PROFESSOR: Welcome back to recitation.

In this video, what I'd like us to do is answer the following question. Suppose that f is a continuous, differentiable function. And if it's derivative, if f prime is never 0, and a is not equal to b, then show that f of a is not equal to f of b. I'm going to let you think about it for a while, see if you can come up with a good reason for that, and then I'll be back to explain my reasons.

OK. Our object, again, is to show, if f is continuous and differentiable and its derivative is never 0 and you're looking at two x-values that are different, show that their y-values have to be different. Show that if the inputs are different, the outputs have to be different.

Now, this might remind you of something you saw in lecture about if the derivative has a sign, show the function-- if the derivative is positive, show the function is always increasing, or if the derivative is negative, show the function is always decreasing. So this is a similar type of problem to that. So what we're going to use is actually the mean value theorem.

If you'll notice, I have f. It does satisfy the mean value theorem on an interval from a to b. I haven't even specified which is bigger, a or b. But it doesn't matter in this case.

So what do we know? The mean value theorem tells us that if we look at-- well, let's just write it out-- f of b minus f of a over b minus a is equal to f prime of c-- and what do we know-- for c between a and b.

So we want to know whether or not f of b minus f of a can ever be 0. We're trying to show that it cannot be 0. So we're going to isolate this expression and show that this subtraction cannot be 0. Well, how do we do that? Let me come over here to give us a little more room.

I'm going to rewrite the mean value theorem. I'm going to multiply through by b minus a. So we get f prime of c time b minus a. Now, we just want to show, again, that f of b minus f of a cannot be 0.

What's the only thing-- well, not only thing, we know two things-- what two things do we know? We know f prime of c is not 0. That was given to you. f prime is never 0, so certainly at any fixed value, f prime of c is not 0. So we know this term is not 0. We also know that b is not equal to a, so we know b minus a is not 0.

The only way to get a product of two numbers to be 0 is if one of them is 0. So this in fact, this product is not equal to 0. The fact that this product is not equal to 0 tells us f of b minus f of a is not equal to 0. And that alone is enough to conclude that f of b is not equal to f of a.

So, again, let me just point out of this is probably reminds you very much of the type of thing you've seen where you were showing if f prime had a sign, then you could determine whether f was increasing or decreasing. It's the same type of problem as that. It's exploiting what the mean value theorem tells you.

So I think we'll stop there.