Higher Derivatives

Higher derivatives are derivatives of derivatives. Given a differentiable function $u = u(x)$ its derivative u' is a new function, which we may be able to differentiate again to get $(u')' = u''$.

For example, if $u(x) = \sin x$ then $u' = \cos x$ and $u'' = -\sin x$. We can go on: $(u'')' = u''' = -\cos x$ ($u''' = u^{(3)}$ is called the third derivative of u and u'' is the second derivative) and $u'''' = u^{(4)} = \sin x$. The function $\sin x$ is a special example – we won't usually "come back to" the function we started with.

Since there's more than one way to write derivatives, there's more than one notation for higher derivatives.

Notations

$f'(x)$	Df	$\dfrac{df}{dx}$	$\dfrac{d}{dx}f$
$f''(x)$	$D^2 f$	$\dfrac{d^2 f}{dx^2}$	$\left(\dfrac{d}{dx}\right)^2 f$
$f'''(x)$	$D^3 f$	$\dfrac{d^3 f}{dx^3}$	$\left(\dfrac{d}{dx}\right)^3 f$
$f^{(n)}(x)$	$D^n f$	$\dfrac{d^n f}{dx^n}$	$\left(\dfrac{d}{dx}\right)^4 f$

The symbols D and $\dfrac{d}{dx}$ represent "operators" which can be applied to a function. When you apply one of these operators to a function you get the derivative of that function.

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