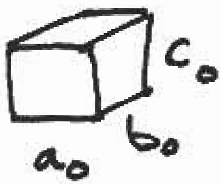
Extension Ratio

$$\lambda = \frac{L}{L_0} = \frac{L_0 + \delta}{L_0} = 1 + \frac{\delta}{L_0} = 1 + \epsilon$$

volume change:



$$\frac{\Delta V}{V} = \frac{abc - a_0 b_0 c_0}{a_0 b_0 c_0}$$

$$= \frac{\lambda_x a_0 \cdot \lambda_y b_0 \cdot \lambda_z c_0 - a_0 b_0 c_0}{a_0 b_0 c_0}$$

$$= \lambda_x \lambda_y \lambda_z - 1$$

for rubber $\Delta V = 0 \rightarrow \lambda_x \lambda_y \lambda_z = 1$

Bungee!

$$WL_o = \frac{A_o L_o}{2} \cdot NkT \left(\lambda^2 + \frac{2}{\lambda} - 3\right)$$

$$\frac{W}{A_o L} = \frac{1}{6}\left(\lambda^2 + \frac{2}{\lambda} - 3\right)$$

for $\lambda = 3$, $E = 1000\,psi$

$A_o = 1.35\ in^2$ $\quad$ ($d_o = 1.31$ in)

- Uniaxial extension

$$\lambda_x = \lambda, \quad \lambda_y = \lambda_z = \frac{1}{\sqrt{\lambda}}$$

$$F = \frac{dW}{dL} = \frac{d\,(V \cdot \Delta W_v)}{L_0\, d\lambda} = A_0 \frac{NkT}{2}\left(2\lambda - \frac{2}{\lambda^2}\right)$$

$$\sigma_{engr} = \frac{F}{A_0} = NkT\left(\lambda - \frac{1}{\lambda^2}\right)$$

- Biaxial extension $\lambda_x = \lambda_y = \lambda$

$$\lambda_z = \frac{1}{\lambda_x \lambda_y}$$

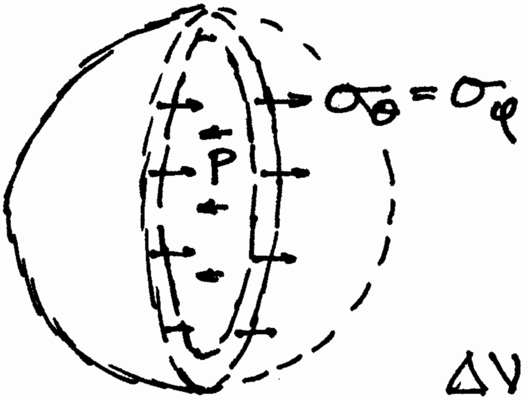
$$\Delta W_v = V \cdot \frac{NkT}{2}\left(\lambda^2 + \lambda^2 + \frac{1}{\lambda^4} - 3\right)$$

$$F_x = \frac{\partial(\Delta W_v)}{\partial x} = \frac{1}{X_0}\frac{\partial(\Delta W_v)}{\partial \lambda} = \frac{1}{X_0} \cdot V \frac{NkT}{2}\left(4\lambda - \frac{4}{\lambda^5}\right)$$

$$= \frac{XYZ}{X\lambda} \cdot 2NkT\left(\lambda - \frac{1}{\lambda^5}\right)$$

$$\sigma_{true} = \frac{F_x}{YZ} = 2NkT\left(\lambda^2 - \frac{1}{\lambda^4}\right)$$

Spherical Balloon



$$\sigma_\theta (2\pi r \cdot b) = p \cdot \pi r^2$$

$$\sigma_\theta = \sigma_\varphi = \frac{pr}{2b}$$

$$\Delta V = 0 \rightarrow 4\pi r^2 b = 4\pi r_0^2 b_0$$

$$\rightarrow b = b_0 \left[\frac{r_0}{r}\right]^2 = \frac{b_0}{\lambda_r^2}$$

$$\sigma = \frac{p}{2} \cdot \frac{r_0 \lambda_r}{b_0 / \lambda_r^2} = \frac{p r_0}{2 b_0} \lambda_r^3 = 2G\left(\lambda_r^2 - \frac{1}{\lambda_r^4}\right)$$