathbf{P} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) \Rightarrow \overrightarrow{\mathbf{CP}} = \frac{1}{2}(\mathbf{B} + \mathbf{A}) - \mathbf{C}$.

Likewise: $\overrightarrow{\mathbf{BQ}} = \frac{1}{2}(\mathbf{A} + \mathbf{C}) - \mathbf{B}$ and $\overrightarrow{\mathbf{AR}} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) - \mathbf{A}$.

$\Rightarrow$ sum of medians is

$$\overrightarrow{\mathbf{CP}} + \overrightarrow{\mathbf{BQ}} + \overrightarrow{\mathbf{AR}} = \left(\frac{1}{2}(\mathbf{B} + \mathbf{A}) - \mathbf{C}\right) + \left(\frac{1}{2}(\mathbf{A} + \mathbf{C}) - \mathbf{B}\right) + \left(\frac{1}{2}(\mathbf{B} + \mathbf{C}) - \mathbf{A}\right) = 0.$$



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