## Review of Trigonometric Identities

The topic of this segment is the use of trigonometric substitutions in integration. We start by reviewing some basic facts about trigonometry.

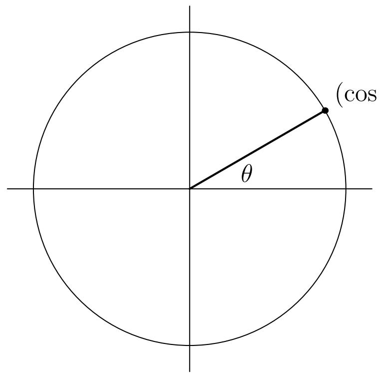


Figure 1: The unit circle.

Trigonometry is based on the circle of radius 1 centered at $(0, 0)$. A point on that circle at angle $\theta$ (see Figure **??**fig:l27g1) has coordinates $(\cos\theta, \sin\theta)$. Because the radius of the circle is 1, the Pythagorean theorem tells us right away that $\sin^2\theta + \cos^2\theta = 1$. (Remember that $\sin^2\theta$ means $(\sin\theta)^2$.) You may also remember some double angle formulas.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ \sin(2\theta) &= 2\sin\theta\cos\theta \end{aligned}$$

From the double angle formula for $\cos(2\theta)$ we can derive the half angle formula:

$$\begin{aligned} \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= \cos^2\theta - (1 - \cos^2\theta) \\ \cos(2\theta) &= 2\cos^2\theta - 1 \\ \Rightarrow \cos^2\theta &= \frac{1 + \cos(2\theta)}{2} \end{aligned}$$

This formula will allow us to rewrite powers like $\cos^2\theta$ in lower degree terms. A similar calculation shows that:

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}.$$