

JOEL LEWIS: Hi. Welcome back to recitation.

In lecture you've started learning about quadratic approximation. So today we're just going to do a quick example of it.

So I have a question written here on the board: What is the quadratic approximation of the function f of x equals e to the x plus x squared-- so here x and plus x squared is the exponent, so it's e to the quantity x plus x squared-- near x equals 0.

So why don't you take a minute or two, pause the video, work this out on your own and then come back and we can do it together.

All right. Welcome back.

So there are two different ways we can do this problem. Let's first just do it the totally straightforward way, which is that you have this formula for quadratic approximations in terms of the derivatives of your function. And so, so we can just apply that formula.

So here, so the formula is that the quadratic approximation of the function f -- so here, near the point 0-- is equal to f of 0 plus f prime of 0 x plus f double prime of 0 over 2 times x squared.

All right. So in order to use this formula we just need to know what the derivatives of our function are and their values at 0. So first we can do the first derivative of f . So for that it's just a straightforward application of the chain rule. Our outer function is e to the x and our inner function is x plus x squared.

So the derivative then, applying the chain rule, is e to the x plus x squared times 1 plus $2x$, which I can also write as e -- well, yeah, let me just reorder it-- $2x$ plus 1 times e to the x plus x squared.

OK. So that's the first derivative. And for the second derivative I can apply just the product rule here. Right? So I've got this-- you know, the second derivative is the derivative of the first derivative, so here I have a product. So f double prime of x is equal to-- well, so we take the derivative of the first one, which is just 2 , times the second plus the derivative of the second one. Well, the second one is e to the x plus x squared. It's actually f of x , so we already computed it once. So the derivative of the second is $2x$ plus 1 e to the x plus x squared times the first. So times another $2x$ plus 1 . OK, and if I multiply these two together and combine all

my terms, this is $4x^2$ plus $4x$ plus 3 times e^x plus x^2 .

So these are the first and second derivatives, and what I need to plug them into my formula is I need their values at 0 . So I need the function value at 0 . So $f(0)$, well that's e^0 plus 0 squared. So it's e^0 , so that's just 1 . f' at 0 , let's see, I go over to my formula for f' and I plug in x equals 0 , so I have 2 times 0 plus 1 , so that's 1 , times e^0 . So that's 1 times 1 , that's also 1 . And for f'' at 0 , I go to my formula for f'' and I put in 0 . And so this is 0 and that's 0 . So I have a 3 times 1 so that's 3 . And now I just take these three values and I plug them right into my formula. So the quadratic approximation is Q of f equals 1 plus x plus $3x^2$ over 2 .

All right. Great. So that was one way to do this problem. Another way to do this problem is the following. And in this case, I'm not sure which way is simpler, but in some cases one way is clearly easier than the other. So if we illustrate both then you'll have twice as many tools to work with.

So the other way is to notice-- so the exponential function is nice. When you have-- you know, one of your exponential rules is that the exponential of a sum is the product of the exponentials. So we can rewrite $f(x)$ equals e^x times e^{x^2} . Now this may seem a little bit silly to you, but if you watched Christine's recitation video you saw that to find the quadratic approximation to a product, it's enough to find the quadratic approximations to each piece separately, multiply together, and then take that quadratic approximation.

So in this case the quadratic approximations are things you might already know. So in particular, we saw in recit-- sorry, in lecture that the quadratic approximation of e^x is 1 plus x plus x^2 over 2 . And I'm going to tell you-- if you haven't seen it in recitation, there are a bunch of different ways you could work it out for yourself-- but the quadratic approximation for e^{x^2} is equal to 1 plus x^2 . All right, so if you don't believe me, by all means work that out for yourself to check it.

So in this case, so that means that the quadratic approximation of f is equal to-- by the rule Christine showed you-- it's the, so it's equal to the quadratic approximation of the product of the quadratic approximations. So that's $(1 + x + \frac{x^2}{2})$ times $(1 + x^2)$. Now if you multiply these two out, that's not hard. There are six terms. So this is the quadratic approximation of $(1 + x + \frac{x^2}{2})(1 + x^2)$. So multiply 1 through here and now x squared. So that's x^2 plus x^3 plus-- OK, some fourth degree term-- x^4 to the

fourth over 2.

And now the quadratic approximation to this we get just by dropping the cubic and quartic terms. So-- and OK, and we can add these two square terms together. So this is equal to 1 plus x plus $3x^2$ over 2.

All right. And luckily, doing the same problem two different ways we get the same answer, which is what should happen.

So that's that.