Session #13: Homework Solutions

Problem #1

Show that green light ($\lambda = 5 \times 10^{-7}$ m) can excite electrons across the band gap of silicon (Si).

Solution

$$\lambda_{crit} = \frac{hc}{E_g} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1.1 \times 1.6 \times 10^{-19}} = 1.13 \times 10^{-6} \text{ m}$$

The critical λ for silicon is 1.1×10^{-6} m; thus radiation of $\lambda = 5 \times 10^{-7}$ m $= 0.5 \times 10^{-6}$ m has even more energy than that required to promote electrons across the band gap.

Problem #2

(a) Electromagnetic radiation of frequency 3.091×10^{14} Hz illuminates a crystal of germanium (Ge). Calculate the wavelength photoemission generated by this interaction. Germanium is an elemental semiconductor with a band gap, E_g, of 0.7 eV.

(b) Sketch the absorption spectrum of germanium, i.e., plot % absorption vs. wavelength, λ.

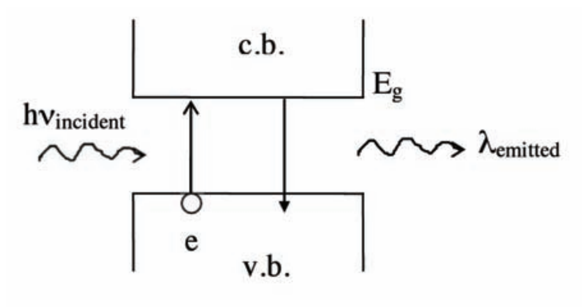
Solution

(a) First compare E of the incident photon with E_g:

$$E_{incident} = h\nu = 6.6 \times 10^{-34} \times 3.091 \times 10^{14} = 2.04 \times 10^{-19} \text{ J}$$

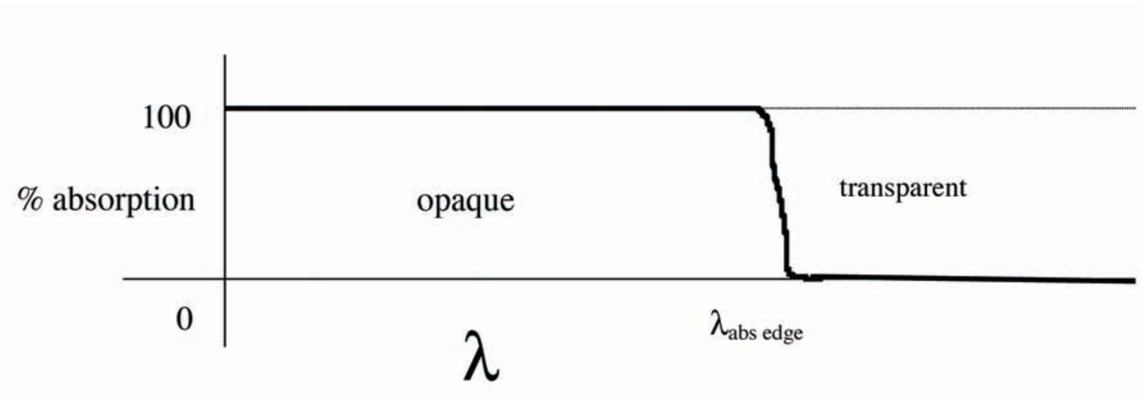
$$E_g = 0.7 \text{ eV} = 1.12 \times 10^{-19} \text{ J} < E_{incident}$$

$\therefore$ electron promotion is followed by emission of a new photon of energy equal to E_g, and energy in excess of E_g is dissipated as heat in the crystal



$$\lambda_{emitted} = \frac{hc}{E_g} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{0.7 \times 1.6 \times 10^{-19}} = 1.77 \times 10^{-6} \text{ m}$$

(b)

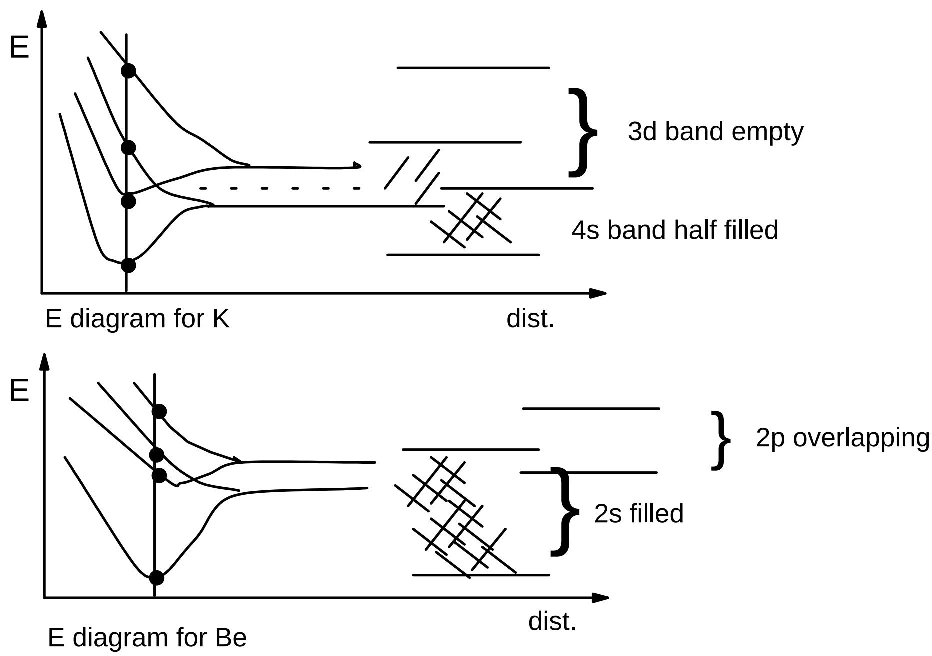


$\lambda_{edge} = \lambda_{emitted}$ as calculated in part (a) = 1.77 μm

Problem #3

Potassium (K) and beryllium (Be) are metals which exhibit good electrical conductivity. Explain for both elements the reasons for the observed conductivity on the basis of the band structure.

Solution



In K, each atom contributes one electron and one orbital to the conduction band (4s band). According to the Pauli exclusion principle, each “molecular orbital” formed in the band (energy state) can accommodate two electrons. As a consequence, the conduction band is only half–filled with electrons – which provides for electrical conduction.

In Be, each atom contributes two electrons and one orbital to the conduction band (2s band). With two electrons per orbital (from each atom) the 2s conduction band is filled. The observed electrical conductivity is due to the overlapping 2p band, which is empty and thus provides empty energy states required for electronic conduction.

Problem #4

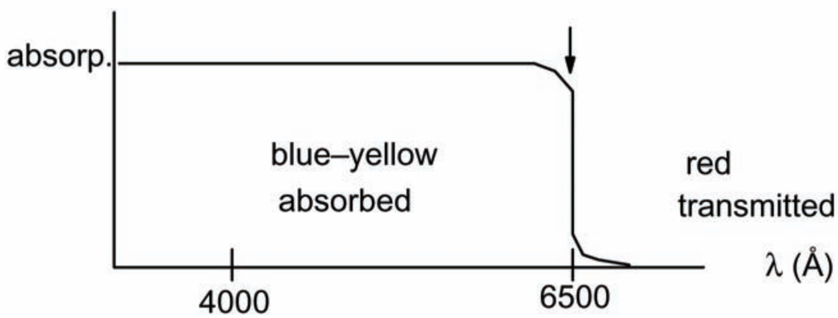
A pure crystalline material (no impurities or dopants are present) appears red in transmitted light.

(a) Is this material a conductor, semiconductor or insulator? Give the reasons for your answer.

(b) What is the approximate band gap (E_g) for this material in eV?

Solution

"White light" contains radiation in wavelength ranging from about 4000 Å (violet) to 7000 Å (deep red). A material appearing red in transmission has the following absorption characteristics:



(a) If the material is pure (no impurity states present), then it must be classified as a semiconductor since it exhibits a finite "band gap" – i.e. to activate charge carriers, photons with energies in excess of "red" radiation are required.

(b) Taking λ = 6500 Å as the optical absorption edge for this material, we have:

$$E = \frac{hc}{\lambda} = 3.05 \times 10^{-29}\ J \times \frac{1\ eV}{1.6 \times 10^{-19}\ J} = 1.9\ eV$$

Accordingly, the band gap for the material is E_g = 1.9 eV.

Problem #5

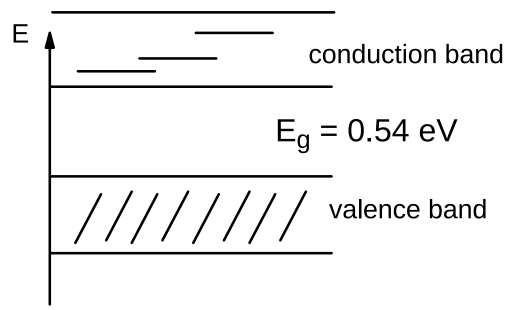
An unknown material is transparent to light of frequencies (ν) up to 1.3 x 10^{14} s^{-1}. Draw a meaningful schematic band structure for this material.

Solution

Since E = hv, we find the material to be transparent to photons with energies less than:

$$(6.63 \times 10^{-34} \text{ J}\cdot\text{s}) \times (1.3 \times 10^{14} \text{ s}^{-1}) \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 0.54 \text{ eV}$$

This constitutes the absorption edge, i.e. the energy band gap.



Problem #6

A material exhibits an "optical band edge" (transition from absorption of light to transmission) at $\nu = 5 \times 10^{14}$ Hz (s^{-1}).

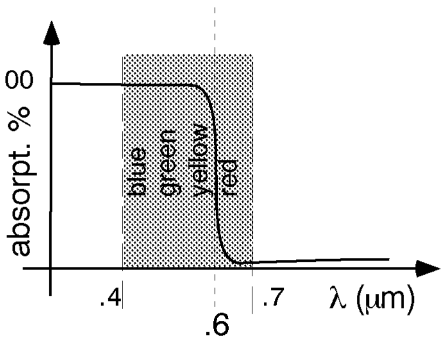
(a) Draw a diagram which reflects the indicated optical behavior.

(b) What do you expect the color of this material to be when viewed in daylight?

(c) What is the band gap (E_g) of this material?

Solution

(a) $\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8} \text{ m/s}}{5 \times 10^{14} \text{ s}^{-1}} = 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$



(b) The color will be orange or "reddish".

(c) $E_g = E_{photon} = h\nu = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} \times 5 \times 10^{14} \text{ Hz}$

$$= 3.315 \times 10^{-19} \text{ J} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 2.07 \text{ eV}$$