LORNA GIBSON: OK, so we should probably start. So last time we finished up talking about energy absorption in foamy cellular materials. And today I wanted to start a new topic. We're going to talk about sandwich panels. So sandwich panels have two stiff, strong skins that are separated by some sort of lightweight core. So the skins are typically, say, a metal like aluminum, or some sort of fiber composite. And the core is usually some sort of cellular material. Sometimes it's an engineering honeycomb. Sometimes it's a foam. Sometimes it's balsa wood.

And the idea is that what you're doing with the core is you're using a light material to separate the faces, and if you think about an I-beam-- so if you remember when we talk about bending and we talk about I-beams, the whole idea is that in bending, you want to increase the moment of inertia. So you want to make as much material as far away from the middle of the beam as possible to increase the moment of inertia.

So if you think about an I-beam, you put the flanges far apart with the web, and that increases the moment of inertia. And the sandwich panels and the sandwich beams essentially do the same thing, but they're using a lightweight core instead of a web. And so the idea is you use a lightweight core. It separates the faces. It increases the moment of inertia. But you don't add a whole lot of weight because you've got this lightweight core in the middle.

So I brought some examples that I'll pass around and we can play with. So these are some examples up on the screen, and some of those I have down here. So for instance, the top-- turn my little gizmo on-- the top left here, this is a helicopter rotor blade, and that has a honeycomb core in it. This is an aircraft flooring panel that has a honeycomb core and has carbon fiber faces. So that's this thing here. I'll pass that around in a minute.

This is a downhill ski. This has aluminum faces and a polyurethane foam core. And that's the ski here. And it's quite common in skis now to have these sandwich panels. This is a little piece of a small sailing boat. It had, I think, glass fiber faces and a balsa wood core. And I don't know if any of you sail, but MIT has new sailing boats. Do you sail?

AUDIENCE: I do not much these days.

LORNA GIBSON: OK, but those little tech dinghies that you see out in the river, those have sandwich panel holes to them. So those are little sandwich panels. This is an example from a building panel. This has a dry wall face and a plywood face and a foam core, and the idea with panels for buildings is that usually they use a foam core because the foam has some thermal insulation. So as well as sort of separating the faces and having a structural role, it has a role in thermally insulating the building.

The foams are a little less efficient than using a honeycomb core. So for the same weight, you get a stiffer structure with a honeycomb core than a foam core. But if you want thermal insulation as well as a structural requirement, then the foam cores are good.

And these are a couple examples of sandwiches in nature. This is the human skull. And your skull is a sandwich of two dense layers of the compact bone, and you can see there's a little thin layer of the trabecular bone in between. So your head is like a sandwich, your skull is like a sandwich. And I don't know if I'll get to it next time, but in the next couple of lectures, I'm going to talk a little bit about sandwich panels in nature, sandwich shells in nature. You see this all the time.

And this is a bird wing, here. And so you can see there's got the dense bone on the top and the bottom, and it's got this kind of almost trust-like structure in the middle. And obviously birds want to reduce their weight because they want to fly, so reducing the weight's very important. And so this is one of the ways that birds reduce their weight, is by having a sandwich kind of structure.

So I have a couple of things here. These are the two panels at the top there. This is the ski, and you can yank those around. I also have a few panels that people at MIT have made. And I have the pieces that they're made from. So you can see how effective the sandwich thing is. So this was made by a guy called Dirk Moore. He was a graduate student in ocean engineering. And it has aluminum faces and a little thin aluminum core.

So you can see, if you try and bend that with your fingers, you really can't bend it any noticeable amount. And this panel here is roughly the same thickness of the face on that. And you can see how easy it is for me to bend that-- very easy. And this is the same kind of thing as the core. It's thicker than that core, but you can see how easy it is for me to bend this, too.

So each of the pieces is not very stiff at all. But when you put them all together, it's very stiff.

So that's really the beauty of this. You can have lightweight components, and by putting them together in the right way, they're quite stiff. So here's another example here. This is a panel that one of my students, Kevin Chang made. And this has actually already been broken a little bit, so it's not quite as stiff as it used to be. And you can kind of hear, it squeaks. But you can feel that and see how stiff that is.

And this is the face panel here. And you can see, I can bend that quite easily with my hands. Doodle-doot. And then this is the core piece here. And again, this is very flexible. So it's really about putting all those pieces together. So you get this sandwich construction and you get that effect, OK? [? Oop-loo. ?]

All right, so what we're going to do is first of all look at the stiffness of these panels, calculate their deflection. We're going to look at the minimum weight design of them. So we're going to look at how for, say, given materials in a given span, how do we minimize the weight of the beam for a given stiffness? And then we're going to look at the stresses in the sandwich beams. So there's going to be one set of stresses in the faces, and a different kind of stress distribution in the core. So we'll look at the stress distribution.

And then we'll talk about failure modes, how these things can fail, and then how to figure out which failure mode is dominant, which one occurs at the lowest load. And then we'll look at optimizing the design, minimizing the design for a certain strength and stiffness. So we're not going to get all that way through everything today, but we'll kind of make a start on that.

OK. So let me start. So the idea here is we have two stiff, strong skins, or faces, separated by a lightweight core.

And the idea is that by separating the faces, you increase the moment of inertia with little increase in weight.

So these are particularly good if you want to resist bending, or if you want to resist buckling. Because both of those involve the moment of inertia.

And they work like an I-beam. So the faces of the sandwich are like the flanges of the I-beam, and the core is like the web.

And the faces are typically made of either fiber reinforced composites or metals. So typically, something like aluminum, usually you're trying to reduce the weight if you use these things, so a lightweight metal like aluminum is sometimes used. And the cores are usually honeycombs, or foams, or balsa.

And when they use balsa wood, what they do is-- I brought a piece of balsa here-- what they do is they would take a block like this and chop it into pieces around here. And then they would lay those pieces on a cloth mat. So typically the pieces are maybe two inches by two inches. They lay them on a cloth mat, and because they're not one monolithic piece, they can then shape that mat to curved shapes. So it doesn't have to be just a flat panel. They can curve it around a curved surface if they want.

So we'll say the honeycombs are lighter than the foams for a given stiffness or strength.

But the foams provide thermal insulation as well as a mechanical support.

And the overall mechanical properties of the honeycomb depend on the properties of each of the two parts, of the faces and the core, and also the geometry of the whole thing-- how thick's the core, how thick's the face, how dense is the core? That kind of thing.

And typically, the panel has to have some required stiffness or strength. And often what you want to do is minimize the weight for that required stiffness or strength.

So often these panels are used in some sort of vehicle, like we talked about the sailboat, or like a helicopter, or like an airplane. They're also used in like refrigerated trucks-- they would have a foam core because they'd want the thermal insulation. So if you were going to use it in some sort of a vehicle, you want to reduce the mass of the vehicle and you want to have the lightest panel that you can. Yup?

AUDIENCE: So if you saw the base material and you'd have the [INAUDIBLE] sandwich panel, that piece [INAUDIBLE] the sandwich panel with something [? solid ?] in the middle?

LORNA GIBSON: Well--

AUDIENCE: So, as we're getting [INAUDIBLE] aluminum piece that's was as thick as a sandwich panel.

LORNA GIBSON: Yeah.

AUDIENCE: [INAUDIBLE].

LORNA GIBSON: Oh, well, if you have the solid aluminum piece that was as thick as the sandwich, it's going to be stiffer, but it's going to be a lot, lot heavier. So the stiffness per unit weight would not be as good. OK? So we're going to calculate the stiffness in just one minute. And then we're going to look at how we minimize the weight, OK? OK.

So what I'm going to do is set this up as kind of a general thing. We're just going to look at sandwich beams rather than plates, just because it's simpler. But the plates, everything we say for the beams basically applies to the plates. The equation's just a little bit more complicated. So we're going to start with analyzing beams.

And I'm just going to start with a beam, say, in three-point bending. So there's my faces there. Boop. And I've made it kind of more stumpy than it would be in real life, just because it makes it easier to draw it. And then if I look at it the other way on, it would look something like that. So say there's some load P here. Say the span of the beam is l. Say the load's in the middle, so each of the supports just sees a load of P/2.

And then let me just define some geometrical parameters here. I'm going to say the width of the beam is b. And I'm going to say the face thicknesses are each t. So the thickness of each face is t. And the thickness of the core is c, OK? So that's just sort of definitions.

And I'm going to say the face has a set of properties, the core has a set of properties, and then the solid from which the core is made has another set of properties. So the face properties that we're going to use are a density of the face. We'll call that row f. The modulus of the face, Ef, and some sort of strength of the face, let's imagine it's aluminum and it yields, that would be sigma y of the face.

And then the core similarly is going to have a density, rho star c. It's going to have a modulus, E star c. And it's going to have some strength, I'm going to call sigma star c. And then the solid from which the core is made is going to have a density row s, a modulus Es, and some strength, sigma ys, OK?

So the core is going to be some kind of cellular material, a honeycomb, or a foam, or balsa. And typically, the modulus of the core is going to be a lot less than the modulus of the face. So I'm just going to say here that the E star c is typically much greater than Ef. And we're going to use that later on.

So we're going to derive some equations, for example, for an equivalent flexural rigidity for the section, an Ei equivalent. And that has several terms. But if we can say the core stiffness is much less than the face thickness, and also if we can say the core-- the stiffness is less and also the thickness of the core is much greater than the thickness of the face, a lot of the expressions we're going to use simplify. So we're going to make those assumptions. So let me just draw the shear diagram here.

So V is shear, so that's the shear diagram. We have some load P/2 at the support. There's no other load applied until we get to here. Than the shear diagram goes down by P, so we're at minus P/2. Then there's no load here, so this just stays constant, and then we go back up to 0. And then let me just draw bending moment diagram. The bending moment diagram for this is just going to look like a triangle.

Remember, if we integrate the shear diagram, we get the bending moment diagram. And that maximum moment there is going to Pl over 4. OK. So initially, I'm going to calculate the deflections. And I don't really need those diagrams for that, but then I'm going to calculate the stresses, and I'm going to need those diagrams for the stresses. So just kind of keep those in mind for now.

So to calculate the deflections, sandwich panels are a little bit different from homogeneous beams. In a sandwich panel, the core is not very stiff compared to the faces. And we've got some shear stresses acting on the thing. And the shear stresses are largely carried by the core. So the core is actually going to shear, and there's going to be a significant deflection of the core and shear as well as the overall bending of the whole panel. So you have to count for that.

So we're going to have a bending term and a shear term-- that's what those two terms are there. So we're going to say there's a bending deflection and a shear deflection. And that shearing deflection arises from the core being sheared and the fact that the core, say, Young's modulus or also the shear modulus, is quite a bit less than the face modulus.

So if you think of the core as being much more compliant than the face, then the core is going to have some deflection from that shear stress. OK, so we're going to start out with this term here, the bending term. And if I just had a homogeneous beam in three-point bending, the central deflections-- so these are all the central deflections I'm calculating here-- with Pl cubed over it turns out to be 48 is the number, and divided by EI.

And because we don't have a homogeneous beam here, I'm going to call that equivalent EI. And to make it a little bit more general, instead of putting 48, that number, I'm just going to put a constant B1. And that B1 constant is just going to depend on the loading geometry. So any time I have a concentrated load on a beam, the deflection's always Pl cubed over EI, and then the sum number in the denominator and that number just depends on the loading configuration.

So for three-point bending, it's 48. For the flexion of a cantilever, B1 would be 3. So think of that as just a number that you can work out for the particular loading configuration. So here we'll say B1 is just a constant that depends on the loading configuration.

And I'll say, for example, for three-point bending, B1 is 48. For a cantilever end deflection, then B1 would be 3. So it's just a number.

So the next thing we have to figure out is what's the EI equivalent. So if this was just a homogeneous beam, and it was rectangular, E would just be E of the material and I would be the width B times the height H cubed divided by 12. So here we don't quite have that because we have two different materials. So here we have to use something called the parallel axis theorem, which I'm hoping you may have seen somewhere in calculus, maybe? But, yeah, somebody is nodding yes.

OK, so what we do, what we want to do is get the equivalent EI-- I'm going to put it back up, don't panic-- of this thing here, right? So I want-- this is the neutral axis here, and I want the EI about that neutral axis there. So, OK, you happy? There.

OK, so I've got a term for the core. OK, the core, that is the middle of the core, right? So for the core, it's just going to be E of the core times bc cubed over 12. Remember, for a rectangular section, it's bh cubed over 12 is the moment of inertia. And here our height for the core is just c, OK? And then if I took the moment of inertia for, say, one face about its own centroidal axis, I would get E of the face now times bt cubed over 12. So that's taking the moment of inertia of one face about the middle of the face. And I have two of those, right? Because I have two faces.

And the parallel axis theorem tells you what the moment of inertia is going to be if you move it, not to the-- you don't use the centroid of the area, but you use some other parallel axis. And what that tells you to do is take the area that you're interested in-- so the area of the face is bt, and you multiply by the square of the distance between the two axes that you're interested in. Oop, yeah.

Let me change my little brackets. Boop. So, oop-a-doop-a-doop. Maybe I'll stick this, make a little sketch over here again. OK, all right. So this term here, Ef bt cubed over 12, that would be the moment of inertia of this piece here, about the axis that goes through the middle of that, right? Its own centroidal axis.

But what I want to do is I want to know what the moment of inertia of this piece is about this axis here. This is the neutral axis. So let's call this the centroidal axis. And the parallel axis theorem tells me what I do is I take the area of this little thing here, so that's the b times t, and I multiply by the square of the distance between those two axes. So the distance between those axes is just c plus t over 2, and I square it. And then I multiply that whole thing by 2 because I've got two faces. Are we good?

AUDIENCE: [INAUDIBLE].

LORNA GIBSON: Yeah?

AUDIENCE: The center [INAUDIBLE] and the [INAUDIBLE], are those Ed's or Ef's?

LORNA GIBSON: These are Ef's because this is the face now, right? So this term here is for the core. So here the core is E star c. And these two Ef's are for the face up there, OK? Because you have to account for the modulus of the material of the bit that you're getting the moment of inertia for. Are we good? OK.

So now I'm just going to simplify these guys a little bit. Doodle-doodle-doodle-do-doot. OK? So I've just multiplied the twos, and maybe I'll just write down here this is the parallel axis theorem. Doot-doot-doot. Yes, sorry?

AUDIENCE: So for the term that comes from the parallel axis theorem, why do we only consider Ef and not

[INAUDIBLE].

LORNA GIBSON: Because I'm taking-- what I'm looking at-- so the very first term, this guy, here--

AUDIENCE: Yeah, [INAUDIBLE].

LORNA GIBSON: Accounts for this, right? And these two terms both account for the face.

AUDIENCE: Oh, OK, so the face acting--

LORNA GIBSON: Yeah, about this axis. So the parallel axis theorem says you take the moment of inertia of your area about its own centroidal axis, and then you add this term here. But it's really referring to that face, OK? Let me scoot that down and then scoot over here.

And this is where we get to say the modulus of the face is much greater than the modulus of the core. And also, typically c, the core thickness, is much greater than t, the face thickness. So if that's true, then it turns out this term is small compared to that one. And also this term is small compared to this one. And also this term, instead of having c plus t squared, if c is big compared to t, then I can just call it c squared, OK?

So you can see here, if Ec is small, then this is going to be small compared to these. If t is small, then this guy is going to be small. So even though it looks ugly, many times we can make this simpler approximation.

OK, so we can just approximate it as Ef times btc squared over 2. So then this bending term here, we've got everything we need now to get that bit there. So the next bit we want to get is the shearing deflection. So what's the shearing deflection equal to? So say we just thought about the core, and all we're interested in here is what's the deflection of the core and shear?

And so say that's P/2, that's P/2, that's l/2. We'll say that's-- oops. That's our shearing deflection there. We can say the shear stress in the core is going to equal the shear modulus times the shear strain, so we can say P over the area of the core is going to be proportional to the Young's modulus times delta s over l. And let's not worry about the constant just yet.

So delta s is going to be proportional to-- well, let me [? make it ?] proportional at this point. Delta s is going to equal Pl divided by some other constant that I'm going to call B2, and divided by the shear modulus of the core, and essentially the area of the core. And here B2 is another constant. So again, B2 just depends on the loading configuration.

Yeah, this is a little bit of an approximation here, but I'm just going to leave it at that. OK, so then we have these two terms and we just add them up to get the final thing. Start another board.

OK. So that would give us an equation for the deflection. And one thing to note here is that this shear modulus of the core, if the core is a foam, then we have an equation for that. We also could use an equation if it's a honeycomb. But I'm just going to write for foam cores. Whoops.

This is for-- that will be for open-cell foam cores. Oops, don't want to-- and get rid of that. We won't update just now, thank you.

OK, so the next thing I want to think about is how we would minimize the weight for a given stiffness. So say if we're given a stiffness, we're given P over delta, so I could take out the two P's here. If I divide it through by P, delta over P would be the compliance, P over delta would be the stiffness.

So imagine that you're given the face and core materials, and you're told how long the span has to be, you're told how wide the beam is going to be, and you're told the loading configuration. So you know if it's three-point bending, or four-point bending, or a cantilever-- whatever it is. And you might be asked to find the core thickness, the face thickness, and the core density that would minimize the weight.

So I have a little schematic here. I don't know if you're going be able to read it. So I'm going to walk through it and then I'll write things on the board. Whoops, hit the wrong button. OK, so we start with the weight equation here. The weight's obviously the sum of the weight of the faces, the weight of the core, so those two terms there. So I'll write that down in a minute.

And then we have the stiffness constraint here. So this equation here is just this equation that I have down here on the board, OK? Then what you do is you solve that stiffness constraint for the density of the core. So this equation here just solves-- we're solving this equation here in terms of the density, and we get the density by substituting in this equation here for the shear modulus of the core.

So you substitute that there. It's kind of a messy thing, but you solve that in terms of the density. Then you put that version of the density here in terms of this weight equation up here. So then you've eliminated the density out of the weight equation, now you've just got it in terms of the other variables.

And then you take the partial derivative of the weight with respect to the core thickness c, set that equal to 0, and you take the partial derivative of the weight with respect to the face thickness, t, and you set that equal to 0. And that then gives you two equations and two unknowns. You've got the core thickness and the face thickness are the two unknowns. And you've got the two equations, so then you solve those. So the value you get for the core thickness is then the optimum, so it's going to be some function of the stiffness, the material properties you started with in the beam geometry.

And similarly, you get some equation for the optimum face thickness, t. And again, it's a function of the stiffness and the material properties in the beam geometry. Then you take those two values for c and t, those two optimum values, and plug it back into this equation here, and get the optimum value of the core density. And so what you end up are three equations for the optimal values of the core thickness, the face thickness, and the core density in terms of the required stiffness, the material properties, and then the loading geometry.

So I'm going to write down some more notes, because I'll put this on the Stellar site. But it's hard to read just here. So let me write it down and I'll also write out the equations so that you have the equations for calculating those optimum values. So before I do that, though, one of the interesting things though is if you figure out the optimal values of the core thickness and the face thickness and the core density, and you substitute it back into the weight, and you calculate this is the weight of the face relative to the weight of the core, no matter what the geometry is, and what the loading configuration is, the weight of the face is always a quarter of the weight of the core.

So the ratio of how much material is in the core and the face is constant, regardless of the core-- of the loading configuration. And this is the bending deflection relative to the total deflection. It's always 1/3. And the shearing deflection relative to the total deflection is always 2/3. So regardless of how you set things up, the ratio of what weight the face is relative to the core and the amount of shearing and bending deflections is always a constant at the optimum.

OK, so let's say we're given the face and the core materials. So that means we're given their material property, too. And say we're given the beam length and width and the loading configuration. So that means we're given those constants, B1 and B2. If I told you it was three-point bending, you would know what B1 and B2 are.

So then what you need to do is find the core thickness, c, the face thickness, t, and the core density, rho c, to minimize the weight of the beam. So there's two faces, so the weight of the face is 2 rho f g times btl. And then the weight of the core is rho c g times bcl. So I'm going to write down the steps and then I'll write down the solution.

So you solve. So you put this equation for the shear modulus of the core into here, and then you rearrange this equation in terms of the density of the core here. So you have an equation for the core density in terms of that stiffness, and then you solve the partial derivatives of the weight equation with respect to the core thickness, c, and put that equal to 0. And then the partial of dw [? over ?] dt and set that to 0.

And if you do that, you can then solve for the optimal values of the face and core thicknesses. Yes?

AUDIENCE: [INAUDIBLE] for weight, what is g?

LORNA GIBSON: Gravity.

AUDIENCE: OK.

LORNA GIBSON: Just density is mass, mass times gravity-- weight. That's all it is.

And then you've got a version of this that's in terms of the core density. You can substitute those values of the optimum face and core thicknesses into that equation and get the optimum core density. And then in the final equations, you get, when you do all that, and I'm going to make them all dimensionless, so this is the core thickness normalized by the span of the beam is equal to this thing, here.

So you can see each of these parameters here, the design parameters that we're calculating the optimum of. I've grouped the constants B1 and B2 together that describe the loading configuration so you'd be given those. C2 is this constant-- oop, which I just rubbed off-- that relates the shear modulus of the foam core. So you'd be given that. These are the material properties of the-- you know, say, it's a polyurethane foam core, this would be the density of the polyurethane. Say it's aluminum faces, that would be the density of the aluminum. so you'd be given that.

You'd be given the stiffnesses of the two materials, the solid from which the core is made and the face material. And then this is the stiffness here that you're given, just divided by the width of the beam, B. So the stiffness, you'd be given the width B. So you're given all those things, then you could calculate what that optimum design would be.

So the next slide here just shows some experiments. And these were done on sandwiches with aluminum faces and a rigid polyurethane foam core. And here we knew what the relationship was for the shear modulus. We measured that. And what we did here was we designed the beams to all have the same stiffness, and they all had the same span in the width, B, then we kept one parameter at the optimum value and we varied the other ones.

So here, on this beam, this set of beams here, the density was at the optimum. And we varied the core thickness, and we varied the face thickness, and the solid line was our model or our sort of optimization. And the little X's were the experiments. So you can see there's pretty good agreement there. Then the second set here, we kept the face thickness at the optimum value and we varied the core thickness, we varied the core density. So the same thing, the solid line is the sort of theory and the X's are the experiments. And here we had the core thickness of the optimum value, and we varied the face thickness and the core density.

So you can kind of see how you can see this here. And over here, just because I forgot to say it, this is the stiffness per unit weight, over here, OK? So these are the optimum designs here, all right? So there was pretty good agreement between these calculations and what we measured on some beams. Do I need to write anything down? Do you think you've got that? Yeah?

AUDIENCE: I was just going to ask, for the optimum design column that you have there, do those numbers like fall out of these equations if you do the math?

LORNA GIBSON: They do, yeah. I mean, it's-- yeah, exactly. So if you remember the equation we had for the weight, so the weight is equal to 2 rho f gbtl plus the density of the core, bcl, so if you plug these things into there, then-- so this is the way to the face, that's the way to the core, then it drops out to be a quarter. So it's kind of magical. I mean, you have this big, long, complicated gory thing, and then, poof, everything disappears except a factor of 1/4.

And the same for the bending deflection. So we had those two terms, so there was the bending and the shear. If you just calculate each of those terms and take the ratio of 1 over the total, or the one over the other, everything drops out except that number. So that's why I pointed it out, because it seemed kind of amazing that everything would drop out except for that one thing.

OK, so then the next thing-- so that's the stiffness in optimizing the stiffness. Are we happy-ish? Yeah? OK. So the next thing-- oh, well, let's see. I don't think I need to write any. I think if you have that graph, I don't really need to write much down.

So the next thing then is the strength of the sandwich beams. So let me get rid of that.

You guys OK? Yeah?

AUDIENCE: Yeah.

LORNA GIBSON: Yeah, but you're shaking your head like this is very, very helpful for me.

AUDIENCE: [INAUDIBLE].

LORNA GIBSON: Oh OK, that's OK.

AUDIENCE: [INAUDIBLE].

LORNA GIBSON: That's OK, you can do that. I don't mind. But as long as you don't have questions for me. OK, and so the first step in trying to figure out about this strength is we need to figure out the stresses in the beams.

So we need to find out about the stresses. And we're going to have normal stresses and we're going to have shear stresses. So I'm going to do the normal stresses first and then we'll do the shear stresses.

So you do this in a way that's just analogous to how you figure out the stresses in a homogeneous beam. So we'll say the stresses in the face-- normally it would be My over I. M is the moment, y is the distance from the neutral axis, I is the moment of inertia. So this time, instead of having a moment of inertia, we have this equivalent moment of inertia. And we multiply by E of the face. So you can think of this as being the strain essentially. And then you multiply by E of the face to get the stress.

The maximum distance from the neutral axis, we can call c/2. So that's y. Then EI equivalent we had Ef btc squared over 2. And then I have a term of Ef here. c squared. So one of the c's goes, the 2's go, the Ef's go. Then you just get that the normal stress in the face is the moment at that section divided by the width, b, the face thickness, t, the core thickness, c.

And I can do the same kind of thing for the stress in the core, except now I multiply by the core modulus. So if I go through the same kind of thing, it's the same factor of M over btc, but now I multiply times E of core over E of the face. And since E of the core is a lot smaller than E of the face, typically these normal stresses in the core are much smaller than the normal stresses in the face.

So the faces carry almost all of the normal stresses. And if you look at an I-beam, the flanges of the I-beam carry almost the normal stresses.

So I want to do one more thing here. I want to relate the moment to some concentrated load. So let's say we have a beam with a concentrated load, P. So for example, something in three-point bending, typically we're interested in the maximum stresses, so we want the maximum moment. So M max is going to be P times l over some number. And this B3 is another constant that depends on the loading configuration.

So if it was three-point bending, B3 would be 4. If it was a cantilever, B3 would be 1. So if I put those things together, the normal stress in the face is Pl B3 divided by btc.

OK, so that's the normal stresses. And then the next thing is the shear stresses, and the shear stresses are going to be carried largely by the core. And if you do all the exact calculations, they vary parabolically through the core. But if we make those same approximations that the face is stiff compared to the core, and that the face is thin compared to the core, then you can say that the shear stress is just constant through the core.

So we'll say the shear stresses vary parabolically through the core. But if the face is much stiffer than the core and the core is much thicker than the face, then you can say that the shear stress in the core is just equal to the sheer force over the area of the core, bc. So here, V is the shear force of the cross-section you're interested in. And bc is just the area of the core.

And we could say the maximum shear force is just going to be V over-- actually, let's make it P, P over yet another constant. And B4 also depends on the loading configuration. So if I was giving you a problem, I would give you all these B1, B2, B3, B4's and everything.

So the maximum shear stress in the core is in just the applied load, P, divided by this B4 and divided by the area of the core.

OK, so this next figure up here just shows those stress distributions. So here's a piece of the cross-section here. So there's the face thickness and the core thickness. You can think of that as a piece along the length, if you want. This is the normal stress distribution, here. So this is all really from saying plane sections remain plane. These are the stresses, the normal stresses in the core. And you can see they're a lot smaller in this schematic than the ones in the face.

And then this is the parabolic stress in the core. And similarly, there'd be a different parabola in the face. And these are the approximations. Typically these approximations are made so the normal stress in the face is just taken as a constant. The normal stress in the core is often neglected. And here the shear stress in the core is just a constant here.

So the two things you need to worry about are the normal stress for the face and the shear stress for the core. Are we good? We're good? Yeah, good-ish.

OK, so if we want to talk about the strength of the beam, we now have to talk about different failure modes. And the next slide just shows some schematics of the failure modes. So there's different ways the beam can fail. Say it's in three-point bending just for the sake of convenience. One way it can fail is, say it had aluminum faces. This face here would be in tension, and the face could just yield. So you could just get yielding of the aluminum. That would be one way.

It could be a composite face and you could have some sort of composite failure mode. You can get more complicated failure modes for composites, but there could be some sort of failure mode. This face up here is in compression, and if you compress that face, you can get something called face wrinkling. You get sort of a local buckling mode. So imagine you have the face, that you're pressing on it, but the core is kind of acting like an elastic foundation underneath it. And you can get this kind of local buckling, and that's called wrinkling. That's another mode of failure.

You can also get the core failing in shear. So here's these two little cracks, denoting shear failure in the core. And there's a couple of other modes you can get, but we're going to not pay much attention to those. The whole thing can delaminate, and, as you might guess, if the whole thing delaminates, you're in deep doo-doo. Because, remember when I passed those samples around, how flexible the face was by itself and how flexible the core is by itself. If the whole thing delaminates, you lose that whole sandwich effect and the whole thing kind of falls apart. We're going to assume we have a perfect bond and that we don't have to worry about that.

The other sort of failure mode you can get is called indentation. So imagine that you apply this load here over a very small area. The load can just transfer straight through the face and just kind of indent the core underneath it. We're going to assume that you distribute this load over a big enough area here, that you don't indent the core. So we're going to worry about these three failure modes here-- the face yielding, the face wrinkling, and the core failing and shear, OK? So let me just write that down.

And then you also can have debonding or delamination, and we're going to assume perfect bond. And then you can have indentation, and we're going to assume the loads are applied over a large enough area that you don't get--

So you can have different modes of failure, and the question becomes which mode is going to be dominant? So whichever one occurs at the lowest load is going to be the dominant failure mode. So you'd like to know what that lowest failure mode is. So we want to write equations for each of these failure modes and then figure out which one occurs first.

So we'll look at the face yielding here. And face yielding is going to occur just when the normal stress in the face is equal to the yield stress of the face. So this is fairly straightforward. So this was our equation for the stress in the face. And when that's equal to the face yield strength, then you'll get failure.

And the face wrinkling occurs when the normal compressive stress in the face equals a local buckling stress.

And people have worked that out by looking at what's called buckling on an elastic foundation. So the core acts as elastic support. You can think that as the face is trying to buckle into the core, the core is pushing back on the face. And so the core is acting like a spring that pushes back, and that's called an elastic foundation. So people have calculated this local buckling stress, and they found that's equal to 0.57 times the modulus of the face to the 1/3 power times the modulus of the core to the 2/3 power.

And here, if we use our model for open cell foams, we can say the core modulus goes as the relative density squared times the solid modulus. And so you can plug that in there.

So then the wrinkling occurs when the stress in the face, the Pl over the B3 btc is equal to this thing here.

OK, so one more failure mode that's the core shear, and that's going to occur when the shear stress in the core is just equal to the sheer strength of the core. So the shear stress is P over B4 times bc, and the shear strength is some constant, I think it's C11, times the relative density of the core to the three halves power times the yield strength of the solid. And here, this constant is about equal to 0.15, something like that.

So now we have a set of equations for the different failure modes, and we could solve each of them, not in terms of a stress, but in terms of a load P. The load P is what's applied to the beam, right? So we could solve each of these in terms of the load, P. And then we can see which one occurs at the lowest load, P. And that's going to be the dominant failure mode.

So one way to do it would be to, for every time you wanted to do this, to work out all these three equations and figure out which one's the lowest load. But there's actually something called a failure mode map, which we're going to talk about. So let me just show you it and we'll start now. I don't know if we'll get finished this.

But there's a way that you can manipulate these equations and plot the results as this failure mode map. And you'll end up plotting the core density on this plot, on this axis here, and the face thickness to span ratio here, and so this will kind of tell you, for different configurations of the beam, different designs, for these ones here, the face is going to wrinkle, for those ones there, the face is going to yield, and for these ones here, the core is going to shear.

So I'm going to work through these equations, but I don't think we're going to finish it today. So this is just kind of where we're headed is to getting this map. So we'll say the dominant failure mode is the one that occurs at the lowest load.

So the question we're going to answer is how does the failure mode depend on the beam design? And we're going to do this by looking at the transition from one failure mode to another.

So at the transition from one mode to another, the two modes occur at the same load.

So I'm going to take those equations I had for each of the failure modes, and instead of writing this in terms of, say, the stress in the face, I'm going to write it in terms of the load, P. So using that first one over there, the load for face yielding, I'm just rearranging that. It's B3 times bc times t/l times the yield strength of the face.

And similarly for face wrinkling, I can take this equation down here and solve it for this P here, OK?

And then I can take that equation at the top and solve that for P2 for the core shear, and that's equal to C11 times B4 times bc times sigma ys times-- oops, wrong thing-- times the relative density to the 2/3 power. OK?

And then the next step is to equate these guys. So you get a transition from one mode to the other when two of these guys are equal to each other, right? So there's going to be a transition from face yielding to face wrinkling when these guys are equal. And I'm not going to start that because we're going to run out of time.

But let me just say that I can pair these two up and say there's a transition between those two. And that transition is going to correspond to this line here, OK? So at this line here, that means you get face yielding and face wrinkling at the same load, OK? And then if I paired up-- let's see here. If I paired up face wrinkling and core shear, these two guys here, I'm going to get this equation here on that plot. And then if I paired up these two guys here, the face shielding and the core shear, I would get that line there, OK?

So once I have those lines, that tells me, you know, anything with a lower density core and a smaller face thickness is going to fail by face wrinkling. Anything with a bigger density is going to fail by face yielding. And anything with a larger face thickness and a larger density is going to fail by core shearing. And so you can start to see that it-- I'll work out the equations next time, but you can start to see that it kind physically makes sense.

Intuitively, this face wrinkling, it depends on the normal stress in the face, in compression. So obviously the thinner the face gets, the more likely that's going to be to happen. So it's going to happen at this end of the diagram. And it also depends on that elastic foundation, on how much spring support the foundation has, right? So the lower the core density, the more likely that is to happen.

Then if you, say you have small t, so the face is going to fail before the core, as you increase the core density, you're making that elastic foundation stiffer and stiffer, and you're making it harder for the buckling to occur. It can't buckle into the elastic foundation, so then you're going to push it up to the yielding. And then as you make the face thickness bigger, as t gets bigger, then the face isn't going to fail and the core is going to fail. So you can kind of see just looking at the relative position of those things, they all kind of make physical sense.

So I'm going to stop there for today and I'll finish the equations for that next time. And we'll also talk about how to optimize for strength next time. And we'll talk about a few other things on sandwich panels.