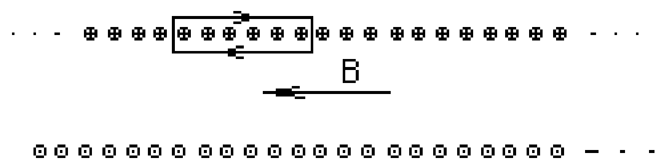
CHAPTER 1 SOLUTIONS

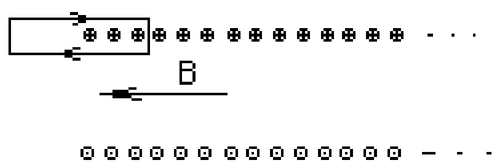
1.1 $B = \mu_o NI/l$

$= \mu_o \, 200 \times 0.5/0.2$

$B = 6.3 \times 10^{-4}$ T in middle. [$H = 500$ A/m or 6.3 Oe (cgs)]

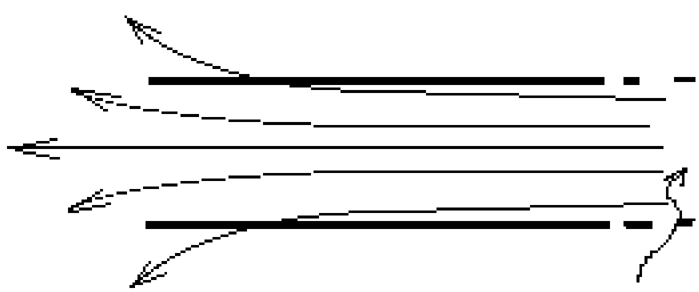


For the field at the end, remove all the turns to the left of the center of the Ampere circuit so that it is now centered on the end of the solenoid:



$B_{end} = \mu_o(N/2)\, I/l$

$B_{end} = 3.15 \times 10^{-4}$ T. $H = 250$ A/m (3.15 Oe) .



Field at end = (1/2) Field at center.

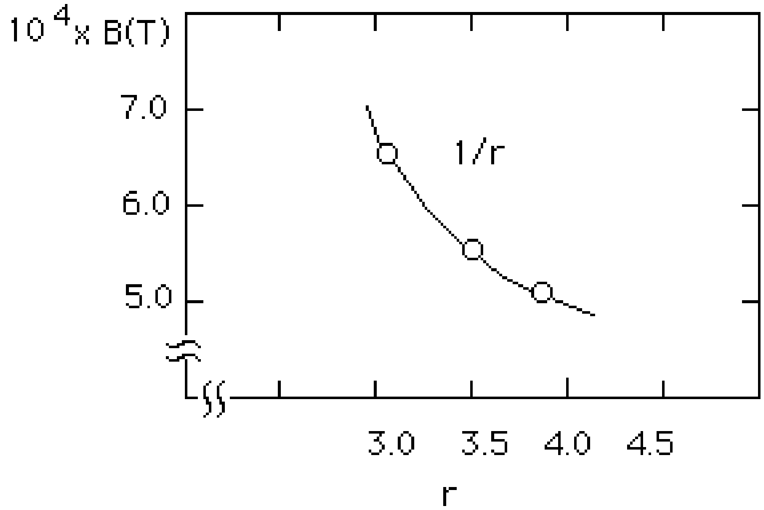
1.2 Use $\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_o \boldsymbol{J}$ in the form $\int \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_o \, 100\, I$ and integrate $\boldsymbol{B}$ over a circular path inside the toroid.

$B = 100\mu_o/2\pi r$ for $r = 3.1$ cm

$B_{3.1} = 6.45 \times 10^{-4}$ T [$H = 514$ A/m $= 6.45$ Oe (cgs)]

$B_{3.8} = 5.13 \times 10^{-4}$ T. [$H = 408$ A/m $= 5.1$ Oe]

$B_{3.5} = 5.7 \times 10^{-4}$ T, etc.



Plot shows B or H inside the toroid goes as r^{-1}. Note that $B(r)$ depends only on r and is independent of the values chosen for r_1 and r_2.

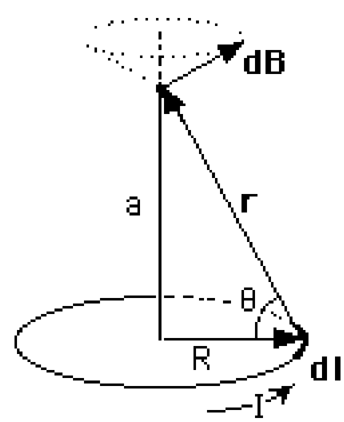
1.3 Differentiate Eq. 1.12.

$$dB = -\mu_o \frac{NI}{2\pi r^2} dr. \quad \text{Require} \quad \frac{dB}{B} = -\frac{dr}{r} < 0.1$$

that is, $\Delta r = r_2 - r_1$ should be less than 10% of $r_{ave} = (r_1 + r_2)/2$.

1.5 The Biot-Savart law is $\boldsymbol{dB} = \left(\frac{\mu_o}{4\pi}\right) I \frac{\boldsymbol{dl} \times \boldsymbol{r}}{\boldsymbol{r}^3}$.

a) From the figure at right,



$$r = \sqrt{a^2 + r_c^r}, \quad \underbrace{dl \times \boldsymbol{r}}_{B=\frac{\pi}{2}} = dl\, r$$

with $\theta = \pi/2$ for $\boldsymbol{dl} \times \boldsymbol{r}$. The field is then given by

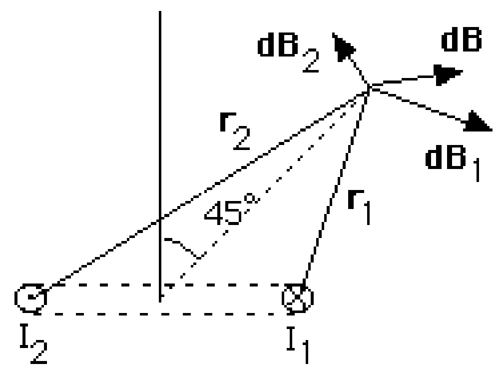
$$B(a) = \frac{\mu_o I}{4\pi} \oint \frac{\sqrt{a^2 + R^2}}{(a^2 + R^2)^{3/2}} dl = \frac{\mu_o I}{4\pi} \frac{2\pi R}{(a^2 + R^2)}.$$

The term inside the integral is independent of l giving:

$$B_z(a) = \frac{\mu_o I}{2} \frac{R}{R^2 + a^2}.$$

Note that the horizontal components of ***dB*** from around the current loop average to zero, while all the axial components add. The resultant is $\boldsymbol{B} = B_z$.

b) For $\theta = 45^\circ$, the components of $d\boldsymbol{B}$ from "near" side (smaller r) are larger. See figure at right which shows a cross section through the current loop.



b) For $\theta = 90^\circ$, the components of $d\boldsymbol{B}$ from near side (r_1) are larger than those from the far side. All contributions to ***dB*** are vertical. See cross-sectional figure below.