3.185 Problem Set 6

Radiation, Intro to Fluid Flow

Solutions

1. Heat transfer in electrostatic levitation

   (a) For a spherical droplet (for which $F_{11} = 0$ becaus it is convex), exactly in the middle of a cube, the viewfactors from the droplet to each side of the cube are equal. Because they form an enclosure, those viewfactors from the droplet to each side sum to 1, so each is $1/6$. Therefore, the viewfactor to two sides is $F_{12} = 2/6 = 1/3$.

   (b) Here we just use the identity:
   $$A_1 F_{12} = A_2 F_{21}$$
   $$F_{21} = \frac{A_1 F_{12}}{A_2} = \frac{\pi(0.5\text{cm})^2 \cdot \frac{1}{3}}{2 \times (6\text{cm})^2} = 0.0036$$

   (c) The graph on p. 397 (curve 2) gives us the viewfactor from one plate to the other. Since these are sides of a cube, the ratio of side length to discance between plates is 1, so the viewfactor from the graph is 0.2.

   The definition of the viewfactor F_{22} is the fraction of power radiated from surface two which arrives at surface two. We have two plates radiating energy, and 20% of each arrives at the other plate. So 20% of the total power radiated by surface 2 reaches surface 2, and $F_{22} = 0.2$.

   (d) This was straightforward application of the equation given in class:
   $$Q_{12} = \epsilon_1 \sigma T_1^4 A_1 F_{12} \alpha_2.$$

	ϵ	T	A	F	$\alpha(=\epsilon)$	Calculated Q
Q_{12}:	0.5	800 K	$\pi(0.005\text{m})^2$	$\frac{1}{3}$	0.8	0.243 W
Q_{21}:	0.8	1000 K	$2 \times (0.06\text{m})^2$	0.0036	0.5	0.594 W

   Typically in these arrangements, droplet heating is by radiation, but not necessarily by the charged plates which suspend it in place. In any case, these numbers indicate that the plates provide a good bit more heat to the droplet than vice-versa, so the rest of the heat is likely radiated to the environment.

   (e) In class, we mentioned electronic, radiative and phonon conduction, so the latter two were accepted here. (Gases can also conduct heat by motion of atoms, especially at low pressure with large mean free path.) Phonons, however, are quantized vibrations in a lattice, and this is a liquid, so instead we would have simple atomic collisions, not quantized particles as such. Radiation is not likely to play a major role in metals, since the penetration depth of photons is so small (but the question didn't ask for magnitude, just mechanism).

   Convection or heat transfer in the surrounding gas might play a role, but laser flash and other related techniques are fast enough to capture just the conduction in the droplet, and the conductivity of gases is extremely small.

2. Radiation in Zirconia Physical Vapor Deposition

(a) To calculate this viewfactor, we'll let S_1 be the liquid zicronia disc and S_2 the inner surface of the heat shield. Then we'll create two additional fictitious surfaces, S_3 and S_4: S_3 is the 20 cm-diameter disc at the top of the heat shield, and S_4 is a 20 cm-diameter disc with a 5 cm-diameter hole in it between the zirconia and the base of the shield.

Since these four surfaces form an enclosure, we know that:

$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

Since S_1, the zirconia, is not concave, $F_{11} = 0$. Since S_1 and S_4 are coplanar, none of the heat from surface 1 reaches surface 4, so $F_{14} = 0$. Therefore,

$$F_{12} + F_{13} = 1$$

We can get F_{13} from the graph on p. 398 of W^{3}R, using $r_1 = 2.5$ cm, $r_2 = 10$ cm and $D = 10$ cm. This gives us $r_2/D = 10/10 = 1$ and $D/r_1 = 10/2.5 = 4$. Based on this, the graph tells us that $F_{13} = 0.5$, so $F_{12} = 0.5$

(b) Heat leaves the zirconia surface with a flux given by:

$$e_1 = \epsilon\sigma T_1^4$$

The total heat leaving the zirconia is that flux times its area. F_{12} gives the fraction of that which reaches S_2, and we divide that by A_2 to estimate the flux at S_2:

$$q_2 \simeq \frac{F_{12}A_1\epsilon\sigma T_1^4}{A_2} = \frac{0.5 \cdot \pi(2.5\text{cm})^2 \cdot 0.3 \cdot 5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4} \cdot (2100\text{K})^4}{2\pi \cdot 10\text{cm} \cdot 10\text{cm}} = 5200\frac{\text{W}}{\text{m}^2}$$

(c) Calculating F_{22} is a bit more complicated than F_{12}. First, we'll create a new S_5 which is the combination of S_1 and S_4, a 20 cm-diameter disk at the base of the shield, which looks a lot like S_3.

Since S_2, S_3 and S_5 now form an enclosure, we can say:

$$F_{22} + F_{23} + F_{25} = 1$$

We can't get F_{23} from a graph, but we can get F_{32} about the same way we got F_{12} above:

$$F_{32} + F_{33} + F_{35} = 1$$

Since S_3 is not concave, F_{33} is zero, so $F_{32} = 1 - F_{35}$. We can get F_{35} from the graph: $r_1 = r_2 = D = 10$ cm, so D/r_1 and r_2/D are both one, $F_{35} = 0.38$, giving us $F_{32} = 0.62$.

Now we can get F_{23} from:

$$A_2F_{23} = A_3F_{32}$$

A_2 is the inner area of the cylinder, $2\pi RL = 2\pi(0.1\text{m} \cdot 0.1\text{m})$; A_3 is the area of the top disk $\pi R^2 = \pi(0.1\text{m})^2$. So A_3 is clearly half of A_2, and $A_3/A_2 = \frac{1}{2}$, and

$$F_{23} = \frac{A_3}{A_2}F_{32} = \frac{1}{2}F_{32} = 0.31$$

Since $F_{25} = F_{23} = 0.31$ by symmetry, this leaves us with

$$F_{22} = 1 - F_{23} - F_{25} = 1 - 0.31 - 0.31 = 0.38$$

3. Shear Stress and Couette Flow

   The oil kinematic viscosity of $0.00037\frac{\mathrm{m}^2}{\mathrm{s}}$ and density of $0.85\frac{\mathrm{g}}{\mathrm{cm}^3}(=850\frac{\mathrm{kg}}{\mathrm{m}^3})$ give a dynamic viscosity of $\mu = \rho\nu = 0.3145\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}$.

   Because the fluid layer is so thin, cylinder curvature and weight of the fluid can be neglected. This is therefore like having two parallel plates of area $\pi \times 36.03cm \times 3.14m = 3.55m^2$ with a fluid layer 0.1 mm thick between them.

   (a) The ram (one "plate") is moving at 0.15 m/s, and the cylinder is fixed. The shear stress is $\tau = \mu U/L = 472$ N/m^2, so the force required to travel at that velocity is that times the area, or 1677 N.

   (b) Now the force is mg (the mass of the ram and car times g), which is 6664 N. Divided by the area, the shear stress is 1875 N/m^2. The velocity of the ram U is thus $U = \tau L/\mu = 0.596$ m/s.

4. Glass Viscosity

   (a) This was a straightforward Arrhenius extrapolation, like you've done a zillion times before in Course 3:

$$\eta = A \exp\left(\frac{\Delta G_{vis}}{RT}\right)$$

   This can be made into a 2-point linear fit by taking the log of both sides:

$$\ln \eta = \ln A + \frac{\Delta G_{vis}}{R}\frac{1}{T}$$

$$1500\mathrm{K}: \ \ln\left(100\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right) = \ln A + \frac{\Delta G_{vis}}{R}\frac{1}{1500\mathrm{K}}$$

$$1700\mathrm{K}: \ \ln\left(20\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right) = \ln A + \frac{\Delta G_{vis}}{R}\frac{1}{1700\mathrm{K}}$$

   Subtracting gets rid of $\ln A$:

$$\ln\left(100\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right) - \ln\left(20\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right) = \frac{\Delta G_{vis}}{R}\left(\frac{1}{1500\mathrm{K}} - \frac{1}{1700\mathrm{K}}\right)$$

$$\frac{\Delta G_{vis}}{R} = \frac{\ln\frac{100}{20}}{\frac{1}{1500\mathrm{K}} - \frac{1}{1700\mathrm{K}}} = 20520\mathrm{K}$$

   Now solve the 1500K equation for $\ln A$:

$$\ln A = \ln\left(100\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right) - \frac{\Delta G_{vis}}{R}\frac{1}{1500\mathrm{K}} = -9.075 + \ln\left(\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right)$$

   So, $A = 1.14 \times 10^{-4}\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}$. Now just plug 1450K into the original equation:

$$\eta = A \exp\left(\frac{\Delta G_{vis}}{RT}\right) = 1.14 \times 10^{-4}\frac{\mathrm{N \cdot s}}{\mathrm{m}^2} \exp\left(\frac{20520\mathrm{K}}{1450\mathrm{K}}\right) = 160\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}$$

   (b) This kind of extrapolation downward is very dangerous to do with molten glasses, as you might hit the glass transition temperature, which would send the viscosity upward by orders of magnitude! So, okay for this problem set, but be careful when doing this in "real life".

5. Plate Glass Casting

For this problem, a useful analogy is that of steady-state conduction through a multilayer wall. In that steady-state zero-generation problem, the general solution is $T = Ax + B$, but each layer has a different A and B. Other similarities are described below.

(a) For flow in the x-direction with z normal to the plate, the general solution is:

$$u_x = -\frac{g \sin\theta}{2\nu} z^2 + Az + B.$$

This solution will hold in both the tin and the glass, but with different A and B, as mentioned above.

(b) Just as with heat conduction, where in a multilayer wall the temperatures and normal fluxes are matched at the boundary, here the velocities and shear stresses are equal for the glass and tin at their interface:

$$u_{x,tin} = u_{x,glass}; \tau_{zx,tin} = \tau_{zx,glass}.$$

(c) Now things start to get ugly. We have two pairs of constants A_{tin}, B_{tin} and A_{glass}, B_{glass} for the two layers. And we have two interface equations (part 5b), and two boundary conditions: zero shear stress at the top of the glass, and zero velocity at the bottom of the tin.

Let's start by setting $z = 0$ at the bottom of the tin, $z = z1$ at the tin-glass interface, and $z = z2$ at the top of the glass. Then we can just plug in the boundary conditions:

$$z = z_2 \quad \Rightarrow \quad -\mu_{glass} \frac{\partial u_x}{\partial z}_{glass} = 0, \tag{1}$$

$$z = z_1 \quad \Rightarrow \quad u_{x,tin} = u_{x,glass}, \tag{2}$$

$$z = z_1 \quad \Rightarrow \quad -\mu_{tin} \frac{\partial u_x}{\partial z}_{tin} = -\mu_{glass} \frac{\partial u_x}{\partial z}_{glass}, \tag{3}$$

$$z = 0 \quad \Rightarrow \quad u_{x,tin} = 0. \tag{4}$$

Right away, the last of these gives us $B_{tin} = 0$. Condition 1, with only A_{glass}, expands to:

$$-\mu_{glass} \frac{\partial u_x}{\partial z}_{glass}\bigg|_{z=z2} = \frac{\mu_{glass} g \sin\theta}{\nu_{glass}} z_2 - \mu_{glass} A_{glass} = 0,$$

$$A_{glass} = \frac{g z_2 \sin\theta}{\nu_{glass}}.$$

Since we have A_{glass}, and condition 3 deals with A_{tin} and A_{glass}, we can calculate A_{tin} from that boundary condition:

$$\frac{\mu_{tin} g \sin\theta}{\nu_{tin}} z_1 - \mu_{tin} A_{tin} = \frac{\mu_{glass} g \sin\theta}{\nu_{glass}} z_1 - \mu_{glass} A_{glass},$$

$$A_{tin} = \frac{\mu_{glass}}{\mu_{tin}} A_{glass} + \frac{g z_1 \sin\theta}{\mu_{tin}} (\rho_{tin} - \rho_{glass}) = g \sin\theta \frac{\rho_{glass}(z_2 - z_1) + \rho_{tin} z_1}{\mu_{tin}}.$$

This is the x-component of the weight of the glass-tin sandwich, divided by the tin viscosity. Now we need just B_{glass}, which we can get from condition 2 (remembering $B_{tin} = 0$):

$$-\frac{g \sin\theta}{2\nu_{tin}} z_1^2 + A_{tin} z_1 = -\frac{g \sin\theta}{2\nu_{glass}} z_1^2 + A_{glass} z_1 + B_{glass},$$

$$B_{glass} = g z_1^2 \sin\theta \left(\frac{1}{2\nu_{glass}} - \frac{1}{2\nu_{tin}} \right) + g \sin\theta \frac{\rho_{glass}(z_1 z_2 - z_1^2) + \rho_{tin} z_1^2}{\mu_{tin}} - \frac{g z_1 z_2 \sin\theta}{\nu_{glass}},$$

$$B_{glass} = \frac{g z_1^2 \sin\theta}{2}\left(\frac{1-2z2/z1}{\nu_{glass}} + \frac{1}{\nu_{tin}} + \frac{2\rho_{glass}(z_2/z_1-1)}{\mu_{tin}}\right)$$

So we have the velocity profile in the glass and tin layers:

$$u_{x,glass} = \frac{g\sin\theta}{2}\left(\frac{-z^2+2z_2z+z_1^2-2z_1z_2}{\nu_{glass}} + \frac{z_1^2}{\nu_{tin}} + \frac{2\rho_{glass}(z_1z_2-z_1^2)}{\mu_{tin}}\right)$$

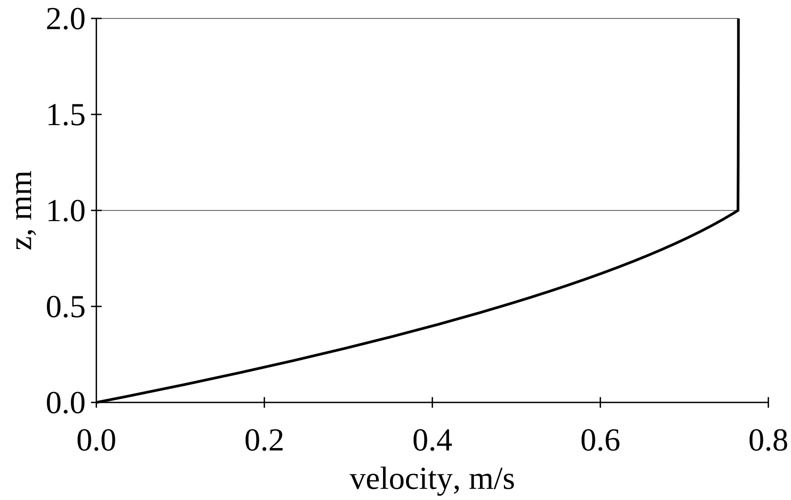
$$u_{x,tin} = \frac{g\sin\theta}{2\mu_{tin}}\left[-\rho_{tin}z^2 + 2(\rho_{glass}(z_2-z_1)+\rho_{tin}z_1)z\right].$$

With all of the constants, layer thicknesses, etc. inserted, this becomes:

$$u_{glass} = -547.22(\mathrm{ms})^{-1}z^2 + 2.18896\mathrm{s}^{-1}z + 0.762192\frac{\mathrm{m}}{\mathrm{s}},$$

$$u_{tin} = -399017(\mathrm{ms})^{-1}z^2 + 1162.85\mathrm{s}^{-1}z$$

and when graphed (which wasn't required), this looks like:



(d) The maximum velocities are easy, just plug the top z for each layer into the velocity equations above:

$$u_{max,glass} = 0.76438,\ u_{max,tin} = 0.76383.$$

The average velocities are a bit less straightforward, we have to integrate the velocity over the layer:

$$u_{av} = \frac{Q}{A} = \frac{\int_{z_a}^{z_b} u_x W dz}{W(z_b-z_a)} = \frac{1}{z_b-z_a}\int_{z_a}^{z_b}\left(-\frac{g\sin\theta}{2\nu}z^2 + Az + B\right)dz,$$

$$u_{av} = \frac{1}{z_b-z_a}\left[-\frac{g\sin\theta}{6\nu}z^3 + \frac{A}{2}z^2 + Bz\right]_{z_a}^{z_b}.$$

Inserting the constants for each material gives:

$$u_{av,glass} = \frac{1}{0.001\mathrm{m}}\left[-182.41(\mathrm{ms})^{-1}z^3 + 1.0944\mathrm{s}^{-1}z^2 + 0.762192\frac{\mathrm{m}}{\mathrm{s}}z\right]_{0.001\mathrm{m}}^{0.002\mathrm{m}} = 0.76420\frac{\mathrm{m}}{\mathrm{s}},$$

$$u_{av,tin} = \frac{1}{0.001\mathrm{m}}\left[-133006(\mathrm{ms})^{-1}z^3 + 581.42\mathrm{s}^{-1}z^2\right]_0^{0.001\mathrm{m}} = 0.4484\frac{\mathrm{m}}{\mathrm{s}}.$$

(e) In the glass, using the thickness of the glass sheet for L, the Reynolds number is about 24, which is teetering on the edge of instability. In the tin, it is about 1050, which would make the tin unstable if on its own. But since it is confined by the much more viscous glass, it acts like Couette flow, with a higher critical Reynolds number, so it can (barely) exist as a stable laminar flow.