Self-Assessment: Structure of the Atom

Weekly Homework Quiz - Solution Outlines

Problem #1

(a) Cerium has many isotopes (8 to be exact), but only ^{140}Ce and ^{142}Ce are present in substantial amounts. Which isotope of cerium is the most abundant?

from the periodic table, you see that the atomic mass of Ce is 140.115, which must be the weighted sum of the isotope masses

here we assume that it is necessary to consider only ^{140}Ce and ^{142}Ce

so, (x mass of ^{140}Ce) + (1-x) mass of ^{142}Ce = 140.115

for the purposes of this decision, we can

approximate the atomic masses of the isotopes

as ~140 for ^{140}Ce and ~142 for ^{142}Ce and solve for x

x = 0.94 which means

that ^{140}Ce is the most abundant isotope of cerium

Production of chromium in an electric arc furnace would involve the reaction of carbon with chromium sesquioxide according to the following reaction:

$$Cr_2O_3 + a\,C = b\,CO + c\,Cr$$

(i) Balance the equation, i.e., specify the values of *a*, *b*, and *c*. Insert the correct values below.

$$Cr_2O_3 + 3C = 3\,CO + 2Cr$$

(ii) Calculate the minimum amount of chromium (in kg) produced if the reaction consumed 333 kg C and produced the stoichiometric amount of Cr. Assume 100% efficiency.

333 kg C = 333000/12.011 = 27725 moles C

the stoichiometric amount of Cr is 2/3

amount of carbon on a molar basis. Therefore, amount of Cr = 27725 x 2/3 moles of Cr

= (27725 x 2/3) x 51.996 = 961 kg Cr

Problem #2

(a) Antimony has two isotopes, ^{121}Sb and ^{123}Sb. Which isotope has the higher natural abundance?

from the periodic table, you see that the atomic mass of Sb is 121.757, which must be the weighted sum of the isotope masses

so, (x mass of ^{121}Sb) + (1-x) mass of ^{123}Sb = 121.757

for the purposes of this decision, we can approximate the atomic masses of the isotopes as ~121 for ^{121}Sb and ~123 for ^{123}Sb and solve for x

x = 0.62 which means that ^{121}Sb is the more abundant isotope of antimony

(b) Production of hafnium by the Kroll Process would involve the reaction of magnesium with hafnium tetrachloride according to the following reaction:

$$HfCl_4 + a\,Mg = b\,MgCl_2 + c\,Hf$$

(i) Balance the equation, i.e., specify the values of *a*, *b*, and *c*. Insert the correct values below.

$$HfCl_4 + 2\,Mg = 2\,MgCl_2 + Hf$$

(ii) Calculate the minimum amount of magnesium (in kg) needed to convert 111 kg $HfCl_4$ into elemental hafnium.

111 kg $HfCl_4$ = $111000/[178.49 + (4 \times 35.45)]$ = 347 moles $HfCl_4$

the stoichiometric amount of Mg is twice the amount of $HfCl_4$ on a molar basis

$\therefore$ amount of Mg = 347×2 moles of Mg = $(347 \times 2) \times 24.305$ = 16.9 kg Mg

Problem #3

(a) Show by means of a calculation that blue light of wavelength, $\lambda = 444$ nm, is not capable of exciting electrons in Li^{2+}(g) from the state $n = 2$ to $n = 4$.

let's equate the energy required to excite electrons in Li^{2+}(g) from the state $n = 2$ to $n = 4$ with the minimum energy needed from an incident photon to cause the excitation

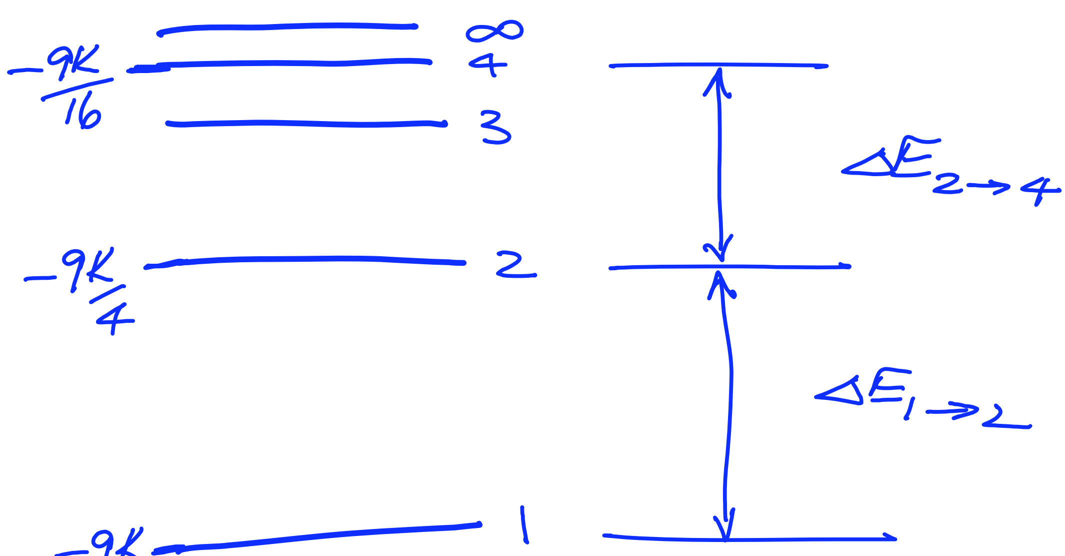
$$\frac{hc}{\lambda} = KZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\therefore \quad \lambda = \frac{hc}{KZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)} = \frac{6.6\times10^{-34}\times3.00\times10^{8}}{2.18\times10^{-18}\times3^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)}$$

$$= 5.38 \times 10^{-8} \text{ m} = 53.8 \text{ nm} < 444 \text{ nm}$$

$\therefore$ since E scales with $1/\lambda$, blue light of wavelength $\lambda = 444$ nm does not have enough energy per photon to cause the excitation

(b) Is the value of the energy of transition from the state $n = 2$ to $n = 4$ in Li^{2+}, $\Delta E_{2\to4}$, greater than or less than the value of the energy of transition from the state $n = 1$ to $n = 2$ in Li^{2+}, $\Delta E_{1\to2}$? Explain with the use of an energy level diagram. There is no need to calculate the values of the two quantities.



clearly, $\Delta E_{1\to2} > \Delta E_{2\to4}$

Problem #4

(a) In a gas discharge tube what is the minimum frequency (v) of a photon capable of ionizing ground-state electrons in Li^{2+}?

here is the central concept: the energy of the incident photon must be at least as great as the ionization energy (I.E.)

Li^{2+} is a one-electron atom, so we can calculate the I.E. using the Bohr Model

$I.E. \; = \; E_\infty - E_1 \; = \; 0 - (-\frac{KZ^2}{n^2})$, where $Z = 3$ and $n = 1$ for the ground-state of Li^{2+} and K is the ground-state energy of atomic hydrogen

energy of incident photon is given by E = hν

$$v \; = \; \frac{KZ^2}{h} \quad \frac{(2.18 \times 10^{-18}\ J)(3)^2}{6.6 \times 10^{-34}} \; = \; 2.97 \times 10^{16}\ Hz$$

(b) Explain with reference to the relevant physical forces why the value of the 1st ionization energy of Li is less than the 3rd ionization energy of Li.

the 1st ionization represents the removal of one of the 3 electrons from neutral Li

the 3rd ionization represents the removal of the single electron from the Li^{2+} ion

in the second case, the single electron ***alone*** feels the pull of the positive charge of the nucleus

in the first case the same positive charge is felt by three electrons; hence, each electron feels a weaker pull than is the case with a lone electron under the influence of the same positive charge