18.02 Multivariable Calculus
Fall 2007

18.02 Problem Set 9

Due Thursday 11/8/07, 12:45 pm.

Part A   (10 points)

Hand in the <u>underlined</u> problems only; the others are for more practice.

Lecture 24. Thu Nov. 1   Green's theorem.
  Read: 15.4 to top of p. 1043.
  Work: 4D/ 1ab<u>c</u>, <u>2</u>, <u>3</u>, <u>4</u>, 5.

Lecture 25. Fri Nov. 2   Flux. Normal form of Green's theorem.
  Read: Notes V3, V4.
  Work: 4E/ <u>1ac</u>, <u>2</u>, 3, <u>4</u>, <u>5</u> (don't use Green's theorem); 4F/ 3, <u>4</u>.

Lecture 26. Tue Nov. 6   Simply-connected regions. Review.
  Read: Notes V5.

Lecture 27. Thu Nov. 8   **Exam 3 covering lectures 18–26.**

Part B   (16 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Write the names of all the people you consulted or with whom you collaborated and the resources you used.

Problem 1. (Thursday, 4 points: 2+2)

a) Show that, if a simple closed (positively oriented) curve C is the boundary of a region R, then $\text{Area}(R) = \oint_C x\,dy = \oint_C -y\,dx$.

b) Use this result to calculate the area of the region between the x-axis and one arch of the cycloid with parametric equations $x = a(t - \sin t)$, $y = a(1 - \cos t)$. (Hint: the second line integral leads to easier calculations).

Problem 2. (Thursday, 6 points: 4+2)

a) For what simple closed (positively oriented) curve C in the plane does the line integral $\oint_C (x^2 y + y^3 - y)\,dx + (3x + 2y^2 x + e^y)\,dy$ have the largest positive value? (use Green's theorem).

b) Determine what this value is.

Problem 3. (Friday, 6 points: 2+2+2)

a) Let C be the unit circle, oriented counterclockwise, and consider the vector field $\vec{F} = xy\,\hat{\boldsymbol{\imath}} + y^2\,\hat{\boldsymbol{\jmath}}$. Which portions of C contribute positively to the flux of $\vec{F}$? Which portions contribute negatively?

b) Find the flux of $\vec{F}$ through C by direct calculation (evaluating a line integral). Explain your answer using (a).

c) Find the flux of $\vec{F}$ through C using Green's theorem.