Probability Example

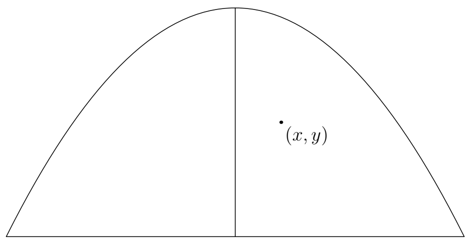

Figure 1: Choose a point at random.

Probability, volumes and weighted averages are three of the most important applications of integration. We'll analyze the probability experiment of picking a point "at random" in the region bounded below by $y = 0$ and above by $y = 1 - x^2$. Inside this parabolic region, the probability of picking a point in a given location is proportional to the area of the location.

What is the chance that $x > 1/2$? In other words, for a point picked at random, what is the probability that $x > 1/2$? Or, what is $P(x > 1/2)$?

$$\begin{aligned} \text{Probability} &= \frac{\text{Part}}{\text{Whole}} \\ &= \frac{\text{Target Area}}{\text{Entire Area}} \\ &= \frac{\text{Success}}{\text{All Possibilities}} \end{aligned}$$

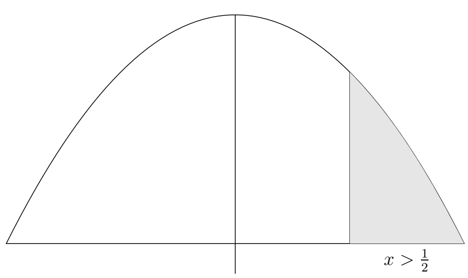

Figure 2: What is the probability that $x > \frac{1}{2}$?

The probability will just be the ratio of the two areas:

$$\frac{\int_{1/2}^{1} (1 - x^2)\, dx}{\int_{-1}^{1} (1 - x^2)\, dx}.$$

If we like, we can think of this as a weighted average with $w(x) = 1-x^2$, $a = -1$, $b = 1$ and:

$$f(x) = \begin{cases} 0 & \text{when } x < 1/2 \\ 1 & \text{when } x \geq 1/2. \end{cases}$$

$$\begin{aligned} P(x > 1/2) &= \frac{\int_{1/2}^{1}(1-x^2)\,dx}{\int_{-1}^{1}(1-x^2)\,dx} \\ &= \frac{(x - \frac{x^3}{3})\Big|_{1/2}^{1}}{(x - \frac{x^3}{3})\Big|_{-1}^{1}} \\ &= \frac{\left(\frac{2}{3} - \frac{11}{24}\right)}{\left(\frac{2}{3} - \left(-\frac{2}{3}\right)\right)} \\ &= \frac{5}{24} \div \frac{4}{3} \\ &= \frac{5}{32}. \end{aligned}$$