3.225 Electronic and Mechanical Properties of Materials
Quiz 2
July 13, 2001

An orthotropic fiber composite has the following elastic moduli:

$E_1 = 100$ GPa	$\nu_{12} = 0.30$	$G_{12} = 30$GPa
$E_2 = 20$ GPa	$\nu_{13} = 0.25$	$G_{13} = 15$ GPa
$E_3 = 10$ GPa	$\nu_{23} = 0.33$	$G_{23} = 5$ GPa

and note that the Poisson's ratio is defined as:

$$\nu_{ij} = -\frac{\varepsilon_j}{\varepsilon_i}$$

Calculate the strain in the composite under the following stress state:

$$\sigma_{ij} = \begin{bmatrix} 50 & 15 & 5 \\ 15 & 20 & 10 \\ 5 & 10 & 0 \end{bmatrix} MPa$$

Quiz 2

$S_{11} = 1/E_1 = 1/100 = 0.01$

$S_{22} = 1/E_2 = 1/20 = 0.05$

$S_{33} = 1/E_3 = 1/10 = 0.10$

$S_{12} = ?$ apply σ_1 only $\quad \epsilon_1 = S_{11}\sigma_1$

$\epsilon_2 = S_{12}\sigma_1$

$$\nu_{12} = -\frac{\epsilon_2}{\epsilon_1} = -\frac{S_{12}\sigma_1}{S_{11}\sigma_1} = -\frac{S_{12}}{S_{11}}$$

$$S_{12} = -\frac{\nu_{12}}{E_1}$$

$$S_{12} = \frac{-\nu_{12}}{E_1} = -\frac{0.30}{100} = -0.003$$

$$S_{13} = \frac{-\nu_{13}}{E_1} = -\frac{0.25}{100} = -0.0025$$

$$S_{23} = \frac{-\nu_{23}}{E_2} = \frac{-0.33}{20} = -0.017$$

$$S_{44} = \frac{1}{G_{23}} = \frac{1}{5} = 0.2$$

$$S_{55} = \frac{1}{G_{13}} = \frac{1}{15} = 0.067$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{30} = 0.033$$

$\sigma_1 = 50 \qquad \sigma_4 = \sigma_{23} = 10$

$\sigma_2 = 20 \qquad \sigma_5 = \sigma_{13} = 5$

$\sigma_3 = 0 \qquad \sigma_6 = \sigma_{12} = 15$

Hooke's Law.

$$\begin{array}{ccccc} & & \text{GPa}^{-1} & & \text{MPa} \\ \therefore \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix} & = & \begin{bmatrix} 0.01 & -0.003 & -0.0025 & 0 & 0 & 0 \\ -0.003 & 0.05 & -0.017 & 0 & 0 & 0 \\ -0.0025 & -0.017 & 0.10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.067 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.033 \end{bmatrix} & & \begin{bmatrix} 50 \\ 20 \\ 0 \\ 10 \\ 5 \\ 15 \end{bmatrix} \end{array}$$

$$\therefore \epsilon_1 = \left[(0.01)(50) - (0.003)(20) - (0.0025)(0)\right] \times 10^{-3} = 0.44 \times 10^{-3}$$

$$\epsilon_2 = \left[(-0.003)(50) + (.05)(20)\right] 10^{-3} = 0.85 \times 10^{-3}$$

$$\epsilon_3 = \left[(-0.0025)(50) - (0.017)(20)\right] 10^{-3} = -.465 \times 10^{-3}$$

$$\epsilon_4 = \gamma_{23} = 2\epsilon_{23} = (0.2)(10)(10^{-3}) = 0.002$$

$$\epsilon_5 = \gamma_{13} = 2\epsilon_{13} = (.067)(5)(10^{-3}) = 0.335 \times 10^{-3}$$

$$\epsilon_6 = \gamma_{12} = 2\epsilon_{12} = (0.033)(15)(10^{-3}) = 0.495 \times 10^{-3}$$

3.225 Electronic and Mechanical Properties of Materials
Quiz 3
July 27, 2001

1. Relaxation modulus data for PMMA at 115°C indicate that a relaxation modulus at $t = 3 \times 10^{-4}$ hours is 10^9 dynes/cm^2. Calculate the time to reach the same relaxation modulus at a temperature of 130°C. The glass transition temperature for PMMA is 100°C.

2. (a) A component is subject to the stress state below. The material that the component is made from has a yield strength of 250 MPa. Does the component yield?

$$\sigma = \begin{bmatrix} 150 & 10 & 30 \\ 10 & 40 & 20 \\ 30 & 20 & 40 \end{bmatrix} MPa$$

(b) Metals and covalently bonded ceramics have Young's moduli that are of the same order of magnitude but yield strengths that are very different. Explain why covalently bonded ceramics are intrinsically hard while metals are intrinsically soft.

3. A component made from 316 stainless steel (atomic volume = 1.21×10^{-29} m^3 and grain size of 0.1 mm) is loaded to a constant uniaxial stress of 100 MPa at a temperature of 1000°C. Calculate the steady state, secondary creep rate associated with (a) vacancy diffusion and (b) grain boundary diffusion. Note that Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/°K and the gas constant $R = 8.314$ kJ/(mole °K).

Creep data for 316 stainless:
$Q_v = 280$ kJ/mole, $D_{ov} = 3.7 \times 10^{-5}$ m^2/sec
$Q_B = 167$ kJ/mole, $\delta D_{oBv} = 2.0 \times 10^{-13}$ m^3/sec

#1.

$T_g = 100°C$; $T_{data} = 115$; $E_r = 10^9$ dynes/cm^2 $t = ?$ @ 130°C.

(i) shift $\overset{T}{115} \to \overset{T_0}{100}$.

$$t_T = t_{T_0}\, 10^{\frac{C_1(T-T_0)}{C_2+T-T_0}}$$

$C_1 = -17.44$

$C_2 = 51.6$.

$$t_T = t_{T_0}\, 10^{\frac{-17.44(15)}{51.6+115-100}}$$

$$\boxed{t_{115} = 3\times10^{-4} \text{ hours}}$$

$$t_T = t_{T_0}\, 10^{-3.93} = t_{T_0} / 10^{3.93}$$

$$\therefore\ t_{T_0} = t_T\, 10^{3.93} = (3\times10^{-4} \text{ hours})\, 10^{3.93} = 2.55 \text{ hours.}$$

$$t_{T_2} = t_{T_0}\, 10^{\frac{C_1(T_2-T_0)}{C_2+T_2-T_0}}$$

$$= (2.55 \text{ hours})\, 10^{\frac{-17.44(130-100)}{51.6+130-100}}$$

$$= (2.55 \text{ hours})\, 10^{-6.41}$$

$$= 9.9\times10^{-7} \text{ hours.}$$

#2 (a) $\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2\right] + 3\sigma_{12}^2 + 3\sigma_{23}^2 + 3\sigma_{13}^2}$

$= \sqrt{\frac{1}{2}\left[110^2 + 0^2 + 110^2\right] + 3(10)^2 + 3(30^2) + 3(20)^2}$

$= \sqrt{110^2 + 300 + 2700 + 1200}$

$= 127.\ \mathrm{MPa}$

(b) see notes

#3.

D.F. $\dot{\epsilon} = \frac{2\sigma\Omega}{kTd^2}\left(D_v + \frac{2\delta}{d}D_b\right)$ $\qquad T = 700 + 273 = 973.$

$D_v = D_{ov}\exp - Q_v/RT = \left(2\times10^{-4}\ \frac{m^2}{s}\right)\exp - \frac{257{,}000}{(8.314)(973)}$

$D_v = 6.69\times10^{-18}\ m^2/s$

$\delta D_B = \delta D_{ob}\exp\left(-Q_B/RT\right) = \left(1.1\times10^{-12}\ \frac{m^2}{s}\right)\exp - \frac{174\,000}{8.314(973)}$

$\delta D_B = 5.01\times10^{-22}\ m^3/s$

$\dot{\epsilon} = \frac{(2)(100\times10^6\ \cancel{N}/\cancel{m^2})(1.18\times10^{-29}\ \cancel{m^3})}{\left(1.38\times10^{-23}\ \frac{\cancel{Nm}}{°K}\right)(973\cancel{K})(10^{-4})^2\ \cancel{m^2}}\left[6.69\times10^{-18}\frac{m^2}{s} + \frac{2}{10^{-4}\cancel{m}}\ 5.01\times10^{-22}\frac{m^3}{s}\right]$

$= (17.5\times10^6)(6.69\times10^{-18} +$

$= 1.17\ /s.$

#3. $\dot{\epsilon} = \frac{2\sigma\Omega}{kTd^2}\left(D_v + \frac{2\delta}{d} D_b\right)$ $T = 1000°C = 1273°K$

$D_v = D_{ov} \exp - Q_v/RT = 3.7 \times 10^{-5} \exp - \frac{280000}{8.314(1273)} = 1.20 \times 10^{-16}\ m^2/s$

$\delta D_B = \delta D_{ob} \exp - Q_b/RT = 2.0 \times 10^{-13} \exp - \frac{167000}{8.314(1273)} = 2.8 \times 10^{-20}\ m^3/s$

$\dot{\epsilon} = \frac{2\sigma\Omega}{kTd^2}\left(D_v + \frac{2\delta}{d} D_b\right)$

$= \frac{(2)\left(100 \times 10^6 \cancel{\frac{N}{m^2}}\right)\left(1.21 \times 10^{-29} \cancel{m^3}\right)}{\left(1.38 \times 10^{-23} \cancel{\frac{Nm}{°K}}\right)(10^{-8})m^2(1273)\cancel{°K}}\left[1.20 \times 10^{-16} + \frac{2}{10^{-4} m}(2.8 \times 10^{-20})\frac{m^3}{s}\right]$

$= \left[13.7 \times 10^{6} \frac{1}{\cancel{m^2}}\right]\left[1.20 \times 10^{-16} + 5.60 \times 10^{-16}\right] \cancel{m^2}/s$

$= 9.31 \times 10^{-9}\ s^{-1}$

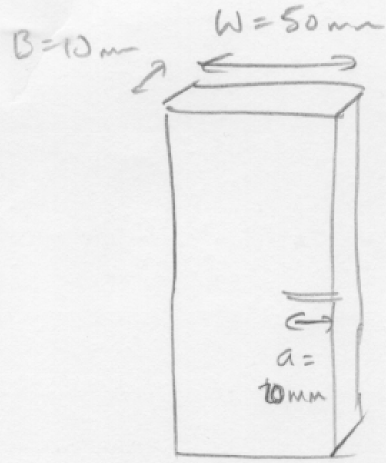
PLC $\dot{\epsilon} = A \exp - Q/RT \left(\frac{\sigma}{\sigma_0}\right)^n$

$= \left[\exp - \frac{270000}{(8.314)(1273)}\right]\left(\frac{1000}{33.5}\right)^{7.9}$

NG

$= (8.33 \times 10^{-12})(4.48 \times 10^{11})$

$= \underline{3.73}/s.$



$\sigma_{ys} = 250$ MPa

$K_{IC} = 15$ MPa$\sqrt{m}$

$a/W = 0.4$

(a) $r_p = ?$ $\quad r_p = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2 = \frac{1}{\pi}\left(\frac{15 \text{ MPa}\sqrt{m}}{250 \text{ MPa}}\right)^2 = 0.0011 \text{ m} = 1.1 \text{ mm}.$

PLANE STRESS

This is the plastic zone size $\longrightarrow$ $r_p = \frac{1}{3\pi}\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2 = \frac{1}{3\pi}\left(\frac{15}{250}\right)^2 = 3.82 \times 10^{-4} \text{ m} = 0.38 \text{ mm}.$

PLANE STRAIN

$r_p^* = \frac{1}{2\pi}\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2 = \frac{1}{2\pi}\left(\frac{15}{250}\right)^2 = 5.73 \times 10^{-4} \text{ m} = .57 \text{ mm}$

(b) PLANE STRAIN FR. TEST : $B > 25 r_p$ ← PLANE STRAIN.

$B > (25)(.38 \text{ mm}) = 9.5 \text{ mm}.$

$\Rightarrow$ SATISFIES PLANE STRAIN FRACT TOUGHNESS REQ'MT.

SMALL SCALE YIELDING (CRITERION REQ'D TO USE GRIFFITH CRITERION)

$r_y/a < 0.02 \qquad \frac{0.38}{20} = 0.019 < 0.02 \Rightarrow$ CAN USE GRIFFITH

(c) $\sigma = ?$ $\quad K_{IC} = Y\sigma\sqrt{a}$

SEN specimen: $Y = 1.99 - (0.41)(a/W) + 18.7(a/W)^2 - 38.48(a/W)^3 + 53.9(a/W)^4$

$= 3.74$

$\sigma = K_{IC}/Y\sqrt{a} = \frac{15 \text{ MPa}\sqrt{m}}{3.74\ \sqrt{.020}\ \sqrt{m}} = \underline{28.4 \text{ MPa}}$

#2 FACE - MAX E/ρ

CORE - max G/ρ.