The Formula for Quadratic Approximation

Quadratic approximation is an extension of linear approximation – we're adding one more term, which is related to the second derivative. The formula for the quadratic approximation of a function $f(x)$ for values of x near x_0 is:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2}(x - x_0)^2 \quad (x \approx x_0)$$

Compare this to our old formula for the linear approximation of f:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) \quad (x \approx x_0).$$

We got from the linear approximation to the quadratic one by adding one more term that is related to the second derivative:

$$f(x) \approx \underbrace{f(x_0) + f'(x_0)(x - x_0)}_{\text{Linear Part}} + \underbrace{\frac{f''(x_0)}{2}(x - x_0)^2}_{\text{Quadratic Part}} \quad (x \approx x_0)$$

These are more complicated and so are only used when higher accuracy is needed.

We'd like to develop a catalog of quadratic approximations similar to our catalog of linear approximations. Let's start by looking at the quadratic version of our estimate of $\ln(1.1)$. The formula for the quadratic approximation turns out to be:

$$\ln(1 + x) \approx x - \frac{x^2}{2},$$

and so $\ln(1.1) = \ln(1 + \frac{1}{10}) \approx \frac{1}{10} - \frac{1}{2}(\frac{1}{10})^2 = 0.095$. This is not the value 0.1 that we got from the linear approximation, but it's pretty close (and slightly more accurate).