PROBLEM SET – 2
Tuesday, November 19th, 2007

Problem 1.
In a 3-D free-electron metal with a Fermi wave vector of 10^{10} m^{-1}, (a) what are the Fermi energy and Fermi velocity? (b) What is the mean speed of the electron at T = 0? (c) What is the root-mean-square speed of the electron at T = 0?

Problem 2.
For a 1-D free-electron metal, calculate the ratio of the density of occupied electron states at the Fermi energy to that at 1/3 the Fermi energy. Assume $k_B T << E_F$. Do the same calculations for (b) 2-D and (c) 3-D free-electron metals.

Problem 3.
(a) From quantum-mechanical free-electron theory, derive expressions for the Fermi energy and for the plasma frequency of alkali metals as function of the body-centered cubic (bcc) lattice parameter, a. (reminder: there are two atoms per unit cell in a bcc crystal.) (b) The lattice parameters of Li and Cs (both are bcc) are, respectively, 0.351 and 0.614 nm. Calculate the Fermi energy and plasma frequency for these two metals. (c) Would either of these metals be transparent to any visible light? Assume $\varepsilon = \varepsilon_o$.

Problem 4.
The energy-momentum relationship for a one-dimensional crystal of inter-atomic spacing *a*, within the band corresponding to $0 < k < \pi/a$, is given by

$$E(k) = Ck^2 \left(1 - \frac{2ak}{3\pi} \right)$$

(a) Derive expressions for the group velocity and effective mass of electrons in this band as a function of k. (b) If the width of this energy band is 5 eV, and the inter-atomic spacing is 0.3 nm, calculate the maximum value of group velocity and the effective masses at the top and bottom of the band.

Problem 5.
In the highest energy band of a 1-D crystal with inter-atomic spacing a = 0.3 nm, the group velocity of electrons in m/s is $10^6 \sin ka$. (a) Calculate the effective mass in kilograms at the top and bottom of this band. (b) Calculate the energy width of the band in eV. (c) This crystal has a total length of 300 μm. How many electron states are in this band? (d) If these atoms are divalent, will this crystal be a conductor or an insulator?

Problem 6.
The Fermi energy of Li from x-ray emission spectra is 3.9 eV. Setting this equal to the Fermi energy predicted from free electron theory, what average effective electron mass does this correspond to? The atomic weight of Li is 6.94 and its density is 0.53 g/cc.