

PROFESSOR: Hi. Welcome back to recitation. You've been talking about differentials. And one of the things that you've shown is that some of the stuff that we've been doing with derivatives can also be phrased in terms of differentials. That for many purposes they form a just a different language to compute the same things. So in particular one example of that is linear approximation. So I have a problem here that's a linear approximation problem, but I'd like you to do it using the method of differentials.

So in particular I'd like you to approximate the square root of 21. And the way I'd like you to do that is by using a linear approximation to the function f of x equals the square root of $10x$ minus x squared based at the point 2, x_0 equals 2. Note that, you know, why are we using this function to approximate square root of 21? Well because at 3, the function value at 3, f of 3, is exactly equal to the square root of 21. So to approximate the square root 21 means approximate the function value at 3. And so, you know, this function has a nice value at x equals 2, so we can use linear approximation at 2 to try and compute an approximation to the function at x equals 3. So why don't you take a couple of minutes, work this out, come back and we can work it out together.

All right, welcome back. So we're doing linear approximation here. And we're doing it with differentials. So, in general, when we use the method of differentials to compute a linear approximation, what we have is that if y equals f of x , so if y is given as a function of x , then we have that-- so if we want to change x a little bit and figure out what the change in y is, we have this formula, which is that f of x plus dx -- so the function value at a nearby point-- is approximately equal to y plus dy . So this is our formula for linear approximation in terms of differentials.

So in our case we have that our-- that the point we want is x equals 2 and y equals 4. And so we have, we know what dx is also, because we want the approximation at the point 3. So we want dx to be equal to 1 for this approximation. And so the question is, what is dy going to be? So that's the value that we're after. That tells us how much the function value changes.

So to compute dy , well dy is the differential of the function. So this is d of the square root of $10x$ minus x squared, quantity. OK, so to compute this differential now, we just use our straight-forward rules for doing this. So we take a derivative and then we have dx 's where necessary. So this is d of square root of $10x$ minus x squared. So the outermost function is the

square root function. So this is going to be $\frac{1}{2}$ times $10x$ minus x squared to the minus $\frac{1}{2}$. So that's $\frac{1}{2}$ over the square root of $10x$ minus x squared. And now I need to multiply by the derivative of the inside function, which is $10 - 2x$ dx . So really it's $10dx$ minus $2x \cdot dx$ but I just, you know, pulled that dx out in front. So we have a differential equal to a differential.

OK, and so, but we want this value at this particular point in question. So at this particular point in question we have x equals 2 and dx is equal to 1 . So the value of dy that we want is equal to $\frac{1}{2}$ times-- well this is again, OK, 10 times 2 minus 2 squared, that's 16 . To the minus $\frac{1}{2}$, times-- so here x equals 2 . 10 minus 4 is 6 . Times dx is just this 1 that we're interested in.

OK so 16 to the minus $\frac{1}{2}$, 16 to the $\frac{1}{2}$ is 4 , so 16 to the minus $\frac{1}{2}$ is $\frac{1}{4}$. So this is $\frac{1}{2}$ times $\frac{1}{4}$ times 6 . So that's $\frac{6}{8}$, which is $\frac{3}{4}$. So our dy is $\frac{3}{4}$. So that means our linear approximation that we're after is just f of 3 , f of x plus dx is approximately equal to our value y that we, at our original point, which was 4 , plus the dy . So plus $\frac{3}{4}$.

So the linear approximation that we get from this method of differentials is exactly the same approximation that we would get if we did this the other way. But I think it's kind of nice to do it in this differential form. Somehow it seems a little more straightforward sometimes. And OK, and so the answer that we get out of it is 4 plus $\frac{3}{4}$. All right, so that's that.