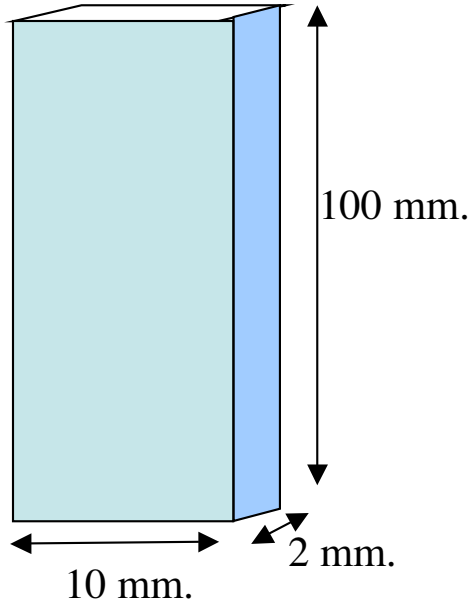
3.45 Mid-term quiz, 4/11/02
Problem 1

1. You want to make an magnetic actuator, that is, a magnetic material that will strain and do work when a magnetic field is applied. You decide to use a rectangular prism of polycrystalline Ni, $\lambda_\sigma \approx -34 \times 10^{-6}$. The prism measures 2 x 10 x 100 mm; assume a modulus, $E = 10^{11}$ N/m^2.

a) What direction will you apply the field along to get the greatest extension?
b) What is the shape anisotropy and demagnetizing field for magnetization in that direction?
c) How does that compare with the anisotropy for single crystal Ni ($K_1 \approx -6 \times 10^3$ J/m^3?
d) What field would you design for; how many turns, how much current to achieve this?
e) Write the expression for the free energy including external field, anisotropy, magnetostriction and an external stress of 100 MPa opposing the Ni extension. Compare the magnitude of the terms.
f) Calculate and plot the field dependence of magnetization.
g) Calculate and plot the field dependence of the strain.



3.45 Mid-term quiz, 4/11/02
Problem 2

2. You have a process for making nanoscale magnetic particles with typical aspect ratios of 5:1. You want to make some very small but thermally stable particles.

a) Describe quantitatively your considerations in choosing between $Fe_{0.6}Co_{0.4}$ and Co assuming you could make either of these compositions with [001] crystal direction along the long particle axis. Consider the following:

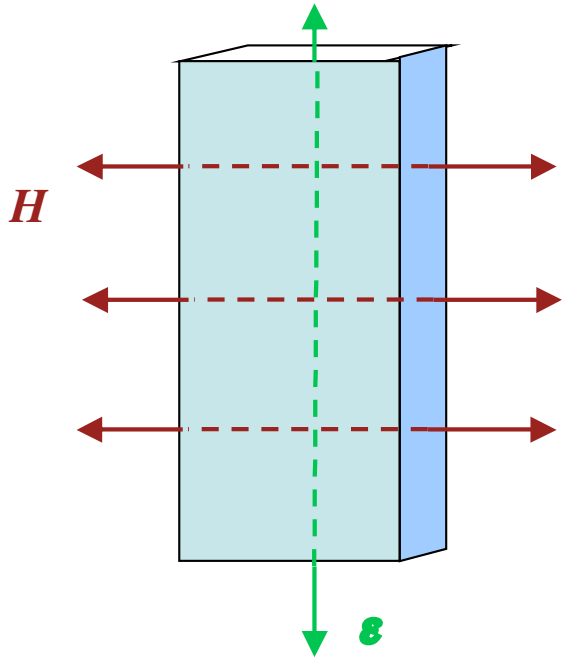
i) Crystal structure, ii) magnetocrystalline anisotropy, iii) shape (magnetostatic) anisotropy, iv) single domain particle size, and v) superparamagnetic limit at lab time scales (1 sec).

b) If the particles were superparamagnetic, compare the susceptibility of particles of the two compositions having the same shape and volume.

3.45 Mid-term quiz, 4/11/02
Solution 1

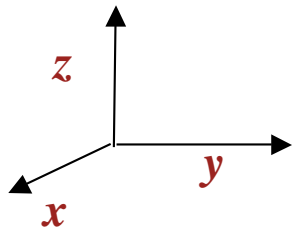
a) What direction will you apply the field along to get the greatest extension?

Largest extension will be in longest direction.The easiest direction to magnetize in to achieve this is the next longest direction, y = 10 mm. The *free* elongation in y is λ x 100 mm = -3.4 microns, while x and z dimensions extend out by 1.7 microns each.



b) What is the demagnetizing field and shape anisotropy for magnetization in that direction?

Eqs. 2.22 and 2.23 indicate $N_y \approx 2t/(\pi \text{ x } 10) = 0.13$ and $N_z \approx 2t/(\pi \text{ x } 100) = 0.013$. The demagnetizing field is $H_{demag} \approx -N_y M_s = 61$ kA/m (less than 760 Oe). The shape anisotropy is, therefore $K_{shape} \approx -\mu_0 M_s(H_y - H_z)_{demag}/2 = \mu_0(N_y - N_z)M_s^2/2 = 17$ kJ/m^3 (using $\mu_0 M_s = 0.6$ T for Ni).

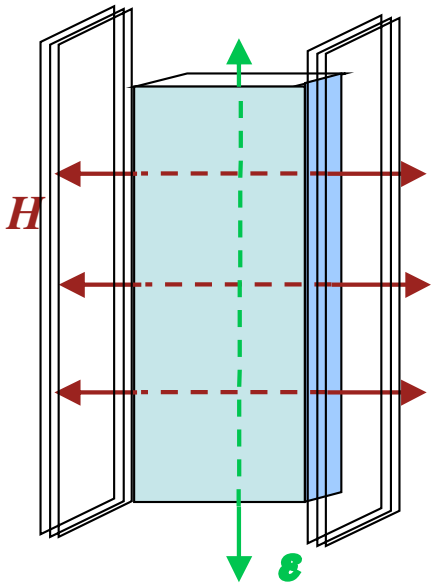


3.45 Mid-term quiz, 4/11/02
Solution 1

c) How does that compare with the anisotropy for single crystal Ni, $K_1 \approx -6 \times 10^3$ J/m^3?

The shape anisotropy, 1.7×10^4 J/m^3, is greater than the crystal anisotropy (which in turn is greater than the anisotropy of the polycrystallne Ni). So shape dominates the magnetization process.

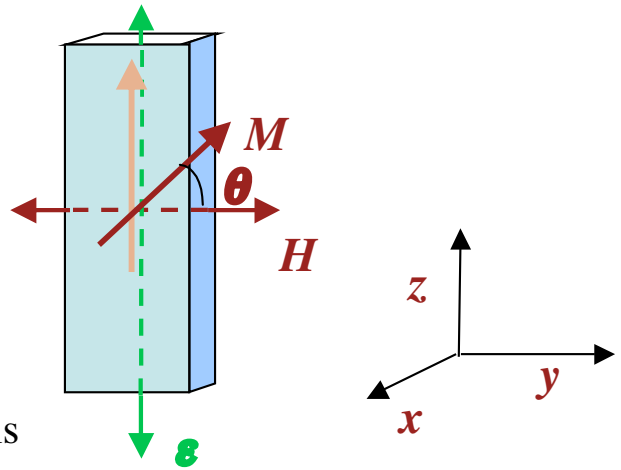
d) What field would you design for; how many turns, how much current to achieve this?



I would design for at least 1 kOe (80 kA/m) applied along *y*. Two air coils of rectangular shape as pictured right, could suffice if heating is not a problem. Otherwise you will need a soft iron core, fewer turns at the cost of more weight. A first guess at the Ampere-turns needed to generate the field would be from the field at the center of a circular loop having the same circumference as the loops at right. This gives $H \approx NI/0.2$. Thus, to get 80 kA/m would require 16 Amps through about 1000 turns. A more careful calculation would involve piecewise consideration of field from the different legs of the rectangular loop.

3.45 Mid-term quiz, 4/11/02
Solution 1

e) Write the expression for the free energy including external field, anisotropy, magnetostriction and an external stress of 100 MPa opposing the Ni extension. Compare the magnitude of the terms.



$$f_{mag} = -\mu_0 M_s H\cos\theta + \frac{1}{2}\mu_0 \Delta N M_s^2 \cos^2\theta + B_1(\varepsilon_x \alpha_x^2 + cycl)$$

Assume the magnetization has no component along x the ME term is

$$B_1(\varepsilon_x\alpha_x^2 + cycl) = -\frac{3}{2}\lambda_s \frac{E}{1+\upsilon}\left[(\varepsilon_y + \upsilon\frac{|\sigma|}{E})\cos^2\theta + (-\frac{\varepsilon_y}{2} - \frac{|\sigma|}{E})(1-\cos^2\theta)\right] = -\frac{3}{2}\lambda_s\left[\frac{3E}{2(1+\upsilon)}\varepsilon_y + |\sigma|\right]\cos^2\theta$$

$$f_{mag} = -\mu_0 M_s H\cos\theta + [\frac{1}{2}\mu_0\Delta N M_s^2 - \frac{3}{2}\lambda_s\{\frac{3E}{2(1+\upsilon)}\varepsilon_y + |\sigma|\}]\cos^2\theta$$

Check that each of these terms makes the energy change in proper direction as field and stress change. The shape anisotropy was calculated to be 17 kJ/m^3, the magnetostrictive anisotropy is about -(34 x 10^{-6})^2 x 10^{11} ≈ 0.12 kJ/m^3 and the external stress anisotropy is of order 1.5 x 34 x 10^{-6} x 10^8 ≈ 5 kJ/m^3. The shape and external stress anisotropies govern $M(H)$.

3.45 Mid-term quiz, 4/11/02
Solution 1

f) Calculate and plot the field dependence of magnetization.

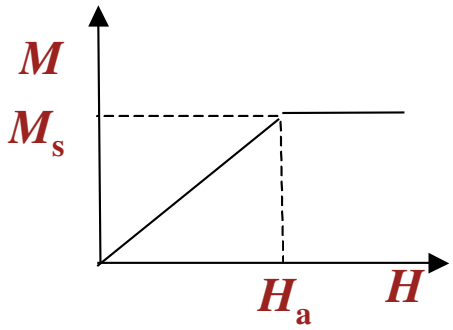
We know the solution to

$$f_{mag} = -\mu_0 M_s H\cos\theta + K\cos^2\theta = -\mu_0 M_s H\cos\theta + [\frac{1}{2}\mu_0 \Delta N M_s^2 - \frac{3}{2}\lambda_s\{\frac{3E}{2(1+\upsilon)}\varepsilon_y + |\sigma|\}]\cos^2\theta$$

is $\cos\theta = m = h$ but now K, the term in [ ], is influenced by the shape and external stress.

$$H_a = \frac{2K}{\mu_0 M_s} = 2\left[\mu_0 \Delta N M_s^2 - \frac{3}{2}\lambda_s\{\frac{3E}{2(1+\upsilon)}\varepsilon_y + |\sigma|\}\right]/(\mu_0 M_s)$$

$$M = \frac{M_s}{H_a}H$$



With $\lambda_s < 0$, increased stress makes it harder to saturate.

g) Calculate and plot the field dependence of the strain.

To calculate the actual strain, you need to add an elastic energy term, $(1/2)E\varepsilon_y^2$, to f. The total energy density can be simplified using $\cos\theta = h = H/H_a = \mu_o M_s H/2K$ so $\mu_o M_s H = 2Kh$:

$$f_{mag} = \underbrace{-\mu_0 M_s H\cos\theta}_{-2Kh\ \ h} + \underbrace{[\frac{1}{2}\mu_0 \Delta N M_s^2 - \frac{3}{2}\lambda_s\{\frac{3E}{2(1+\upsilon)}\varepsilon_y + |\sigma|\}]}_{+K}\underbrace{\cos^2\theta}_{h^2}$$

$$f_{mag+elast} = -Kh^2 + \frac{1}{2}E\varepsilon_y^2$$

3.45 Mid-term quiz, 4/11/02
Solution 1

When taking derivative of f with respect to strain, keep in mind that $h = H/H_a$ is also a function of strain.

$$\frac{\partial h}{\partial \varepsilon_y} = \frac{\frac{\mu_0 M_s H}{2}\frac{9}{4}\frac{\lambda_s E}{1+\upsilon}}{\left[\mu_0 \Delta N M_s^2 - \frac{3}{2}\lambda_s\{\frac{3E}{2(1+\upsilon)}\varepsilon_y + |\sigma|\}\right]^2} = \frac{\mu_0 M_s}{2}\frac{9}{4}\frac{\lambda_s E}{1+\upsilon}\frac{H}{K^2} = \frac{9\lambda_s E}{4(1+\upsilon)}\frac{h}{K}$$

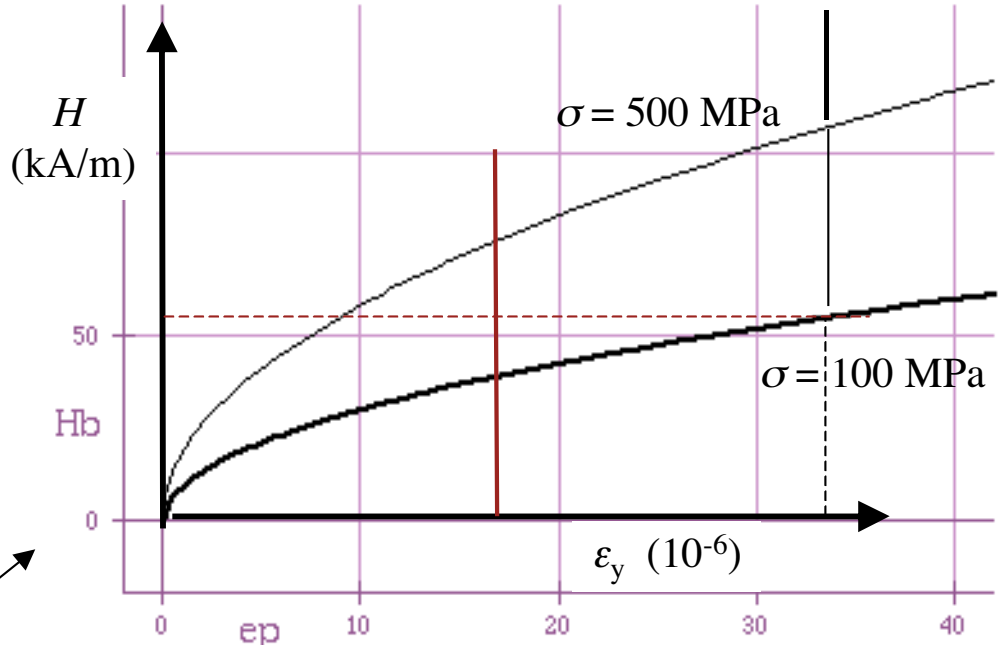
So

$$\frac{\partial f}{\partial \varepsilon_y} = 0 = -2Kh\frac{\partial h}{\partial \varepsilon_y} + E\varepsilon_y$$

$$\varepsilon_y = \frac{2Kh}{E}\frac{\partial h}{\partial \varepsilon_y} = \frac{9\lambda_s}{2(1+\upsilon)}h^2$$

$$\varepsilon_z = -\frac{\varepsilon_y}{2} = -\frac{9\lambda_s}{4(1+\upsilon)}h^2 > 0$$

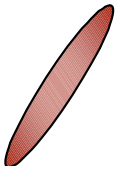
Plot $H = H_a(\varepsilon_y)\sqrt{\frac{2(1+\upsilon)\varepsilon_y}{9\lambda_s}}$



3.45 Mid-term quiz, 4/11/02
Solution 2

2. You have a process for making nanoscale magnetic particles with typical aspect ratios of 5:1. You want to make some very small but thermally stable particles.

a) Describe quantitatively your considerations in choosing between $Fe_{0.7}Co_{0.3}$ and Co assuming you could make either of these compositions with [001] crystal direction along the long particle axis:



$Fe_{0.6}Co_{0.4}$ or Co

i) Crystal structure

Fe-Co alloys have BCC structure up to abut 90% Co. Co is hexagonal except when impure or processed by non-equilibrium methods, in which case it is usually FCC.

ii) magnetocrystalline anisotropy: 70% Fe-Co has $K_1 = +3.5 \times 10^4$ J/m^3 (easy <100>) whereas Co has $K_u = +4.1 \times 10^5$ J/m^3 with an easy *c* axis. Based on magnetocrystalline anisotropy, HCP Co would be more thermally stable, (as also would be FCC Co, $K_1 = -1.2 \times 10^5$ J/m^3*)*.

iii) shape (magnetostatic) anisotropy: 70% Fe-Co has $\mu_0 M_s \approx 2.4$ T, $\mu_0 M_s^2 = 4.6 \times 10^6$ kJ/m^3, whereas Co has $\mu_0 M_s = 1.7$ T, $\mu_0 M_s^2 = 2.3 \times 10^6$ kJ/m^3. Based on magnetostatic energy in a low-symmetry particle, Fe-Co would be more thermally stable.

A prolate ellipsoid with a 5:1 aspect ratio has demagnetizing factors given by Eqs. 2.17 and 18 $N_{||} \approx 0.056$ and $N_{perp} \approx 0.50$ so $\mu_0(N_{perp} - N_{||})M_s^2/2 = 1.02$ and 0.51 MJ/m^3 for Fe-Co and Co, respectively. The net of crystal and shape anisotropies are 1.06, 0.92 and 0.63 MJ/m^3 for Fe-Co, HCP and FCC Co, respectively. Slight advantage for Fe-Co in net anisotropy.

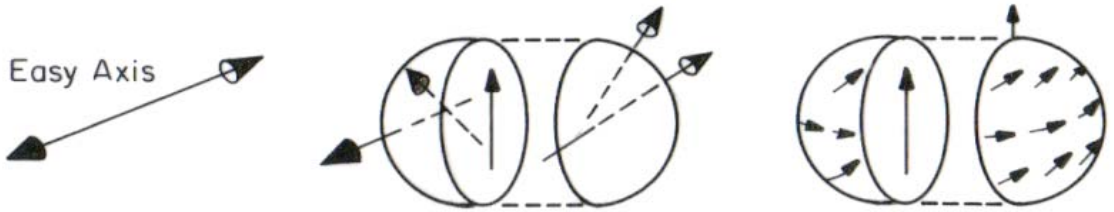
(One might want to consider the strong tendency of iron to oxidize, weakening its magnetostatic energy, and the tendency of Co to oxidize to an antiferromagnetic layer that actually promotes anisotropy.)

3.45 Mid-term quiz, 4/11/02
Solution 2

iv) single-domain particle size,

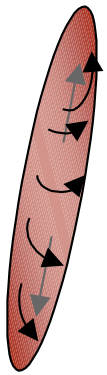
Assuming $A = 10^{-11}$ J/m for both of these high T_C materials, the superparamagnetic limit for a low-anisotropy, *spherical* particle is: $r_c = \sqrt{\frac{9A}{\mu_0 M_s^2}\left[\ln\left(\frac{2r_c}{a}\right) - 1\right]}$ This gives for Fe-Co and Co particles,

respectively r_c = 4.4 and 6.2 nm ignoring the *ln* term (iteration shows very slow convergence).
Fig. 8.24 on the other hand gives r_c = 22 and 31 nm, respectively. These values are more accurate.
But does Eq. 8.32 apply to prolate ellipsoids? No! Consider how it is derived.
What exchange energy is weighed against what magnetostatic energy in a sphere?

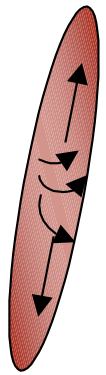


$$\langle f_{ex} \rangle = \frac{3A}{R^2}\left[\ln\left(\frac{2R}{a}\right) - 1\right]$$

$$\mu_0 N_z M_s^2/2 = (1/6)\mu_0 M_s^2$$



But for a prolate ellipsoid, the magnetostatic energy driving domain formation is reduced by the much smaller demag factor, $N_{||} = 0.056$. The exchange energy cost for magnetization curling around the small waist ($b = a/5$) is much larger. So the single-domain particle size (volume) for a prolate ellipsoid is much larger than that for a sphere. In fact the ellipsoid may demagnetize by the "domino" mechanism shown at right (smaller number of atoms at high exchange energy).



v) superparamagnetic limit at lab time scales (1 sec). $r_0^{1s} \approx \left(\frac{6k_BT}{K_u}\right)^{1/3} \approx 3$ nm

More exactly, 2.9, 3.0, and 3.4 nm for FeCo, HCP Co and FCC Co, respectively. To make stable particles you would want an ellipsoidal volume at least equal to that of a sphere of radius 4 nm. That corresponds to particles having semi-minor and semi-major axes of about 2.7 and 13 nm, respectively.

b) If the particles were superparamagnetic, compare the susceptibility of particles of the two compositions having the same shape and volume.

The magnetization vs. temperature in superparamagnetic particles is described by the Langevin (classical) form of paramagnetism because the moment of the particle is so large, the energy states are close together. The susceptibility is proportional to the square of the particle moment:

$$\chi_{classical} = 2\frac{\mu_m^2 N_v \mu_0}{3k_BT} = \frac{(N\mu_m)^2 N_v \mu_0}{3k_BT}$$

Here N is the number of moments per particle (assumed the same in particles of either composition), but the moment for FeCo is greater than that for Co by the ratio 2.4/1.7 so the former has about twice the susceptibility of the latter.