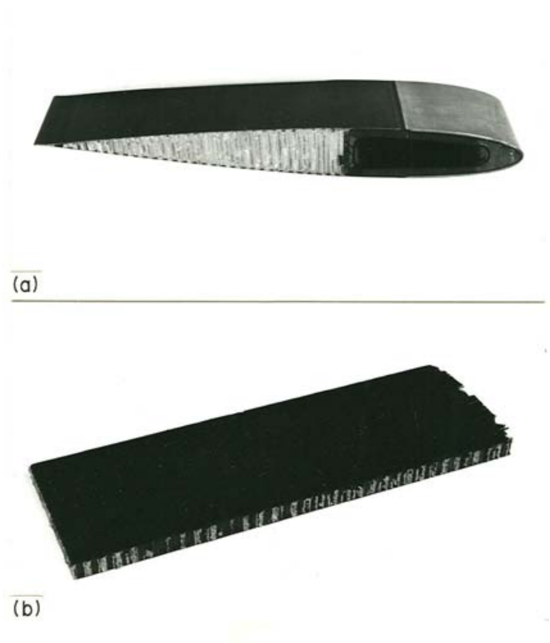
Sandwich Panels

- two stiff strong skins separated by a light weight core
- separation of skins by core increases moment of inertia, with little increase in weight
- efficient for resisting bending + buckling
- like an I beam: faces = flanges - carry normal stress
  core = web - carries shear stress
- examples: engineering + nature

- faces: composites, metals
  cores: honeycombs, foams, balsa
  honeycombs: lighter than foam cores for req'd stiffness, strength
  foams: heavier, but can also provide thermal insulation
- mechanical behavior depends on face + core properties + on geometry
- typically, panel must have some required stiffness and/or strength
- often, want to minimize weight - optimization problem
  eg. refrigerated vehicles; sporting equipment (sail boats, skis)



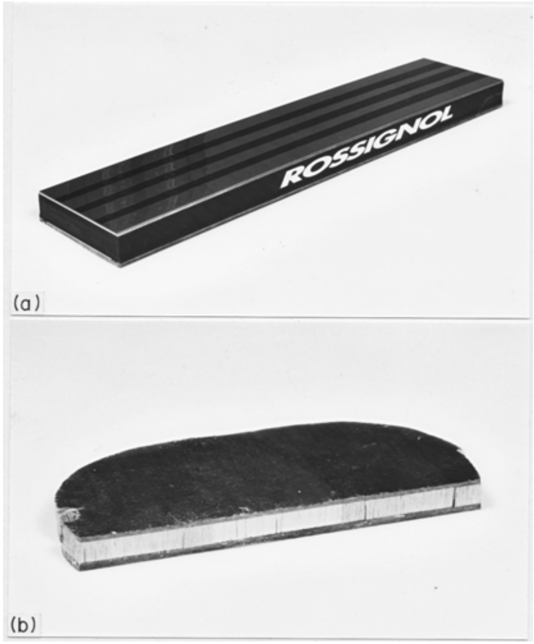
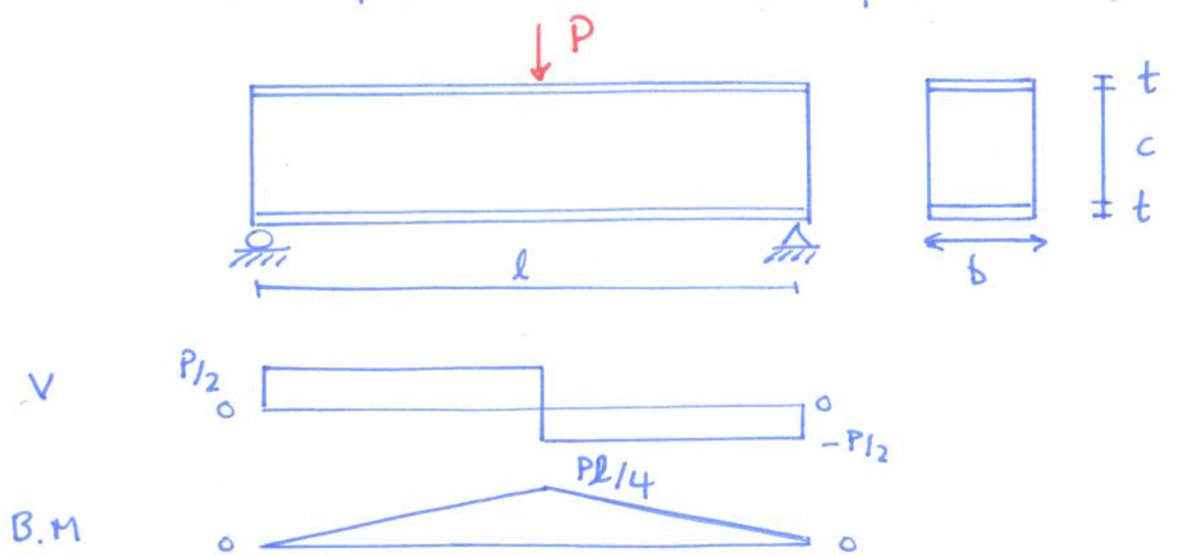


Figure removed due to copyright restrictions. See Figure 9.4: Gibson, L. J. and M. F. Ashby. *Cellular Solids: Structure and Properties*. Cambridge University Press, 1997.

Sandwich beam stiffness

- analyze beams here (simpler than plates; same ideas apply)



Face: ρ_f, E_f, σ_{yf}

Core: ρ_c^*, E_c^*, σ_c^*

(Solid: ρ_s, E_s, σ_{ys})

Typically $E_c^* \ll E_f$

$\delta = \delta_b + \delta_s$: Bending deflection δ_b + shear deflⁿ (of core) δ_s

since $G_c^* \ll E_f$, core shear deflections significant

$\delta_b = \dfrac{P l^3}{B_1 (EI)_{eq}}$ $\quad$ B_1 = constant, depending on loading configuration

3 pt bend, $B_1 = 48$

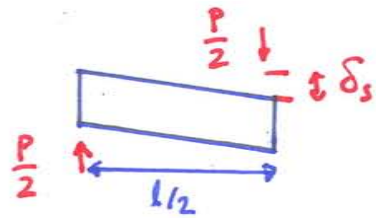
$$(EI)_{eq} = \left(\frac{E_f b t^3}{12} \times 2\right) + \frac{E_c b c^3}{12} + E_f b t \left(\frac{c+t}{2}\right)^2 2 \qquad \text{parallel axis theorem}$$

$$= \frac{E_f b t^3}{6} + \frac{E_c b c^3}{12} + \frac{E_f b t}{2} (c+t)^2$$

sandwich structures: typically $E_f \gg E_c^*$ & $c \gg t$

approximate $(EI)_{eq} \approx \frac{E_f b t c^2}{2}$

$\delta_s = ?$ core



$\tau = G\gamma$

$\frac{P}{A} \propto G \frac{\delta_s}{l}$

$$\delta_s = \frac{Pl}{B_2 (AG)_{eq}}$$

$$(AG)_{eq} = \frac{b(c+t)^2}{c} G_c \approx b c G_c$$

$$\delta = \delta_b + \delta_s$$

$$\boxed{\delta = \frac{2Pl^3}{B_1 E_f b t c^2} + \frac{Pl}{B_2 b c G_c^*}}$$

AND note:

$$G_c^* = C_2 E_s (\rho_c^*/\rho_s)^2 \quad \text{(foam model)}$$

$$C_2 \approx 3/8$$

Minimum weight for a given stiffness

- given: face + core materials
    - beam length, width, loading geometry (eg. 3pt bend, B_1, B_2)
- find: face + core thicknesses, t + c, + core density ρ_c^* to minimize weight

$$W = 2\rho_f g b t l + \rho_c^* g b c l$$

- solve (P/δ) eqn for ρ_c^* & substitute into weight eqn
- solve $\partial W/\partial c = 0$ & $\partial W/\partial t = 0$ to get t_{opt}, c_{opt}
- substitute t_{opt}, c_{opt} into stiffness eqn (P/δ) to get $\rho_c^*{}_{opt}$

- note that optimization possible by foam modelling $G_c = C_2 (\rho^*/\rho_s)^2 E_s$

$$\left(\frac{c}{l}\right)_{opt} = 4.3 \left\{ \frac{C_2 B_2}{B_1^2} \left(\frac{\rho_f}{\rho_s}\right)^2 \frac{E_s}{E_f^2} \left(\frac{P}{\delta b}\right) \right\}^{1/5}$$

$$\left(\frac{t}{l}\right)_{opt} = 0.32 \left\{ \frac{1}{B_1 B_2^2 C_2^2} \left(\frac{\rho_s}{\rho_f}\right)^4 \frac{1}{E_f E_s^2} \left(\frac{P}{\delta b}\right)^3 \right\}^{1/5}$$

$$\left(\frac{\rho_c^*}{\rho_s}\right)_{opt} = 0.59 \left\{ \frac{B_1}{B_2^3 C_2^3} \left(\frac{\rho_s}{\rho_f}\right) \frac{E_f}{E_s^3} \left(\frac{P}{\delta b}\right)^2 \right\}^{1/5}$$

Note: $\dfrac{W_{face}}{W_{core}} = \dfrac{1}{4}$ $\quad \dfrac{\delta_b}{\delta} = \dfrac{1}{3}$ $\quad \dfrac{\delta_s}{\delta} = \dfrac{2}{3}$

Table 9.3 Optimum design of a sandwich panel subject to a stiffness constraint

Formulate objective function for the weight of the beam

$$W = 2\rho_f gblt + \rho_c^* gblc \qquad (\text{Eqn. } (9.7))$$

↓

Formulate the stiffness constraint

$$\delta/P = \frac{2l^3}{B_1 E_f btc^2} + \frac{l}{B_2 bcG_c^*} \qquad (\text{Eqn.}(9.6))$$

↓

Solve stiffness constraint for one variable (e.g. ρ_c^*)

$$\rho_c^* = \left\{\frac{B_1}{C_2 B_2}\,\frac{E_f}{E_s}\left(\frac{ltc}{B_1 btc^2 E_f(\delta/P) - 2l^3}\right)\right\}^{1/2}\rho_s \qquad (\text{Eqn.}(9.8))$$

and substitute this into the objective function

↓

Form $\partial W/\partial c = 0$ *and* $\partial W/\partial t = 0$

and solve to give

$c_{opt} = f(\delta/P$, *material properties, beam geometry*)

$t_{opt} = f(\delta/P$, *material properties, beam geometry*)

Substitute these into Eqn. (9.8) for core density to give optimum density, ρ_{copt}^*.

↓

Final Result:

Optimum	*As function of*
Core thickness, c_{opt}	Design stiffness, δ/P
Face thickness, t_{opt}	Material properties, $\rho_f, \rho_s, E_f, E_s, C_2$
Core density, ρ_{copt}^*	Loading geometry, B_1, B_2, l, b

Table 9.4 Optimization analysis for sandwich panels subject to a stiffness constraint

Geometry	W_f/W_c	δ_b/δ	δ_s/δ
Rectangular beam	1/4	1/3	2/3
Circular plate (distributed load over entire plate)	1/4	1/3	2/3
Circular plate (distributed load over radius r)	1/4	1/3	2/3

Comparison with expeiments

- Al faces with rigid PU foam core
- $G_c = 0.7\, E_s (\rho_c^*/\rho_s)^2$
- beams designed to have same stiffness, P/δ, span l, width, b
- one set had $\rho_c^* = \rho_c^*{}_{opt}$, varied t, c
- " " " $t = t_{opt}$, varied ρ_c^*, c
- " " " $c = c_{opt}$, varied t, ρ_c^*
- Confirms min. weight design; similar results with circular sandwich plates

---

Strength of sandwich beams

- stresses in sandwich beams

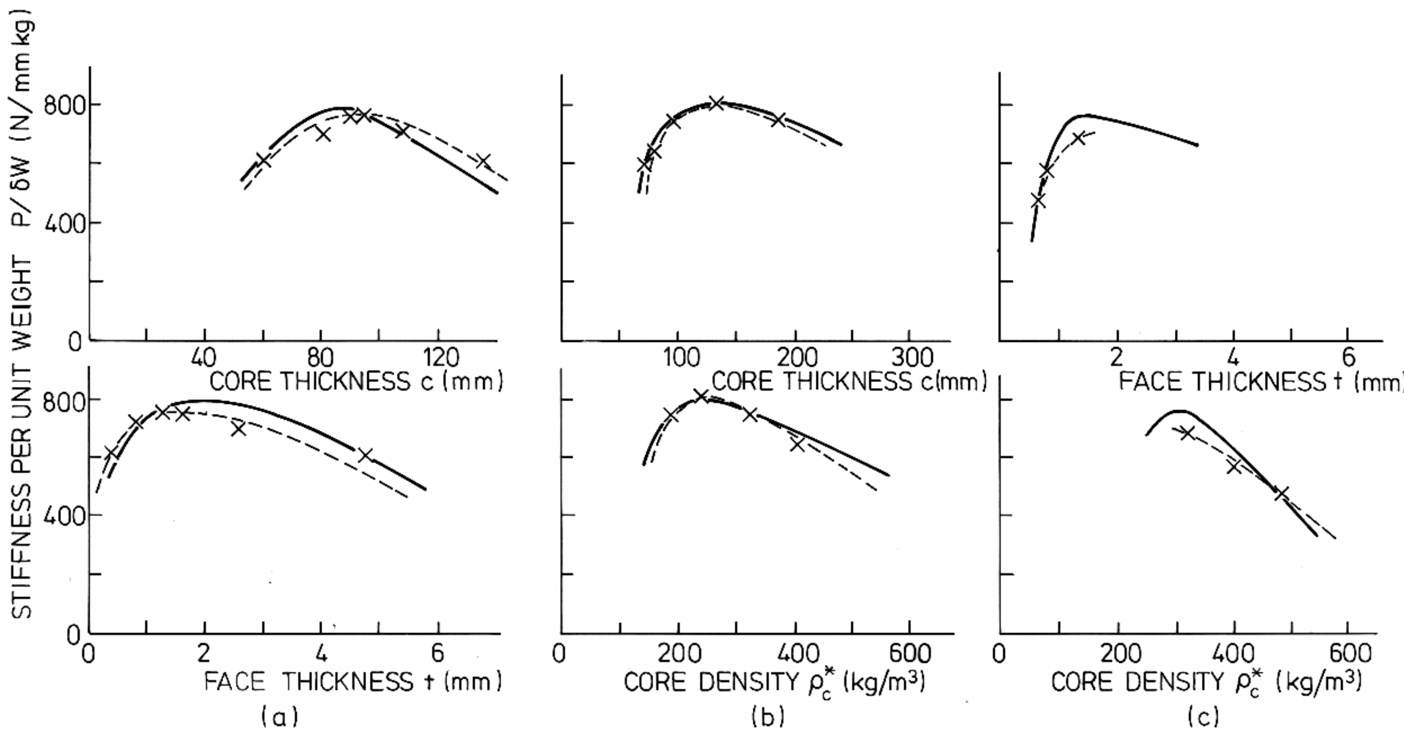
normal stresses

$$\sigma_f = \frac{My}{(EI)_{eq}} E_f = M \frac{c}{2} \frac{2}{E_f b t c^2} E_f = \frac{M}{btc}$$

$$\sigma_c = \frac{My}{(EI)_{eq}} E_c^* = M \frac{c}{2} \frac{2}{E_f b t c^2} E_c^* = \frac{M}{btc} \frac{E_c^*}{E_f}$$

since $E_c^* \ll E_f$ $\quad \sigma_c \ll \sigma_f \Rightarrow$ faces carry almost all normal stress

Minimum Weight Design



Al faces; Rigid PU foam core

Figures 7, 8, 9: Gibson, L. J. "Optimization of Stiffness in Sandwich Beams with Rigid Foam Cores." *Material Science and Engineering* 67 (1984): 125-35. Courtesy of Elsevier. Used with permission.

- for beam loaded by a concentrated load, P (eg. 3 pt bend)

$M_{max} = \frac{Pl}{B_3}$ eg. 3 pt bend $B_3 = 4$; cantilever $B_3 = 1$

$$\boxed{\sigma_f = \frac{Pl}{B_3\, b\, t\, c}}$$

- shear stresses vary parabolically through the cross-section, but if

$E_f \gg E_c^*$ & $c \gg t$ $\tau_c = \frac{V}{bc}$ V = shear force at section of interest

$$\boxed{\tau_c = \frac{P}{B_4\, b\, c}}$$

$V_{max} = \frac{P}{B_4}$ (eg. 3 pt bend $B_4 = 2$)

Failure modes

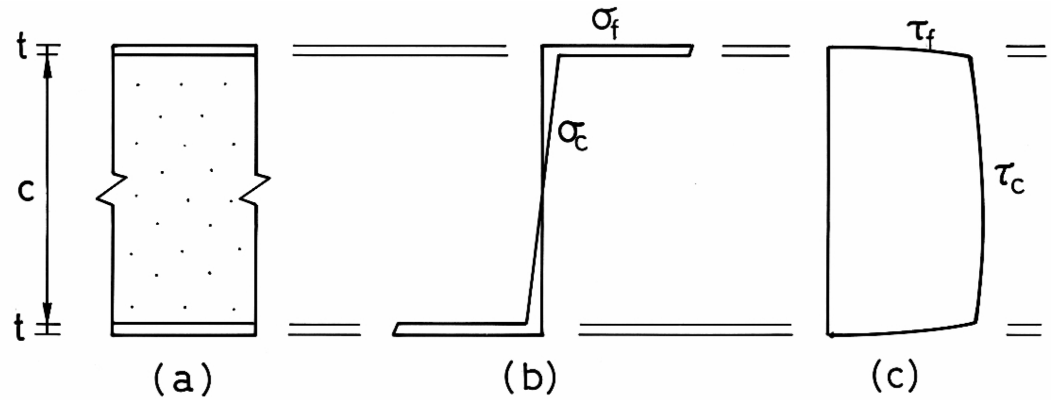
face : · can yield
· compressive face can buckle locally - "wrinkling"

core : can fail in shear

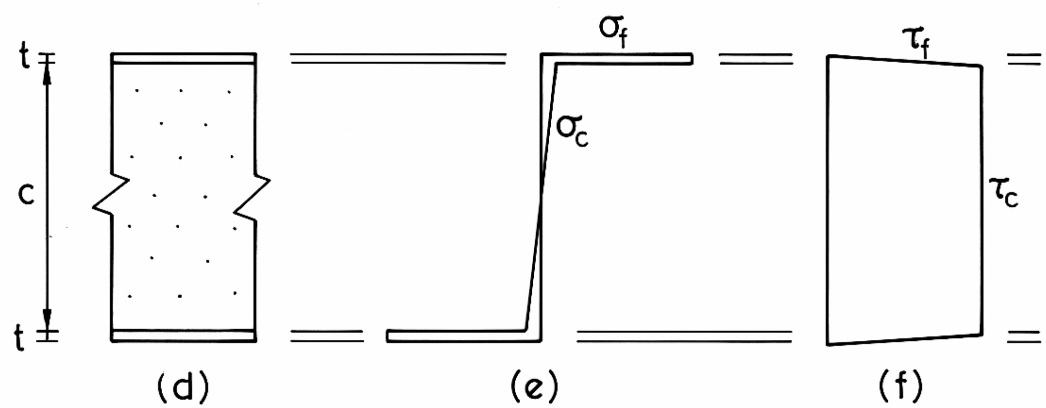
also : can have debonding + indentation

we will assume perfect bond + load distributed sufficiently to avoid indentation

Stresses

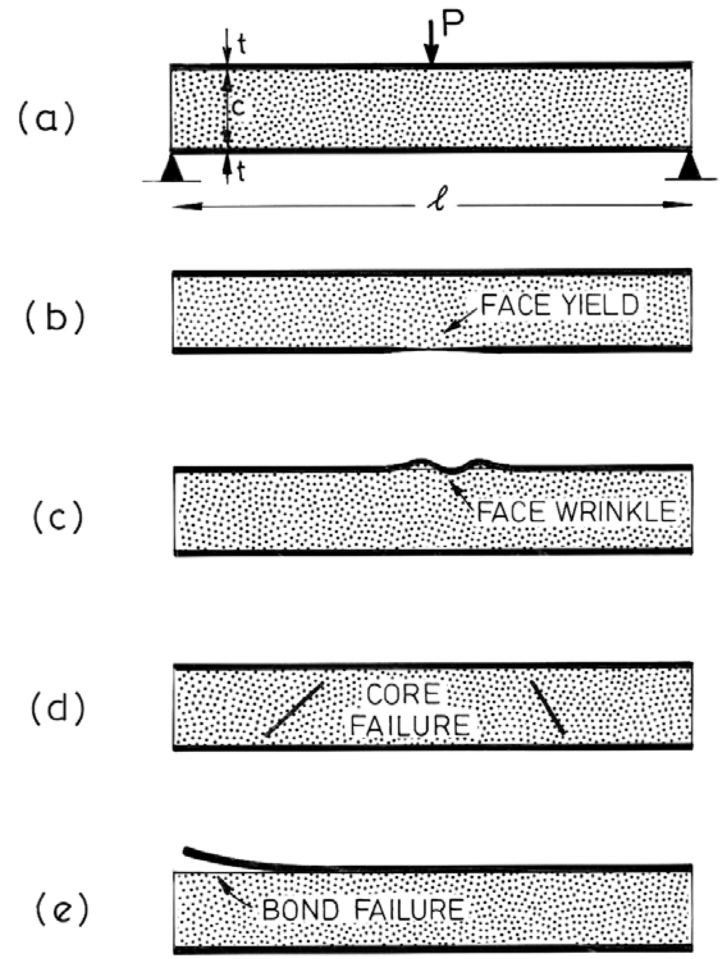


Face: Normal stress
Core: Shear stress



Approximate stress distributions, for: $E_c << E_f$ and $t << c$

Failure Modes



(a) Face yielding

$$\sigma_f = \frac{P\ell}{B_3 b t c} = \sigma_{yf}$$

(b) Face wrinkling : when normal stress in face = local buckling stress

$$\sigma_{buckling} = 0.57\, E_f^{1/3}\, E_c^{*2/3}$$ buckling on an elastic foundation

$$E_c^* = (\rho_c^*/\rho_s)^2 E_s$$

$$\sigma_{buckling} = 0.57\, E_f^{1/3}\, E_s^{2/3} (\rho_c^*/\rho_s)^{4/3}$$

---

wrinkling occurs when $\sigma_f = \frac{P\ell}{B_3 b t c} = 0.57\, E_f^{1/3}\, E_s^{2/3} (\rho_c^*/\rho_s)^{4/3}$

(c) Core shear failure

$$\tau_c = \tau_c^*$$

$$\frac{P}{B_4 b c} = C_{11} (\rho_c^*/\rho_s)^{3/2} \sigma_{ys} \qquad C_{11} \approx 0.15$$

- dominant failure load is the one that occurs at the lowest load
- how does the failure mode depend on the beam design?
- look at transition from one failure mode to another
- at the transition - two failure modes occur at same load

face yielding: $P_{fy} = B_3\, bc\,(t/\ell)\,\sigma_{yf}$

face winkling: $P_{fw} = 0.57\, B_3\, bc\,(t/\ell)\, E_f^{1/3}\, E_s^{2/3}\, (\rho^*_c/\rho_s)^{4/3}$

core shear : $P_{cs} = C_{11}\, B_4\, bc\, \sigma_{ys}\, (\rho^*_c/\rho_s)^{3/2}$

- face yielding + face wrinkling occur at same load if

$$B_3\, bc\,(t/\ell)\,\sigma_{yf} = 0.57\, B_3\, bc\,(t/\ell)\, E_f^{1/3}\, E_s^{2/3}\, (\rho^*_c/\rho_s)^{4/3}$$

or $$(\rho^*_c/\rho_s) = \left(\frac{\sigma_{yf}}{0.57\, E_f^{1/3}\, E_s^{2/3}}\right)^{3/4}$$

i.e. for given face + core materials, at constant ρ^*_c/ρ_s

- face yield - core shear $\frac{t}{l} = \frac{C_{11} B_4}{B_3} \left(\frac{\rho_c^*}{\rho_s}\right)^{3/2} \left(\frac{\sigma_{ys}}{\sigma_{yf}}\right)$

- face wrinkling - core shear $\frac{t}{l} = \frac{C_{11} B_4}{0.57 B_3} \frac{\sigma_{ys}}{E_f^{1/3} E_s^{2/3}} \left(\frac{\rho_c^*}{\rho_s}\right)^{1/6}$

- note: transition eq'n only involve constants (C_{11} B_3 B_4), material properties (E_f, E_s, σ_{ys}) & t/l, ρ_c^*/ρ_s; do not involve core thickness, c
- can plot transition eqn on plot with axes ρ_c^*/ρ_s & t/l

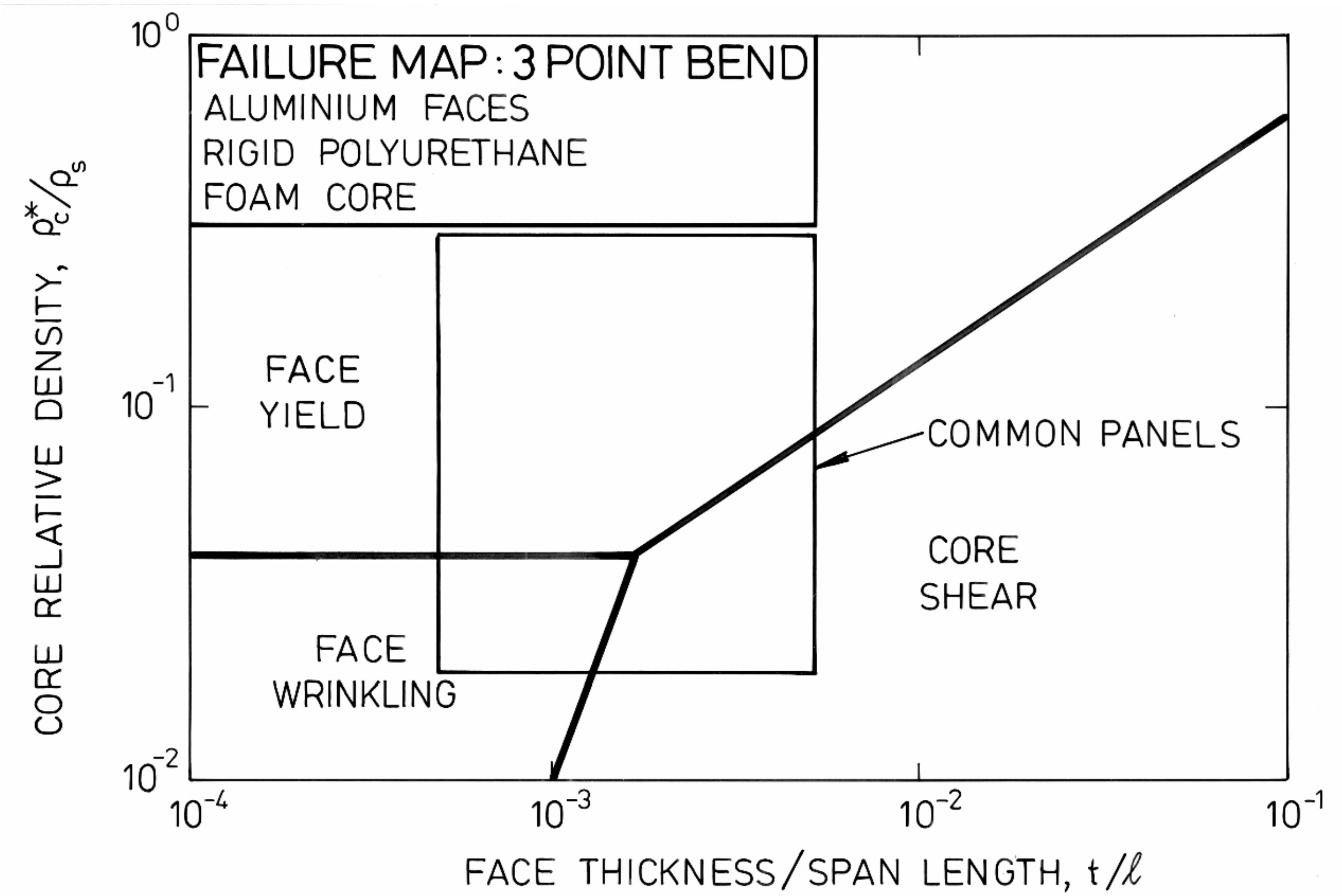
- values of axes chosen to represent realistic values of

  ρ_c^*/ρ_s - typically 0.02 to 0.3

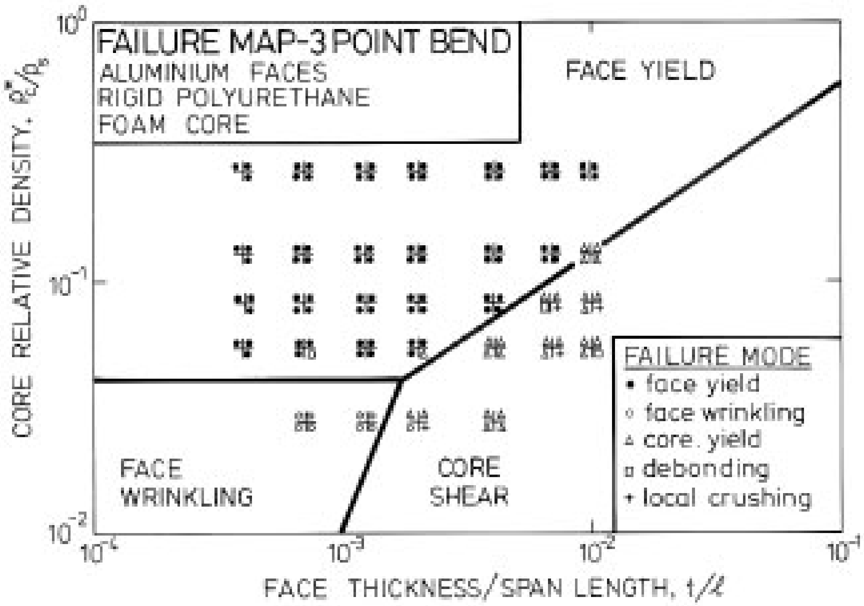
  t/l - " $1/2000$ to $1/200$ = 0.0005 to 0.005

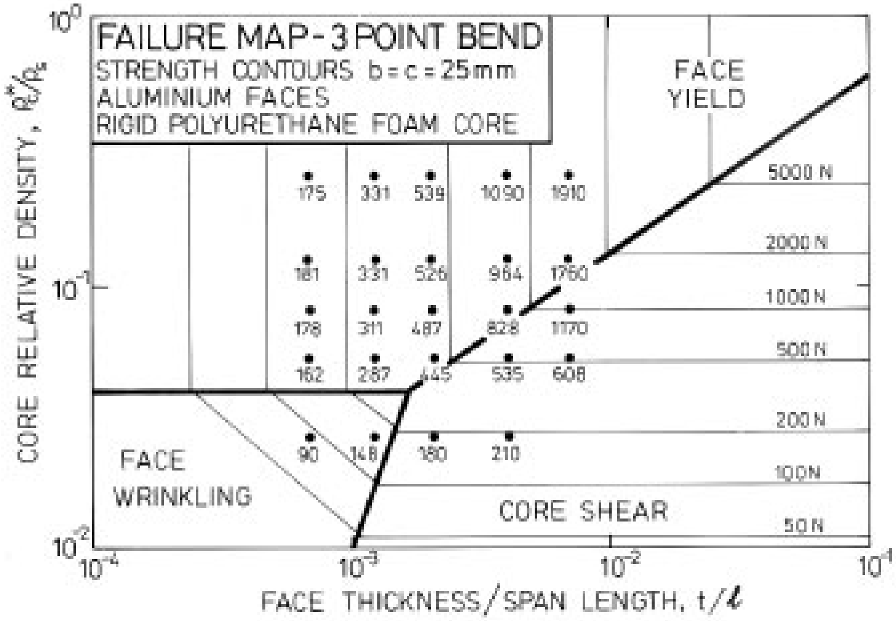
- low values of t/l + ρ_c^*/ρ_s ⇒ face wrinkling
  - t thin & core stiffness, which acts as elastic fndtn, low
- low values t/l, higher values ρ_c^*/ρ_s ⇒ transition to face yielding
- higher values of t/l ⇒ transition to core failure

Failure Mode Map



Failure Map: Expts





Figures 12 and 13: Triantafillou, T. C., and L. J. Gibson. "Failure Mode Maps for Foam Core Sandwich Beams." *Materials Science and Engineering* 95 (1987): 37–53. Courtesy of Elsevier. Used with permission.

- map shown in figure for three point bending ($B_3 = 4$, $B_4 = 2$)
- changing loading config. moves boundaries a little, but overall, picture similar
- expts on sandwich beams with Al faces + rigid PU foam cores confirm eq'n
- if know b, c - can add contours of failure loads.

Minimum weight design for stiffness + strength: t_{opt}, c_{opt}

- given: stiffness P/δ

  strength P_o

  span l width b

  loading configuration (B_1 B_2 B_3 B_4)

  face material (ρ_f, σ_{yf}, E_f)

  core material + density (ρ_s, E_s, σ_{ys}, ρ_c^*)

FIND: face + core thickness, t, c, to minimize weight.

- can obtain solution graphically, axes t/ℓ + c/ℓ
- eq'n for stiffness constraint + each failure mode plotted
- dashed lines - contours of weight
- design limiting constraints are stiffness + face yielding
- optimum point - where they intersect
- could add ρ_c^*/ρ_s as variable on third axis + create surfaces for stiffness + failure eq'n; find optimum in same way

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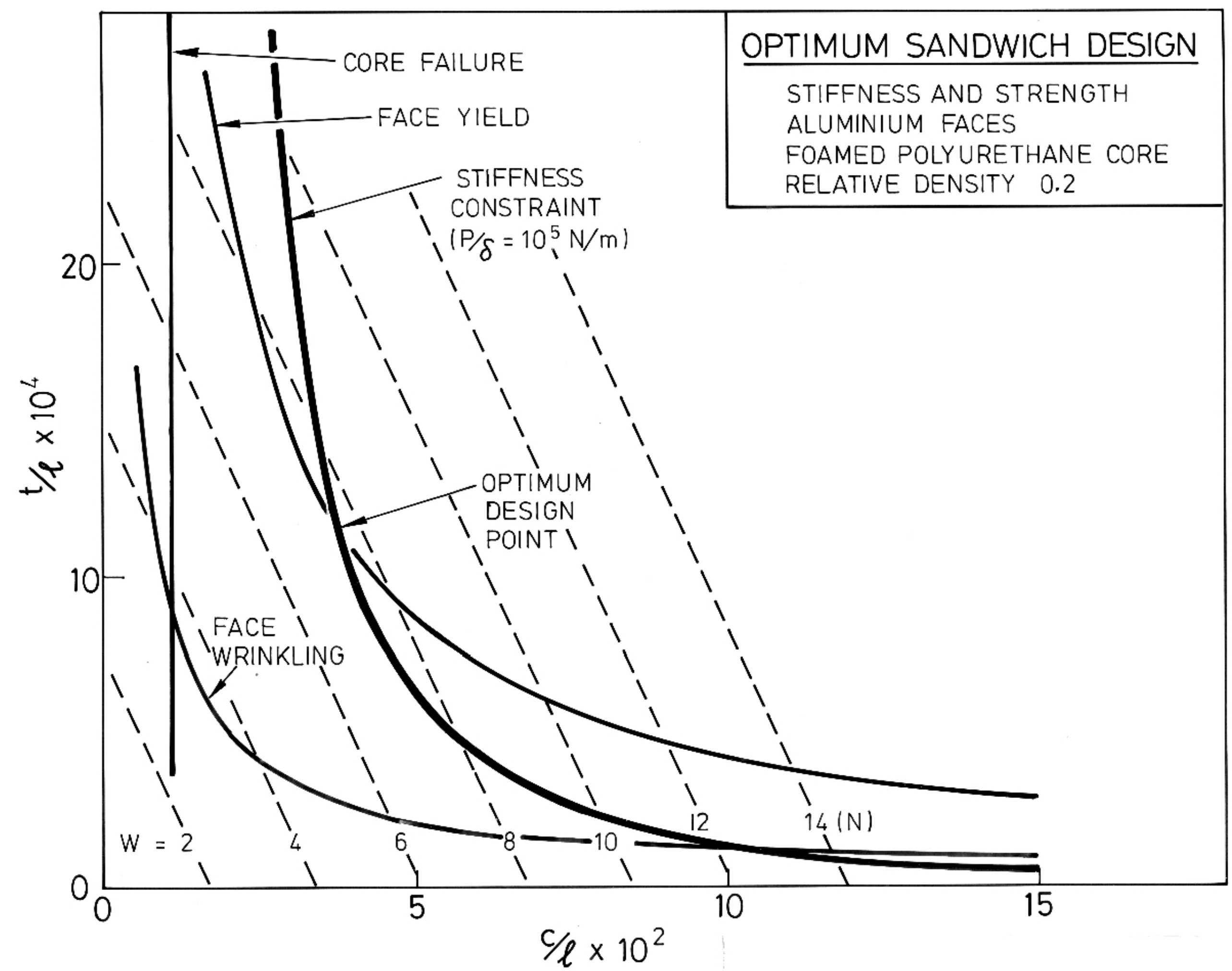
- analytical sol$^{\underline{n}}$ possible but cumbersome
- also, values of c/ℓ obtained this way may be unreasonably large - then have to introduce an additional constraint on c/ℓ (e.g. $c/\ell < 0.1$)



OPTIMUM SANDWICH DESIGN
STIFFNESS AND STRENGTH
ALUMINIUM FACES
FOAMED POLYURETHANE CORE
RELATIVE DENSITY 0.2
CORE FAILURE
FACE YIELD
STIFFNESS CONSTRAINT
(P/δ = 10^5 N/m)
OPTIMUM DESIGN POINT
FACE WRINKLING
W = 2
4
6
8
10
12
14 (N)
t/ℓ x 10^4
c/ℓ x 10^2
0
5
10
15
20

Minimum weight design: materials

- what are best materials for face + core? (stiffness constraint)
- go back to min. wt. design for stiffness
- can substitute $(\rho_c^*)_{opt}$, t_{opt}, c_{opt} into weight eq'n to get min. wt.:

$$W = 3.18\, b l^2 \left[\frac{1}{B_1 B_2^2 C_2^2} \; \frac{\rho_f \rho_s^4}{E_f E_s^2} \left(\frac{P}{\delta b}\right)^3 \right]^{1/5}$$

- faces: W minimized with materials that minimize $\frac{\rho_f}{E_f}$ (or maximize E_f/ρ_f)

- core: W " " " " " ρ_s^4/E_s^2 or max. $E_s^{1/2}/\rho_s$
- note: · faces of sandwich loaded by normal stress, axially

  if have solid material loaded axially, want to maximize E/ρ

  · core loaded in shear + in the foam, cell edges bend

  if have solid material, loaded as beam + in bending + want to minimize weight for a given stiffness, maximize $E^{1/2}/\rho$
- sandwich panels may have face + core same material eg. Al faces Al foam core. integral polymer face + core "structural polymer foams"
  - then want to maximize $E^{3/5}/\rho$

Case study: Downhill ski design

- stiffness of ski gives skier right "feel"
- too flexible - difficult to control
- too stiff - skier suspended, as on a plank, between bumps
- skis designed primarily for stiffness

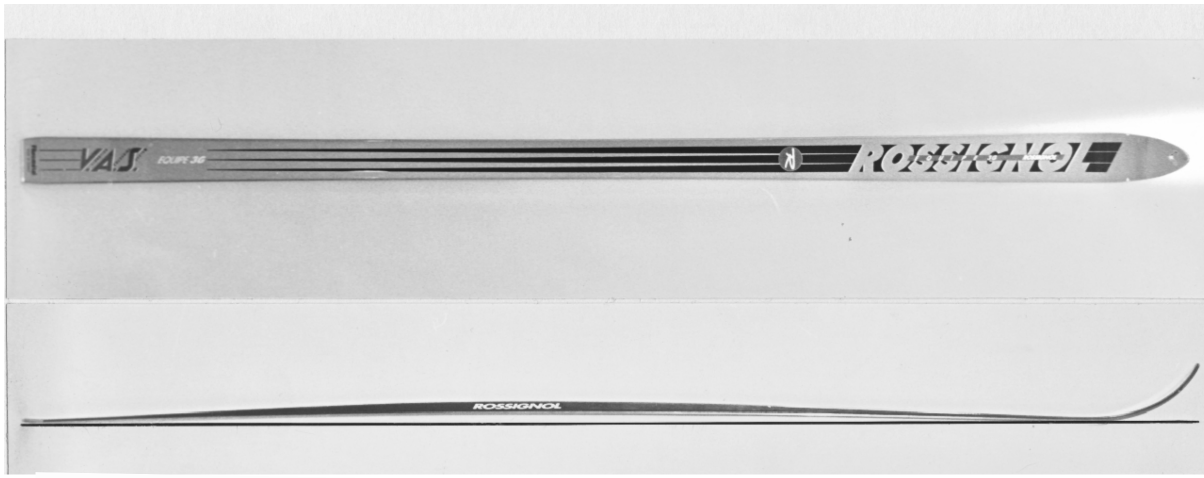
- originally skis made from a single piece of wood
- next - laminated wood skis with denser wood (ash, hickory) on top of lighter wood core (pine, spruce)

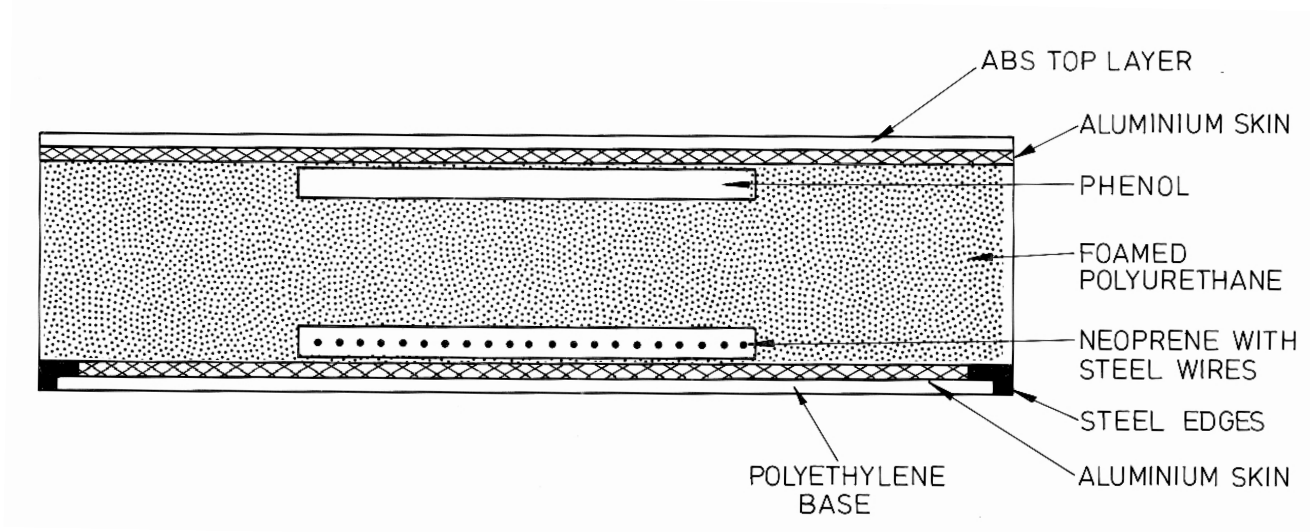
- modern skis - sandwich beams
  - faces - fiber composites or Al
  - core - honeycombs, foams (eg. rigid PU), balsa

  ] controls stiffness.

- additional materials
  - bottom - layer of polyethylene - reduces friction
  - short strip phenol - screw binding in
  - neoprene strip ~ 300 mm long - damping
  - steel edges - better control

Ski Case Study





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Ski case study

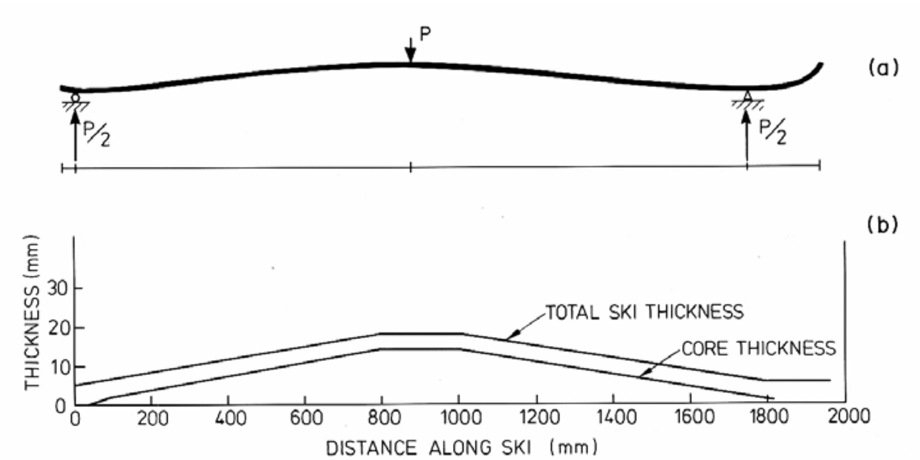
· properties of face + core materials

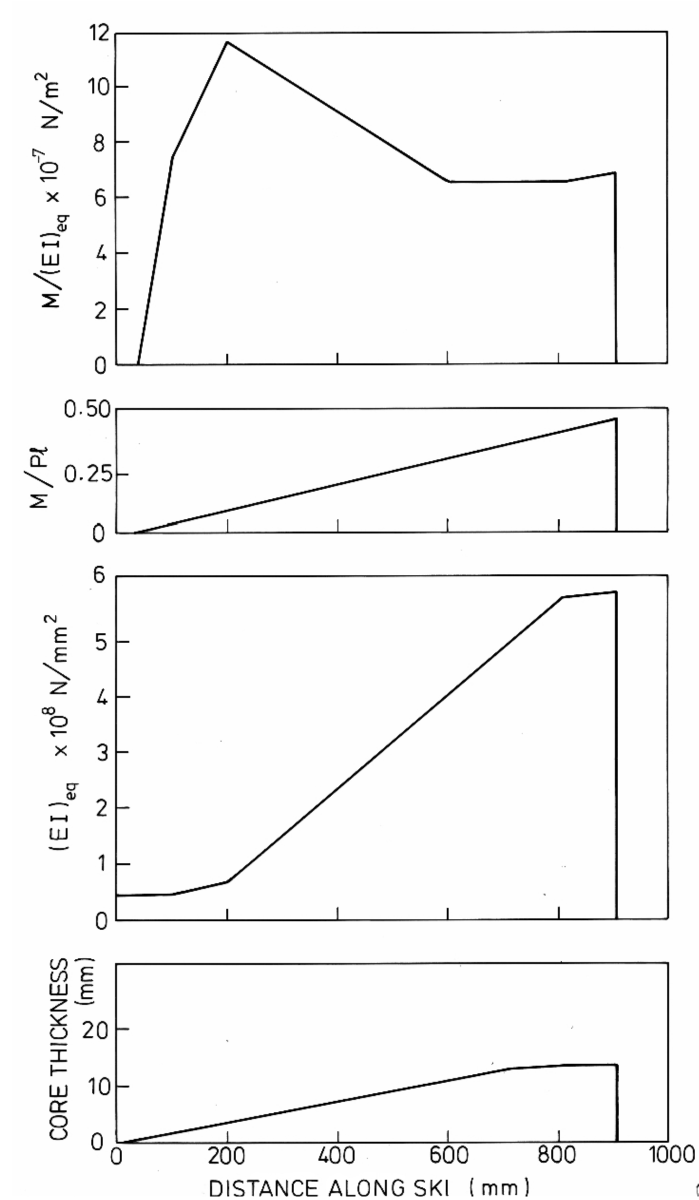
	Al	Sold Pu	Foam Pu
ρ (Mg/m³)	2.7	1.2	0.53
E (GPa)	70	1.94	0.38
G (GPa)	-	-	0.14

· ski geometry

- Al faces constant thickness t
- Pu foam core – c varies along length, thickest at centre, where moment highest
- ski cambered
- mass of ski = 1.3 kg (central 1.7 m, neglecting tip + tail)

Ski Case Study





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Bending stiffness

- plot c vs. x, distance along ski
- calculated $(EI)_{eq}$ vs x
- calculated Moment applied vs x
- get $M/(EI)_{eq}$ vs x
- can then find bending deflection, $\delta_b = 0.28P$
- shear deflection found from avg. equiv. shear rigidity
  $\delta_s = \frac{Pl}{(AG)_{eq}} = 0.0045P$

- $\delta = \delta_b + \delta_s = 0.29P$ $P/\delta = 3.5$ N/mm measured $P/\delta = 3.5$ N/mm.

- note current design $\delta_s \ll \delta_b$; at optimum $\delta_s \sim 2\delta_b$ (constant c)
- can ski be redesigned to give same stiffness, P/δ, at lower weight?
- if use optimization method described earlier (assuming c = constant along length)

$c_{opt} = 70$ mm mass = 0.31 kg ⇒ 75% reduction from current design

$t_{opt} = 0.095$ mm

$\rho^*_{c\,opt} = 29$ kg/m³

<u>But</u> this design impractical
⇒ c too large, t too small

Alternative approach:

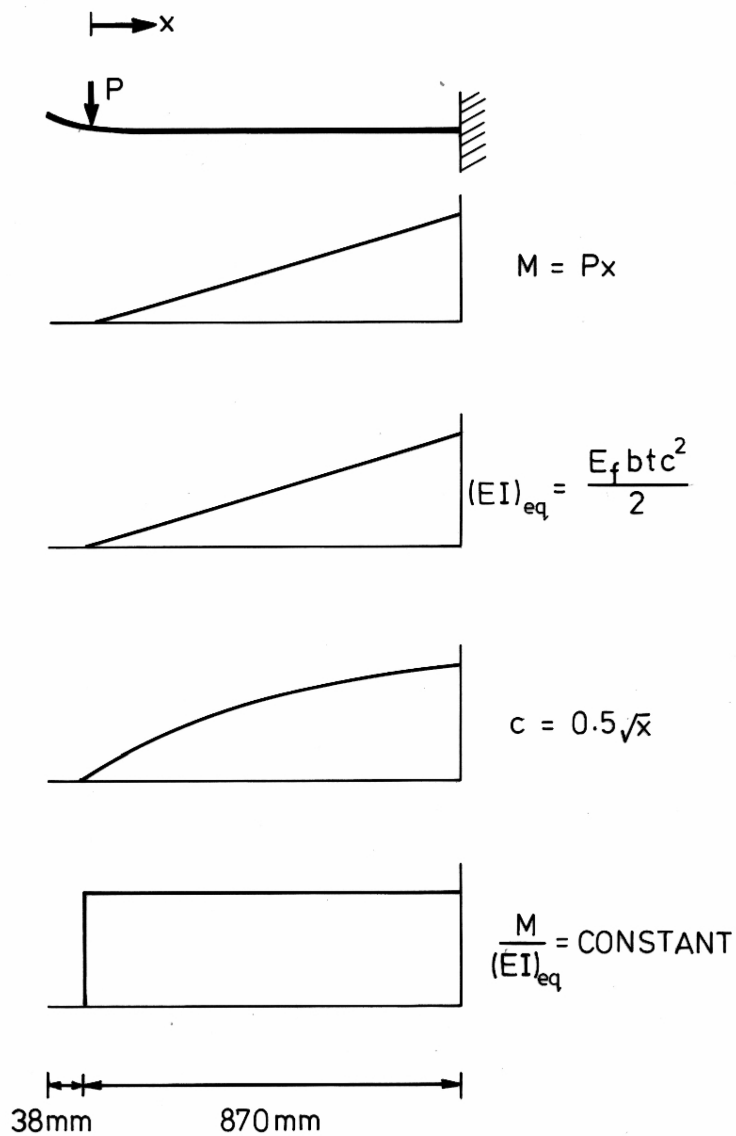
- fix c = max. value practical under binding & profile c to give constant $M/(EI)_{eq}$ along length of ski (use $c_{max} = 15$ mm)
- find values of t, ρ_c^* to minimize wt. for $P/\delta = 3.5$ N/mm.
- Moment M varies linearly along the length of the ski
- want $(EI)_{eq}$ to vary linearly, too; $(EI)_{eq} = E_f b t c^2/2$
- want $c \propto \sqrt{x}$, distance along length of ski

- half length of ski is 870 mm & $c_{max} = 15$ mm

$$c = 15\left(\frac{x}{870}\right)^{1/2} = 0.51\, x^{1/2} \quad \text{(mm)}$$

- can now do minimum weight analysis with

$$\delta = \frac{P l^3\, 2}{B_1 E_f b t (c_{max}+t)^2} + \frac{P l}{B_2 C_2 b c_{max} (\rho_c^*/\rho_s)^2 E_s}$$



x
P
M = Px
$(EI)_{eq} = \frac{E_f btc^2}{2}$
$c = 0.5\sqrt{x}$
$\frac{M}{(EI)_{eq}}$ = CONSTANT
38mm
870mm

- B_1 - corresponds to beam with constant M/EI
- B_2 - cantilever value ($B_2 = 1$) multiplied by avg. value of c divided by maximum value of c $\quad B_2 = 2/3$
- solve stiffness eq'n for ρ_c^*, substitute into weight eqn + take $\frac{\partial W}{\partial t} = 0$
- solve for t_{opt}, then $\rho^*_{c\,opt}$
- find: $\quad c_{max} = 15\text{ mm} \qquad \rho^*_{c\,opt} = 163\text{ kg/m}^3$

  $t_{opt} = 1.03\text{ mm} \qquad$ mass $= 0.88\text{ kg} \Rightarrow 31\%$ less than current design

---

Daedalus

- MIT designed + built human powered aircraft (1980s)
- flew 72 miles in ~4 hrs. from Crete to Santorini, 1988
- Kanellos Kanellopoulos - Greek bicycle champion pedalled + flew

mass	68.5 #	= 31 kg
length	29'	= 8.8 m
wingspan	112'	= 34 m

propeller: kevlar faces, PS foam core (11' long)

wing + trailing edge strips kevlar faces/ rohacell foam core

tail surface struts: carbon composite faces, balsa core

Daedalus

Courtesy of NASA. Image is in the public domain. NASA Dryden Flight Research Center Photo Collection.

Mass = 31 kg

Length = 8.8m

Wingspan = 34m

Propeller blades = 3.4m

Flew 72 miles, from Crete to Santorin, in just under 4 hours

Sandwich panels: propeller, wing and tail trailing edge strips, tail surface struts

Image: MIT Archives

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2014