The Function $\mathrm{sinc}(x)$

The *unnormalized sinc function* is defined to be:

$$\mathrm{sinc}(x) = \frac{\sin x}{x}.$$

This function is used in signal processing, a field which includes sound recording and radio transmission.

Use your understanding of the graphs of $\sin(x)$ and $\frac{1}{x}$ together with what you learned in this lecture to sketch a graph of $\mathrm{sinc}(x) = \sin(x) \cdot \frac{1}{x}$.

Solution

Because $\lim_{x \to 0} \frac{\sin x}{x} = 1$, we know that $\mathrm{sinc}(0) = 1$.

Because $\sin(x)$ oscillates between positive and negative values, $\mathrm{sinc}(x)$ will do so as well. Except at $x = 0$, the x-intercepts of the graph of $\mathrm{sinc}(x)$ will match those of $\sin(x)$.

We know that $-1 \leq \sin(x) \leq 1$, so it must be true that:

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}.$$

The graph of $\mathrm{sinc}(x)$ moves up and down between the graphs of $\frac{1}{x}$ and $-\frac{1}{x}$.

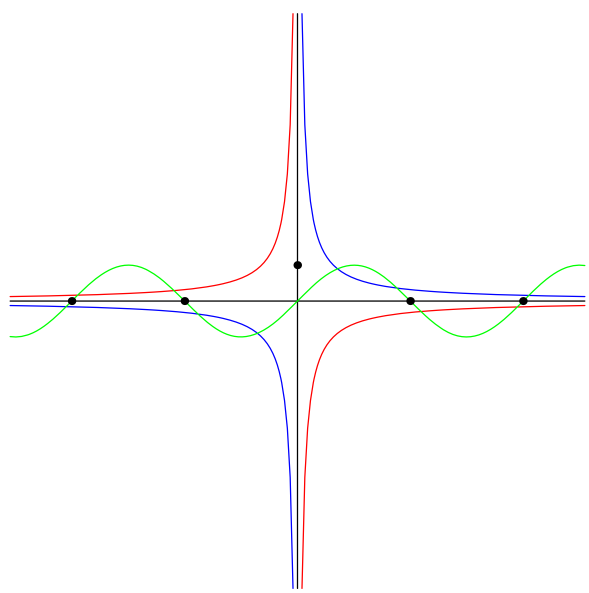

Figure 1: The graphs of $\sin(x)$ (green), $\frac{1}{x}$ (blue) and $-\frac{1}{x}$ (red).

In drawing the graph of $\mathrm{sinc}(x)$ we start by superimposing the graphs of $\sin(x)$, $\frac{1}{x}$ and $-\frac{1}{x}$. (See Figure 1.)

When $x < 0$, both $\frac{1}{x}$ and $\sin(x)$ are negative, so their quotient is positive; $\text{sinc}(x)$ turns out to be an even function. Knowing this, we quickly guess that the graph of $\text{sinc}(x)$ looks like graph shown in Figure 2. (It's not easy to tell what the graph will look like near $x = 0$. We could deal with this by plotting a few points using a calculator or by learning more calculus and then returning to this problem.)

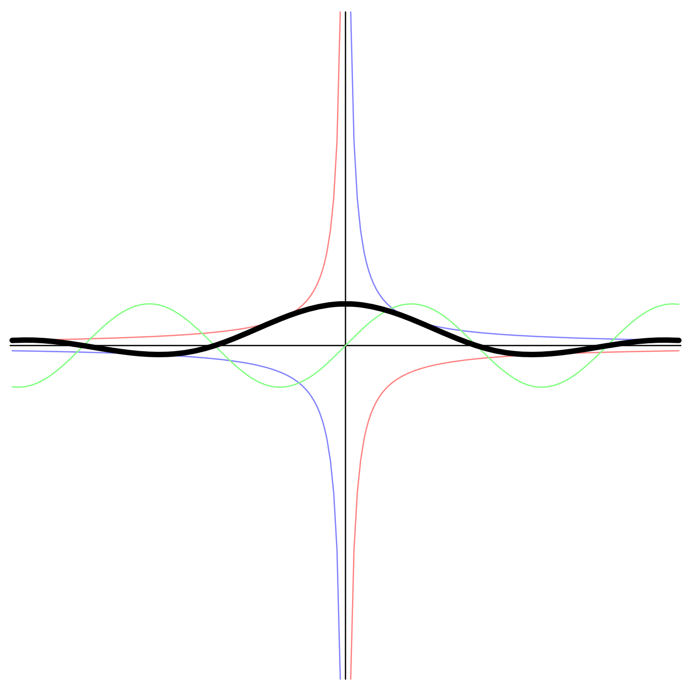

Figure 2: The graph of $\text{sinc}(x)$.