| Month | Balance ($) |
|---|---|
| January | 175 |
| February | 220 |
| March | 255 |
| ⋮ | ⋮ |

## Checking Account Balances

The derivative of a function $f(t)$ describes how the function's output changes as the value of $t$ changes.

Suppose that a checking account has balance $f(t) = -5t^2 + 60t + 120$ dollars $t$ months into the year, and that the derivative (rate of change) of the account balance is given by $f'(t) = -10t + 60$.

1. Compute $f'(1)$. If $f'(1)$ is approximately equal to the rate of change of the account balance during the month of January, will the account balance in February be greater or less than the balance in January?

2. Compute $f'(10)$. Does the account balance continue to increase throughout the year? How do you know?

3. During which month is the account balance greatest? What is the value of $f'(t)$ at this time?

**Solution**

1. Compute $f'(1)$. If $f'(1)$ is approximately equal to the rate of change of the account balance during the month of January, will the account balance in February be greater or less than the balance in January?

$$
\begin{aligned}
f'(1) &= -10(1) + 60 \\
&= 50 \text{dollars per month}
\end{aligned}
$$

In January, the account balance is changing by a positive \$50 per month. The account balance in February should be roughly \$50 greater than the January balance. We can see from the table of values of $f(t)$ that this is a reasonable answer, and also that the approximation is not exact.

2. Compute $f'(10)$. Does the account balance continue to increase throughout the year? How do you know?

$$
\begin{aligned}
f'(10) &= -10 \cdot 10 + 60 \\
&= -40 \text{dollars per month}
\end{aligned}
$$

We expect the account balance to be decreasing by approximately $40/month in October; the account balance will not continue to increase throughout the year. We can check this answer by confirming that $f(11) = \$175$ is approximately $40 less than $f(10) = \$220$.

The value of $f'(10)$ describes the instantaneous rate of change of $f(t)$ when the time elapsed is exactly $t = 10$ months and corresponds to the slope of the line tangent to the graph of $f(t)$ at the point $(10, 220)$. The difference $f(10) - f(11)$ is the average rate of change between time $t = 10$ and $t = 11$ and corresponds to the slope of a secant line of the graph of $f(t)$.

3. During which month is the account balance greatest? What is the value of $f'(t)$ at this time?

We could answer the first part of this problem by calculating balances for the remaining six months of the year, but that's somewhat tedious and unenlightening. Instead, we observe that the account balance is increasing when $f'(t) > 0$ and decreasing when $f'(t) < 0$. By graphing the equation $y = f'(t)$ we find that the account balance should increase until month 6 (June) and then decrease thereafter.

More experienced problem solvers might observe that $f'(t)$ is a continuous function that is always decreasing and find the point where $f(t)$ chanes from increaseing to decreasing by solving the equation $f'(t) = 0$:

$$
\begin{aligned}
f'(t) &= 0 \\
-10t + 60 &= 0 \\
-10t &= -60 \\
\frac{-10t}{-10} &= \frac{-60}{-10} \\
t &= 6
\end{aligned}
$$

The account balance is expected to be greatest in June, at which time:

$$f'(t) = f'(6) = -10(6) + 60 = 0.$$

To check our work, we compute that $f(6) = 300$, $f(5) = 295$, and $f(7) = 295$. Our answer does seem to be correct.

18.01SC Single Variable Calculus
Fall 2010