

PROFESSOR: Welcome back to recitation. In this video we're going to do another solid of revolution problem. So what I'd like to do in this problem is to find the volume of the solid generated by rotating the region bounded by the following curves: y equals 0, x equal 4, and y equals square root of x , around the line x equals 6. And you can choose your favorite method to do this.

What I would like first, when you're doing this kind of problem, is get a rough sketch of the region. So you have some picture of what's actually going to happen. You don't necessarily need a three-dimensional picture. But at least have the two-dimensional region and understand the where the rotation line is, with respect to that region. So I will give you a little bit of time to work on that problem. And when I come back I'll show you how I do it.

OK, welcome back. So again, what we're doing in this video is we're going to be looking, finding the volume of a solid of revolution. And as I mentioned, the first thing you want to do is get a rough picture of what the region looks like that's being rotated. So I'm going to draw a rough sketch of that. Right here. So my axes, here's my x -axis and my y -axis, and I will draw, first the curve y equals 0. That's y equals 0. And I'm next going to draw the curve y equals square root of x to make this a little easier for myself, to do that one first. So y equals square root of x looks something like this, roughly. And then, x equal 4, because it's a very rough sketch, I can just draw this line right here and say that line's x equal 4. And I want the region bounded by those three curves.

So, this region right here. And my rotating line, in this case, is x equals 6. So to give myself some perspective, I remember this is x equals 4, here's x equals 0. So x equals 6, which may be right about here, I'll draw a dotted line. And I'm going to draw a little arrow around there so I know that's the line I'm rotating about.

Now we notice there's a little bit of a gap here. But you can actually work on this problem quite simply. If we use the shell method, we won't have to worry about the gap at all. We will have to be careful in how we determine our radius. That's actually where you'll see this sort of gap coming up. So when I rotate this around, I'm going to get some object that's going to have a nice curve here. And there's going to be a little cut-out, a cylinder missing in the middle when I rotate it.

So I'm going to use the shell method. I like the shell method for this problem. You could use the washer method for this problem, but I like the shells and that's what I'm going to use. So to

do the shell method, remember we are going to be interested in $2\pi r h$. That's really what we need. And now what we want to figure out is, do I want to do this in terms of x or in terms of y ? All right? When I talk about these shells is it better to think about them as functions of x or y ? And it will, you'll see we sort of have to think about them in a certain way.

So if I draw, let's think about if I draw a straight line down and I rotate it around the line $x = 6$, which I should have labeled here, then I get my shell. So I need these lines to go from 0 to 4. I need to have those lines go from 0 to 4. And so I see that it's the x -value I'm varying over. So I know this is all going to be an integral in terms of x . So let's keep that in mind. So r I should write as a function of x , and h I should write as a function of x .

So if I examined this segment again here, the height is fairly simple. Because what is this curve? This is the curve $y = \sqrt{x}$. So the height is just square root of x . So h is equal to square root of x . And so now I just need r in terms of x .

Well let's look at this segment again. It's just a generic position. How would I find that radius? What is the radius? Well I'm going to draw the three-dimensional picture over here. That that's this curve, this segment, rotated about the line $x = 6$. Right? And the radius is not x . It's the distance from $x = 6$ to my segment. That's an important distinction we should understand. If we come back and look at this picture, this is not the radius. Right? The radius is the distance from this value to this value in x .

And how do I measure that? Well that's fairly straightforward. That's the difference in x values between these two. The x value here is 6. And the x value here, this is a generic x . So the difference, the distance, is just $6 - x$. Right? This is just a simple case of the distance formula. Because I'm not interested in the changes in y , at all. I'm just interested in the difference between these two x values. So it's simply $6 - x$. That's our radius.

So now I have the radius in terms of x . And I have the height in terms of x . So I can completely set up my integral. So let me come a little bit further over and set up the integral. So I know I'm integrating-- we even said what we were integrating from and to already. I'm starting x at 0. And I'm stopping it at $x = 4$. And then I want to integrate $2\pi r h$. So I'll put the 2π out in front. The radius is $6 - x$. And the height is square root of x . And then I'm integrating it all in dx .

And at this point, solving the problem is fairly straightforward. I can write square root of x as $x^{1/2}$ to make myself feel a little better about it. I can then distribute. And then I just use

the power rule to do the integration and evaluate to get an actual number, to get that volume.

So I'm going to go back and just make sure we remember what all the pieces are and that we feel comfortable with doing this type of problem. And I'll let you finish and evaluate that if you would like to know an actual number.

So let's come back over. We were finding the volume of a solid generated by rotating a certain region around the line $x = 6$. The way this might be different from some of the problems you've seen before, is it's not the x or y axis. And the region has a gap. The region is not connected to the line of rotation.

So if we come to see our picture, we can see that clearly. The rotating region is the blue region. The line of rotation is the dotted line. So there's a little bit of a gap there. I let you pick your favorite method. I chose shells, because I didn't have to worry about subtracting off pieces, this little piece here. It was inherent in the radius, the way I determined the radius and the x values that I chose. So I used shell method. And so how did I, I had to figure out a couple things. I had to figure out first, what variable I was doing this in, x or y . And then I had to figure out the radius and height in terms of that variable. You figure out what variable you need by saying, OK if I'm doing shells, I'm looking at these kinds of segments. And I'm varying the x -values, then, as I move and look at those different segments. So I know that this is a dx type of problem. I know I'm integrating something in x .

So then I have to find radius and height in terms of x . The height's the freebie. It's just from 0 up to the function. The radius is maybe a little harder. But that's still just the distance between the line of rotation and the point where you are. And so that's $6 - x$. And then I just set up the integral from there.

So that's really what the idea is in doing these types of problems. And because we get to pick our simplest way, for me it was shells. Maybe you picked washers. But for me, this problem, the simplest way is shells. So I guess that's where I'll stop.