

Substitution: $\int x^3(x^4 + 2)^5 dx$

We want to compute $\int x^3(x^4 + 2)^5 dx$.

We already have a formula for $\int x^n dx$, so we could expand $(x^4 + 2)^5$ and integrate the polynomial. That would be messy. Instead we'll use the method of substitution.

Finding the exact integral of a function is much harder than finding its derivative; occasionally it's impossible. In this unit we're only going to use one method, which means that whenever you see an integral, either you'll be able to divine immediately what the answer is or you'll use substitution. The method of substitution is tailor-made for differential notation.

To use the method of substitution, find the messiest function in your integral; in this case that will be $u = x^4 + 2$. The differential of u is $du = u' dx = 4x^3 dx$. Luckily, we can substitute these two expressions into our original integral and simplify it considerably.

The original problem was to find $\int x^3(x^4 + 2)^5 dx$. We can replace $(x^4 + 2)^5$ by u^5 , and $x^3 dx = \frac{1}{4} du$:

$$\begin{aligned} \int x^3(x^4 + 2)^5 dx &= \int \underbrace{(x^4 + 2)^5}_{u^5} \underbrace{x^3 dx}_{\frac{1}{4} du} \\ &= \int \frac{u^5 du}{4} \\ &= \frac{1}{4} \cdot \frac{1}{6} u^6 = \frac{u^6}{24} \end{aligned}$$

This is not the answer to the question, because this answer is expressed in terms of u . The problem was posed in terms of the variable x . We change variables back to x by plugging in our definition of u :

$$\frac{u^6}{24} = \frac{(x^4 + 2)^6}{24}.$$

We conclude that:

$$\int x^3(x^4 + 2)^5 dx = \frac{(x^4 + 2)^6}{24}.$$

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