3.60 Symmetry, Structure and Tensor Properties of Materials

Problem Set 14

1. The trace of a second-order tensor is defined as the sum of its diagonal elements, $T = a_{11} + a_{22} + a_{33}$. Show that the trace of a tensor remains invarient upon a change of reference axes which is specified by a general direction cosine scheme $[C_{ij}]$.

2. Suppose a crystal has electrical conductivity given by the tensor

$$\sigma_{ij} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 7 & -3 \\ 0 & -3 & 10 \end{bmatrix} \times 10^7 \text{ ohm}^{-1} \text{ meter}^{-1}$$

when referred to a coordinate system x_i.

(a) What are the conductivities in the x_1, x_2, x_3 directions, respectively?

(b) What are the maximum and minimum conductivities, respectively, which the crystal may display?

(c) What are the direction cosines for the directions of the principal axes of the representation quadric?

(d) Write the direction cosine scheme for a transformation of axes such that a plate cut from the crystal would have maximum conductivity along x_3 and minimum conductivity along x_1.

3. The linear thermal expansion coefficient for an arbitrarily-oriented chunk of crystal is

$$\alpha_{ij} = \begin{bmatrix} 5 & 2 & -3 \\ 2 & 8 & 4 \\ -3 & 4 & 12 \end{bmatrix} 10^{-6} {}^{o}C^{-1}$$

Use the method of successive approximations to find the magnitude and direction of:

(a) the maximum thermal expansion

(b) the minimum thermal expansion

(c) the intermediate thermal expansion

Can you say anything about the crystal system to which this crystal might belong?