Fatigue:
Total Life Approaches

Fatigue Design Approaches

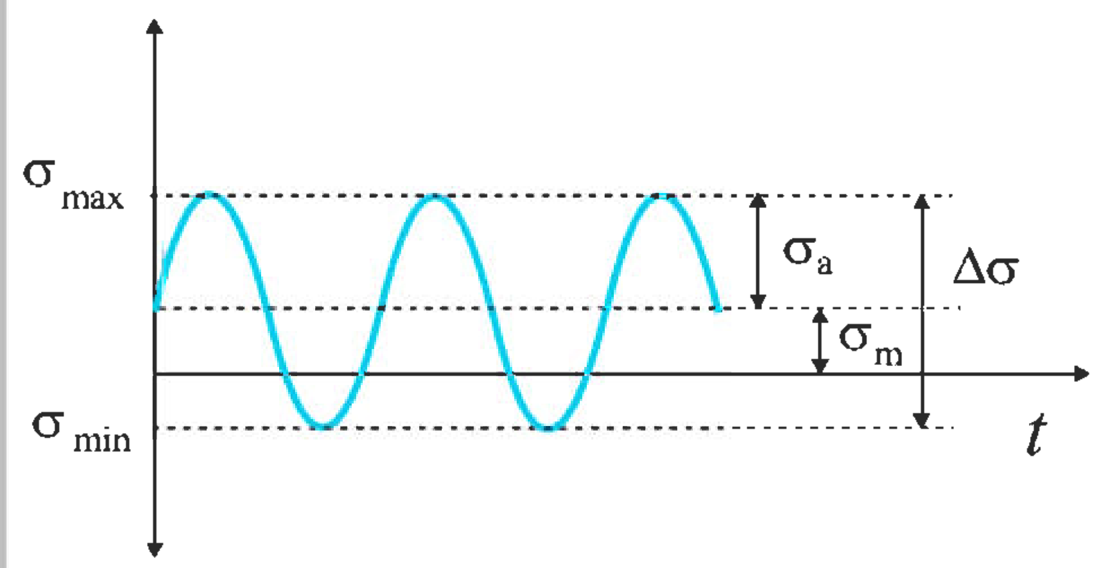
Stress-Life/Strain-Life

Long before the LEFM-based approaches (e.g. Paris Law, 1961) to characterize fatigue failure were developed, the importance of cyclic loading in causing failures (e.g. railroad axles) was recognized. Starting with the work of A. Wöhler (1860), who did rotating bend tests on various alloys, *empirical* methods have been developed. In this lecture we present two empirically-based design approaches, the *stress-life* approach and the *strain-life* approach.

Cyclic Loading

Definitions

A typical stress history during cyclic loading is depicted below.



Cyclic Loading

Definitions

continued

What are the important parameters to characterize a given cyclic loading history?

- **Stress Range:** $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$
- **Stress amplitude:** $\sigma_a = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$
- **Mean stress:** $\sigma_m = \frac{1}{2}(\sigma_{\max} + \sigma_{\min})$
- **Load ratio:** $R = \frac{\sigma_{\min}}{\sigma_{\max}}$

Cyclic Loading

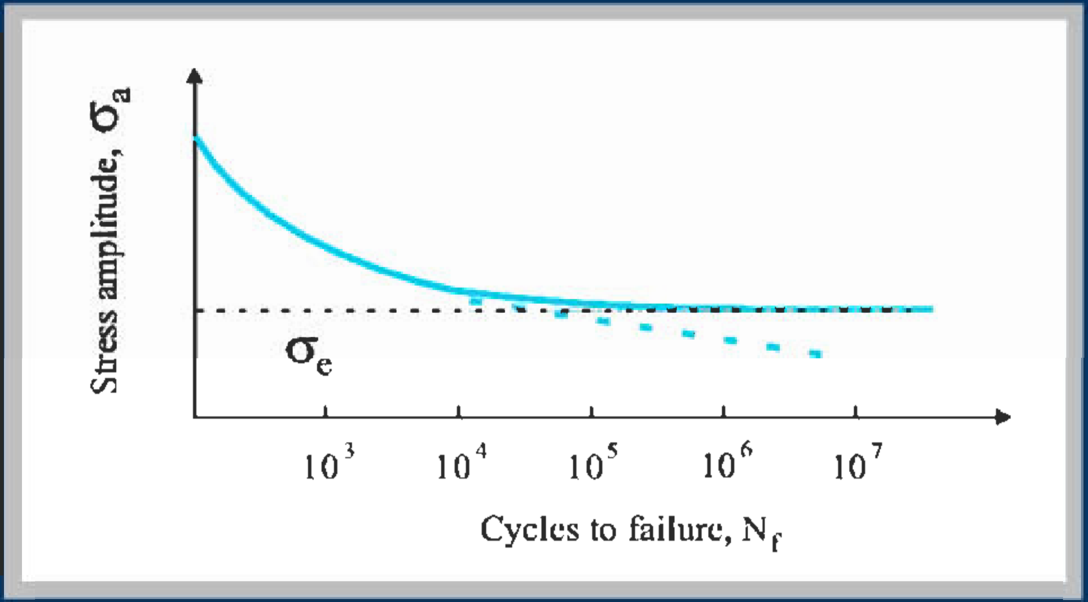
Definitions

continued

- **Frequency:** ν or f in units of Hz. For rotating machinery at 3000 rpm, f = 50 Hz. In general only influences fatigue if there are environmental effects present, such as humidity or elevated temperatures.
- **Waveform:** Is the stress history a sine wave, square wave, or some other waveform? As with frequency, generally only influences fatigue if there are environmental effects.

S-N Curve

If a plot is made of the applied stress amplitude versus the number of reversals to failure (S-N curve) the following behavior is typically observed:



S-N Curve

Endurance Limit

If the stress is below σ_e (the *endurance limit* or *fatigue limit*), the component has effectively *infinite* life.

$\sigma_e \approx 0.35\sigma_{\text{TS}} - 0.50\sigma_{\text{TS}}$ for most steels and copper alloys.

If the material does not have a well defined σ_e, often σ_e is arbitrarily defined as the stress that gives $N_f = 10^7$.

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Stress-Life Approach

If a plot is prepared of $\log(\sigma_a)$ versus $\log(2N_f)$ (where $2N_f$ represents the number of reversals to failure, one cycle equals two reversals) a *linear* relationship is commonly observed. The following relationship between stress amplitude and lifetime (Basquin, 1910) has been proposed:

$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma_f' \left(2N_f\right)^b$$

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Stress-Life Approach

Continued

In the previous expression σ'_f is the fatigue strength coefficient (for most metals $\approx \sigma_f$, the true fracture strength), b is the fatigue strength exponent or Basquin's exponent (≈ -0.05 to -0.12), and $2N_f$ is the number of reversals to failure.

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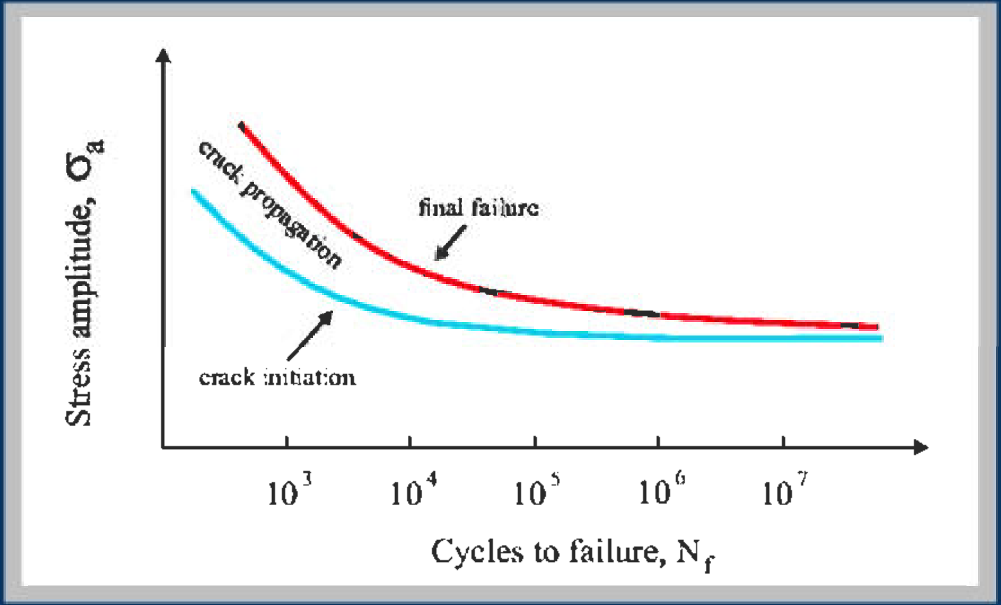
Stress-Life Approach

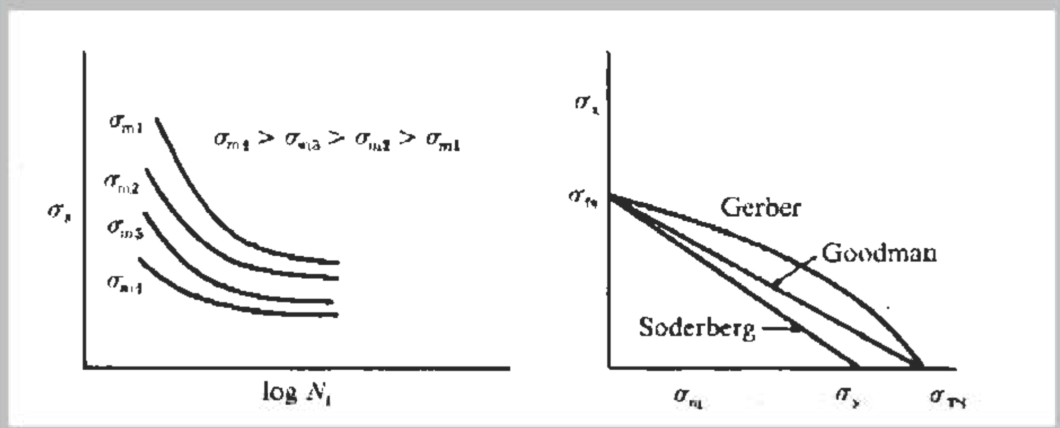
continued

The total fatigue life of a component can be considered to have two parts, the *initiation life* and the *propagation life*, as depicted below.



Mean Stress Effects

The preceding approach to calculate the lifetime assumes fully reversed fatigue loads, so that the mean stress σ_m is zero. How do we handle cases where $\sigma_m \neq 0$?



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Mean Stress Effects

continued

$$\sigma_a = \sigma_a|_{\sigma_m=0}\left\{1 - \frac{\sigma_m}{\sigma_y}\right\} \quad \text{(Soderberg)}$$

$$\sigma_a = \sigma_a|_{\sigma_m=0}\left\{1 - \frac{\sigma_m}{\sigma_{\mathrm{TS}}}\right\} \quad \text{(Goodman)}$$

$$\sigma_a = \sigma_a|_{\sigma_m=0}\left\{1 - \left(\frac{\sigma_m}{\sigma_{\mathrm{TS}}}\right)^2\right\} \quad \text{(Gerber)}$$

Fatigue Design Approaches

Mean Stress Effects

continued

In the previous expressions, σ_a is the stress amplitude denoting the fatigue strength for a nonzero mean stress, $\sigma_a|_{\sigma_m=0}$ is the stress amplitude (*for a fixed life*) for fully reversed loading ($\sigma_m = 0$ and $R = -1$), and σ_y and σ_{TS} are the yield strength and the tensile strength, respectively.

Soderberg gives a conservative estimate of the fatigue life.

Gerber gives good predictions for ductile alloys for **tensile** mean stresses. Note that it cannot distinguish between tensile and compressive mean stresses.

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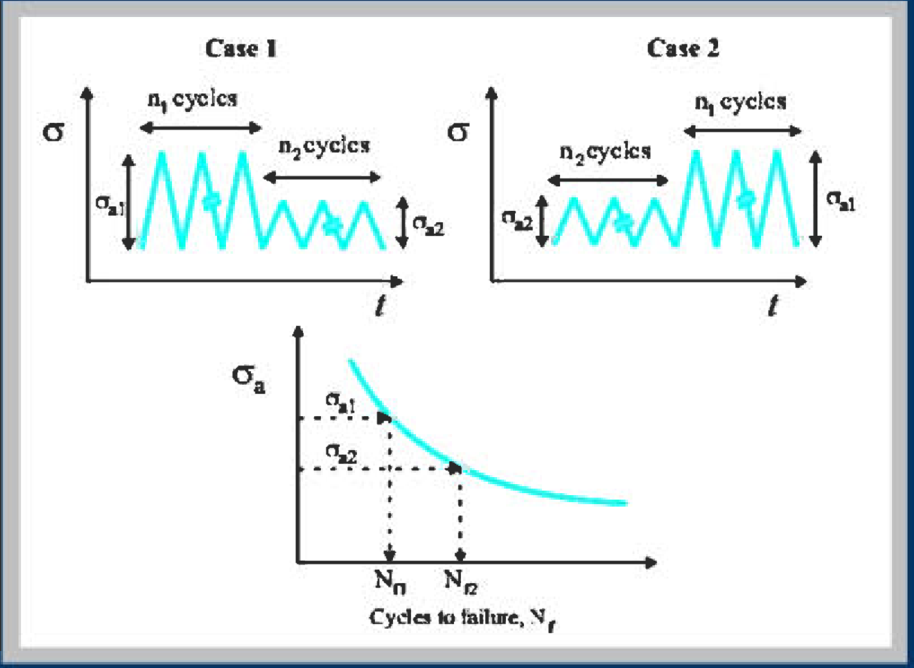
Different Amplitudes

How do we handle situations where we have varying amplitude loads, as depicted below?



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Different Amplitudes

A very common appoach is the *Palmgren-Miner* linear damage summation rule.

If we define $2N_{fi}$ as the number of reversals to failure at σ_{ai}, then the partial damage d for each different loading σ_{ai} is

$$d = \frac{2n_i}{2N_{fi}} = \frac{\text{Reversals at } \sigma_{ai}}{\text{Reversals to failure at } \sigma_{ai}}$$

Fatigue Design Approaches

Different Amplitudes

Continued

The component is assumed to fail when the total damage becomes equal to 1, or

$$\sum_i \frac{n_i}{N_{fi}} = 1$$

It is assumed that the **sequence** in which the loads are applied has no influence on the lifetime of the component. In fact, the sequence of loads *can* have a large influence on the lifetime of the component.

Fatigue Design Approaches

Different Amplitudes

Consider a sequence of two different cyclic loads, σ_{a1} and σ_{a2}. Let $\sigma_{a1} > \sigma_{a2}$.

Case 1: Apply σ_{a1} then σ_{a2}.

In this case, $\sum_i \frac{n_i}{N_{fi}}$ can be less than 1. During the first loading (σ_{a1}) numerous microcracks can be initiated, which can be further propagated by the second loading (σ_{a2}).

Fatigue Design Approaches

Different Amplitudes

Continued

Case 2: Apply σ_{a2} then σ_{a1}.

In this case $\sum_i \frac{n_i}{N_{fi}}$ can be greater than 1. The first loading (σ_{a2}) is not high enough to cause any microcracks, but it is high enough to *strain harden* the material. Then in the second loading (σ_{a1}), since the material has been hardened it is more difficult to initiate any damage in the material.

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Strain-Life Approach

The **stress-life** approach just described is applicable for situations involving primarily elastic deformation. Under these conditions the component is expected to have a long lifetime. For situations involving high stresses, high temperatures, or stress concentrations such as notches, where significant plasticity can be involved, the approach is not appropriate. How do we handle these situations?

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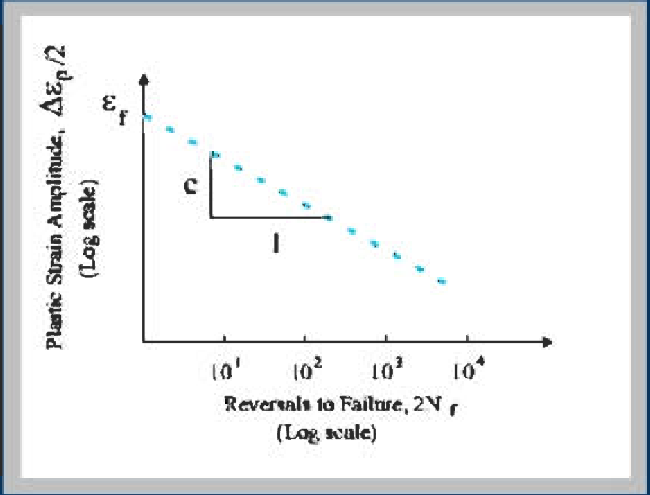
Strain-Life Approach

Rather than the stress amplitude σ_a, the loading is characterized by the plastic strain amplitude $\Delta\epsilon_p/2$. Under these conditions, if a plot is made of $\log(\frac{\Delta\epsilon_p}{2})$ versus $\log(2N_f)$, the following *linear* behavior is generally observed:



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Strain-Life Approach

To represent this behavior, the following relationship (Coffin-Manson, ca. 1955) has been proposed:

$$\frac{\Delta\epsilon_p}{2} = \epsilon'_f \, (2N_f)^c$$

where $\Delta\epsilon_p/2$ is the plastic strain amplitude, ϵ'_f is the fatigue ductility coefficient (for most metals $\approx \epsilon_f$, the true fracture ductility) and c is the fatigue ductility exponent (-0.5 to -0.7 for many metals).

Fatigue Design Approaches

General Approach

In general, how does one know which equation to apply (stress-life or strain-life approach)? Consider a fully reversed, strain-controlled loading. The total strain is composed of an elastic and plastic part, i.e.

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_e}{2} + \frac{\Delta\epsilon_p}{2}$$

The Coffin-Manson expression may used to express the term $\Delta\epsilon_p/2$. What about $\Delta\epsilon_e/2$?

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General Approach

From the Basquin Law (stress-life approach):

$$\frac{\Delta\sigma}{2} = \sigma'_f \left(2N_f\right)^b$$

For 1-D, elastic loading $\Delta\epsilon_e/2 = \Delta\sigma/2E = \sigma_a/E$ and thus

$$\frac{\Delta\epsilon_e}{2} = \frac{\sigma'_f}{E} \left(2N_f\right)^b$$

and we may write that:

Fatigue Design Approaches

General Approach

$$\frac{\Delta\epsilon}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \epsilon'_f\left(2N_f\right)^c$$

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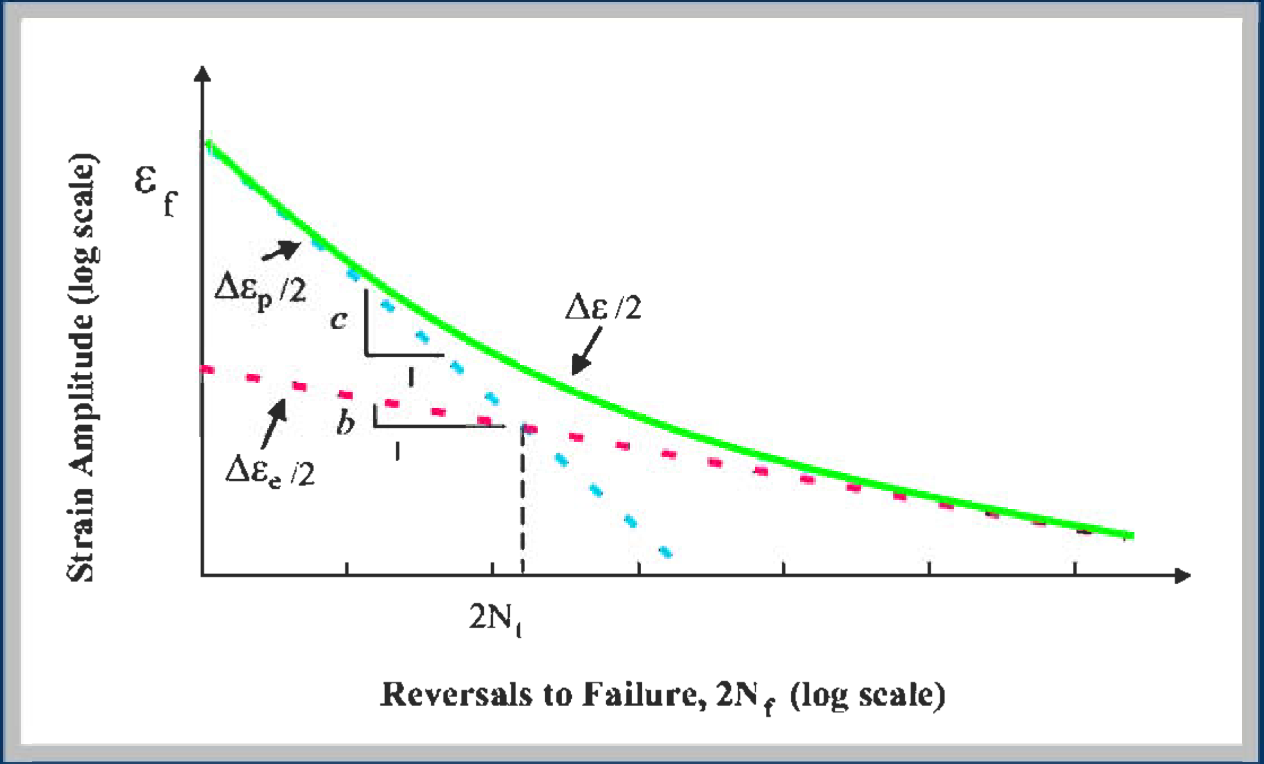
A plot of this expression is depicted below:



Fatigue Design Approaches

HCF/LCF

If the amplitude of the total strain is such that we have significant plasticity, the lifetime is likely to be short (Low Cycle Fatigue or LCF; strain life approach). If the stresses are low enough that the strains are elastic, the lifetime is likely to be long (High Cycle Fatigue or HCF; stress-life approach).

The transition life (at $2N_t$) is found by setting the plastic strain amplitude equal to the elastic strain amplitude.

$$\frac{\sigma'_f}{E}(2N_t)^b = \epsilon'_f\,(2N_t)^c$$

Fatigue Design Approaches

HCF/LCF

$$2N_t = \left(\frac{\epsilon'_f E}{\sigma'_f}\right)^{\frac{1}{b-c}}$$

At long component lifetimes ($N \gg N_t$), the elastic strain is more dominant and *strength* will control performance. At short component lifetimes ($N \ll N_t$), plastic strain dominates, and *ductility* will control performance. Unfortunately, in most materials improvements in strength lead to reductions in ductility, and vice-versa.

Fatigue Design Approaches

HCF/LCF

Important: These total life approaches represent crack initiation life in smooth specimens. However, engineering materials contain inherent defects. Therefore, these approaches can lead to overestimation of useful life.

Fatigue Crack Inhibition

Shot-Peening

Shot peening is a cold working process in which the surface of a part is bombarded with small spherical media called *shot*. Each piece of shot striking the surface acts as a tiny peening hammer, imparting to the surface a small indentation or dimple. The net result is a layer of material in a state of residual compression. It is well established that cracks will not initiate or propagate in a compressively stressed zone.

Fatigue Crack Inhibition

Shot-Peening

Since nearly all fatigue and stress corrosion failures originate at the surface of a part, compressive stresses induced by shot peening provide *considerable* increases in part life. Typically the residual stress produced is at least half the yield strength of the material being peened.

The benefits of shot peening are a result of the *residual compressive stress* and the *cold working* of the surface.

Fatigue Crack Inhibition

Shot-Peening

Residual stress: Increases resistance to fatigue crack growth, corrosion fatigue, stress corrosion cracking, hydrogen assisted cracking, fretting, galling and erosion caused by cavitation.

Cold Working: Benefits include work hardening (strengthening), intergranular corrosion resistance, surface texturing, closing of porosity and testing the bond of coatings.

Fatigue Crack Inhibition

Shot-Peening

Applications

- Blades, buckets, disks and shafts for aircraft jet engines; (e.g., blade roots are peened to prevent fretting, galling and fatigue).
- Crank shafts used in ground vehicles.
- Shot peening of plasma-spray-coated components before and after spraying.
- Shot peening of gears used in automotive and heavy vehicle components, marine transmissions, small power tools and large mining equipment.
- Compression coil springs in automobiles.
- Peening and peen forming of wing skin for aircraft.

Fatigue Crack Inhibition

Shot-Peening

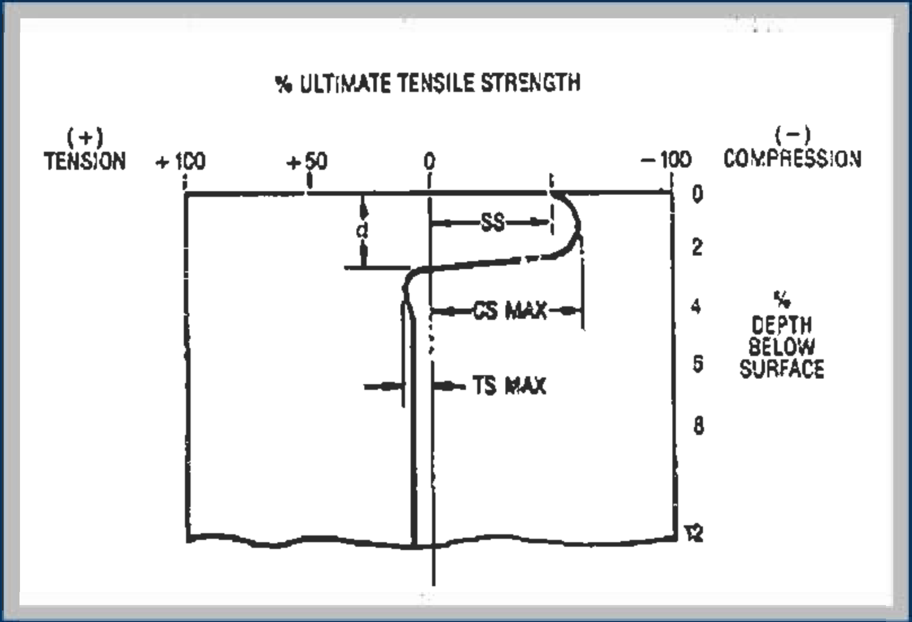
Residual stresses are those stresses remaining in a part after all manufacturing operations are completed, and with no external load applied. In most applications for shot peening, the benefit obtained is the direct result of the residual compressive stress produced.

Fatigue Crack Inhibition

Shot-Peening

continued

A typical residual stress profile created by shot peening is shown below:



Fatigue Crack Inhibition

Shot-Peening

continued

The beneficial effect of shot peening in improving the endurance limit is depicted below: