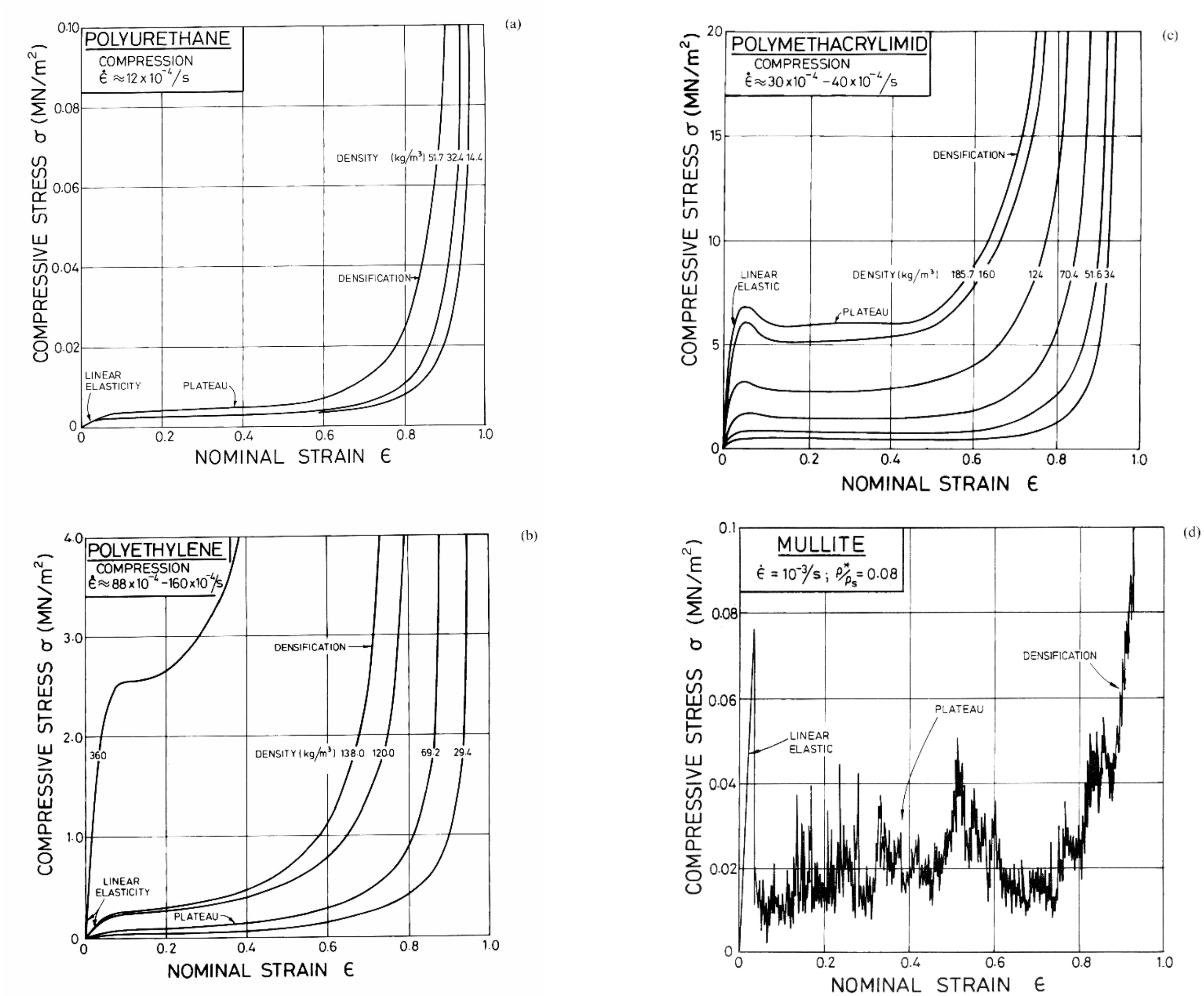
# Lecture 7, Foams, 3.054

## Open-cell foams

- Stress-Strain curve: deformation and failure mechanisms
- Compression - 3 regimes - linear elastic - bending
  - stress plateau - cell collapse by buckling
    yielding
    crushing
  - densification
- Tension - no buckling
  - yielding can occur
  - brittle fracture

## Linear elastic behavior

- Initial linear elasticity - bending of cell edges (small $t/l$)
- As $t/l$ goes up, axial deformation becomes more significant
- Consider dimensional argument, which models mechanism of deformation and failure, but not cell geometry
- Consider cubic cell, square cross-section members of area $t^2$, length $l$

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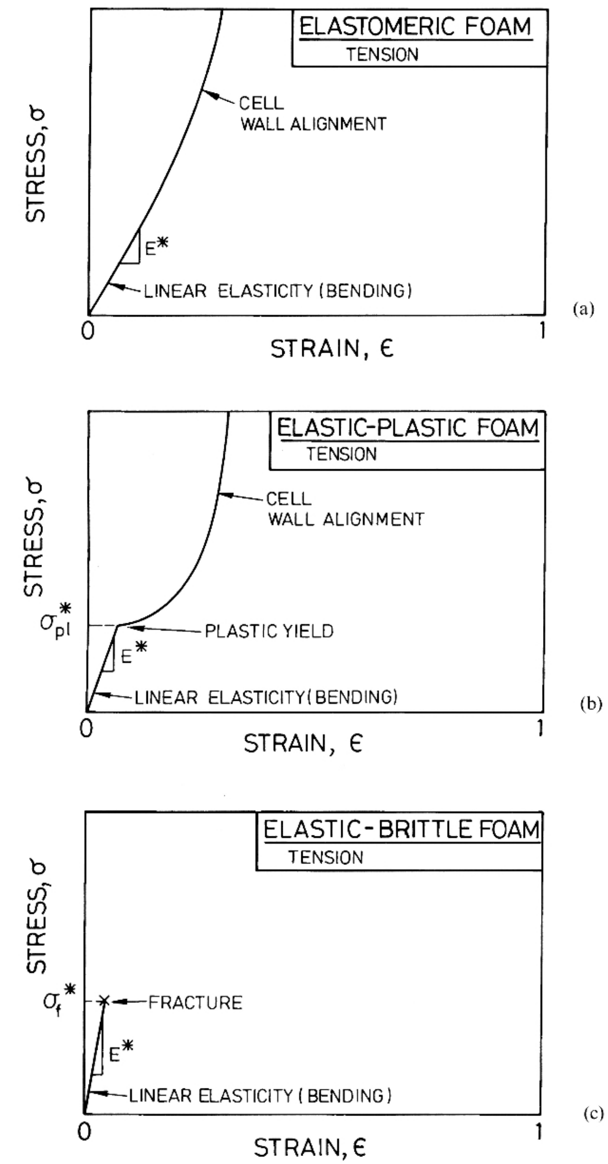


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# Foams: Bending, Buckling

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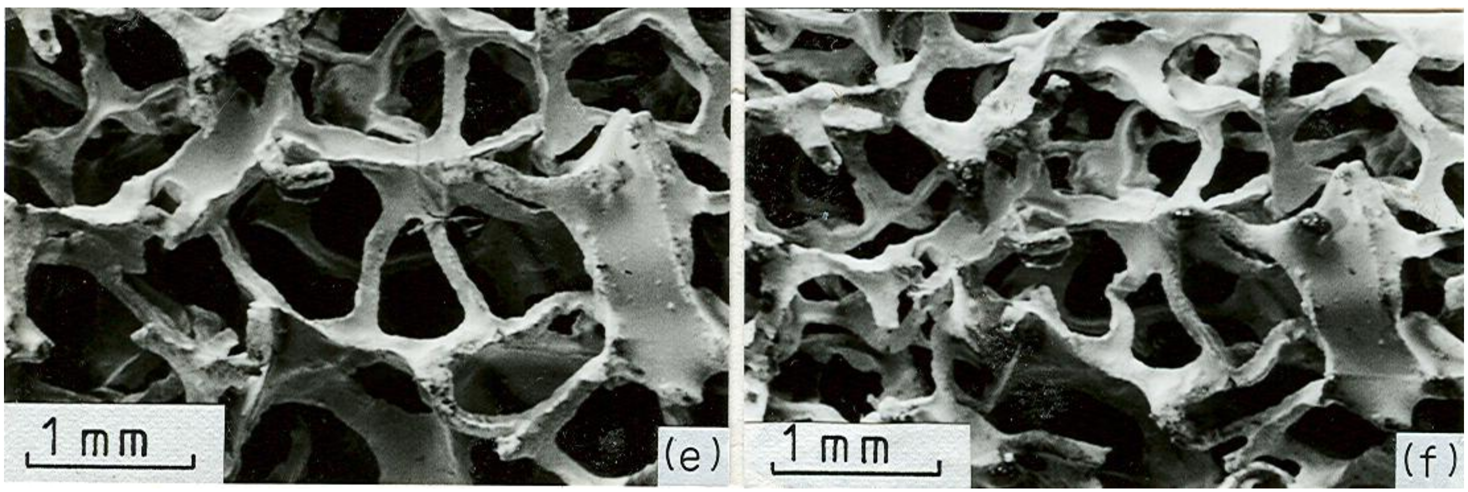
# Foams: Plastic Hinges



# Foams: Cell Wall Fracture

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- Regardless of specific geometry chosen:

$$\rho^*/\rho_s \propto (t/l)^2 \qquad I \propto t^4$$

$$\sigma \propto F/l^2 \qquad \epsilon \propto \delta/l \qquad \delta \propto \frac{Fl^3}{E_S I}$$

$$E^* \propto \frac{\sigma}{\epsilon} \propto \frac{F}{l^2}\,\frac{l}{\delta} \propto \frac{F}{l}\,\frac{E_s t^4}{Fl^3} \propto E_s\left(\frac{t}{l}\right)^4 \propto E_s\left(\frac{\rho^*}{\rho_s}\right)^2$$

$\boxed{E^* = C_1 E_s (\rho^*/\rho_s)^2}$ $C_1$ includes all geometrical constants
Data: $C_1 \approx 1$

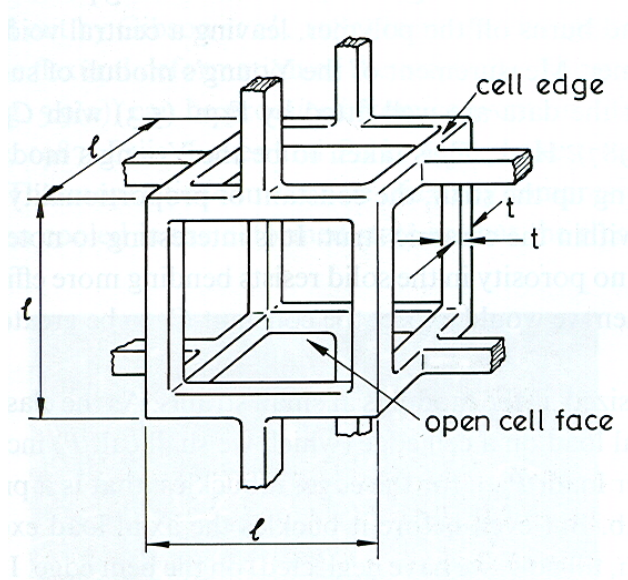
- Data suggests $C_1 = 1$
- Analysis of open cell tetrakaidecahedral cells with Plateau borders gives $C_1 = 0.98$
- Shear modulus $\boxed{G^* = C_2 E_s (\rho^*/\rho_s)^2}$ $C_2 \sim 3/8$ if foam is isotropic

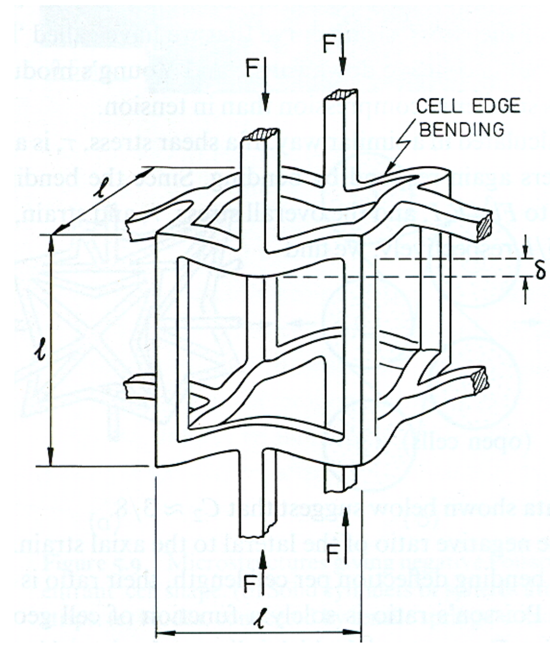
  Isotropy: $G = \dfrac{E}{2(1+\nu)}$

- Poisson's ratio: $\nu^* = \frac{E}{2G} - 1 = \frac{C_1}{2C_2} - 1 =$ constant, independent of $E_s$, $t/l$

  $\boxed{\nu^* = C_3}$ (analogous to honeycombs in-plane)

# Foam: Edge Bending

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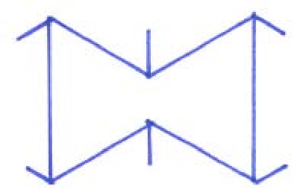
## Poisson's ratio

- Can make negative Poisson's ratio foams
- Invert cell angles (analogous to honeycomb)
- Eg. thermoplastic foams - load hydrostatically

  and heat to $T > T_g$, then cool and release load

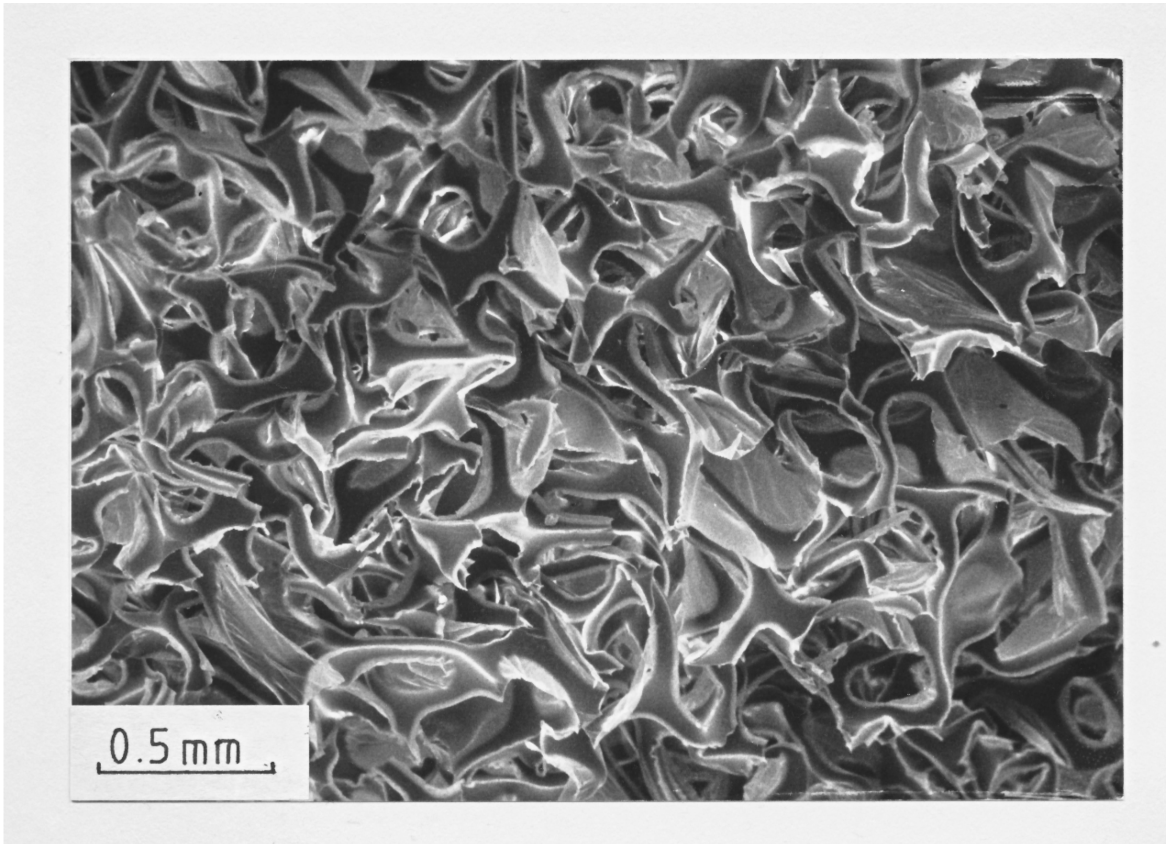
  so that edges of cell permanently point inward



## Closed-cell foams

- Edge bending as for open cell foams
- Also: face stretching and gas compression
- Polymer foams: surface tension draws material to edges during processing
  - define $t_e, t_f$ in figure
- Apply $F$ to the cubic structure

# Negative Poisson's Ratio

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- External work done $\propto F\delta$.
- Internal work bending edges $\propto \frac{F_e}{\delta_e}\delta_e^2 \propto \frac{E_s I}{l^3}\delta^2$
- Internal work stretching faces $\propto \sigma_f\, \epsilon_f\, v_f \propto E_s\, \epsilon_f^2\, v_f \propto E_s\Big(\delta/l\Big)^2 t_f\, l^2$

$$\therefore \quad F\delta = \alpha\, \frac{E_s t_e^4}{l^3}\delta^2 + \beta\, E_s\left(\frac{\delta}{l}\right)^2 t_f l^2$$

$$E^* \propto \frac{F}{l^2}\,\frac{l}{\delta} \rightarrow \quad F \propto E^*\,\delta l$$

$$\therefore \quad E^*\delta^2\, l = \alpha\, \frac{E_s t_e^4}{l^3}\delta^2 + \beta\, E_s\left(\frac{\delta}{l}\right)^2 t_f l^2$$

$$E^* = \alpha E_s\left(\frac{t_e}{l}\right)^4 + \beta E_s\left(\frac{t_f}{l}\right)$$

Note: Open cells, uniform $t$:
$\rho^*/\rho_s \propto (t/l)^2$

Closed cells, uniform $t$:
$\rho^*/\rho_s \propto (t/l)$

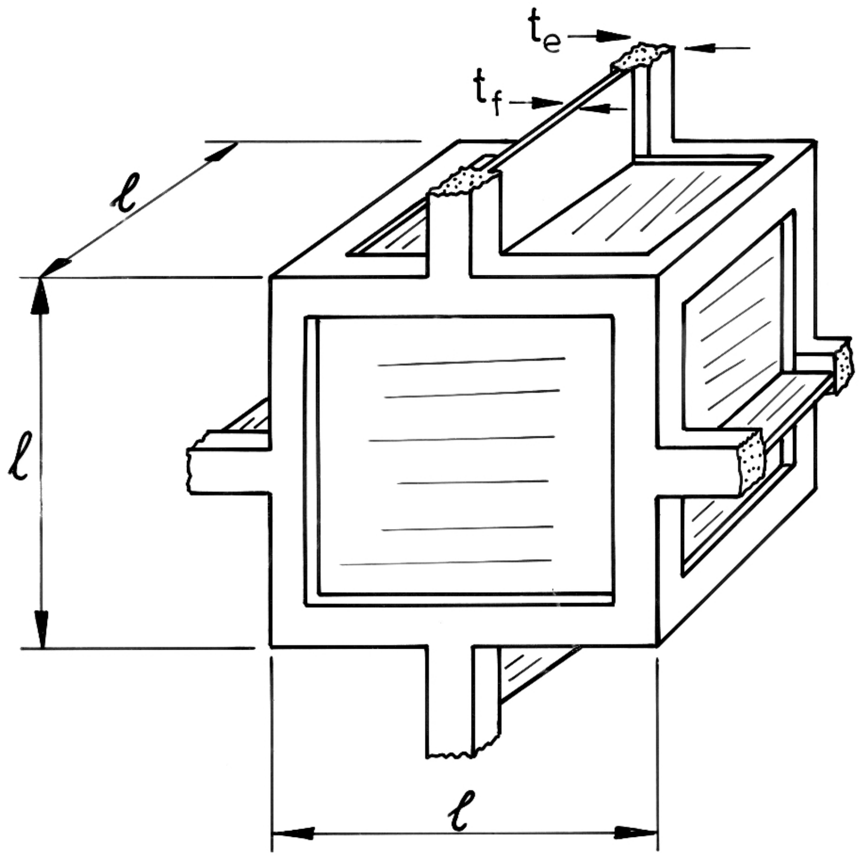
If $\phi$ is volume fraction of solid in cell edges:

$$t_e/l = C\phi^{1/2}(\rho^*/\rho_s)^{1/2}$$

$$t_f/l = C'(1-\phi)\,(\rho^*/\rho_s)$$

$$\boxed{\frac{E^*}{E_s} = C_1\phi^2\Big(\rho^*/\rho_s\Big)^2 + C_1'(1-\phi)\rho^*/\rho_s}$$

# Closed-Cell Foam

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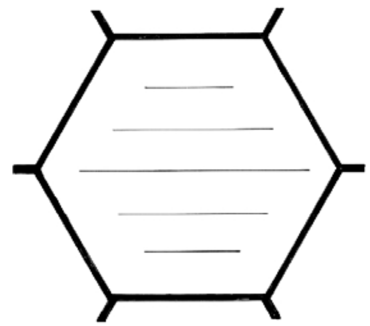
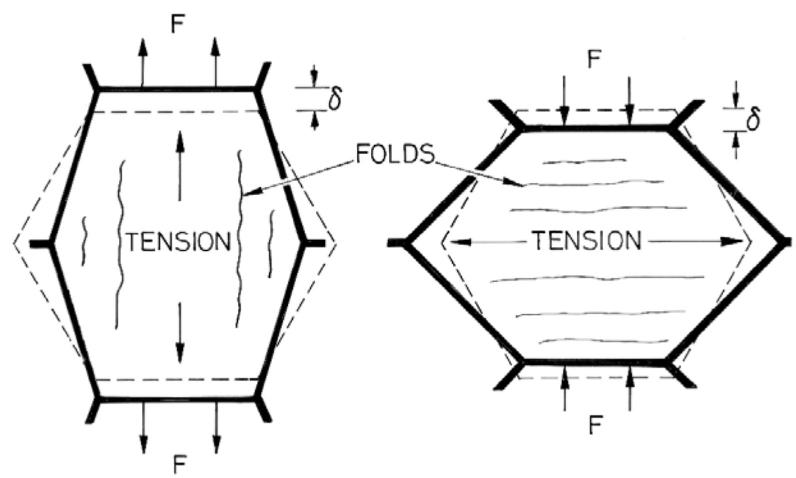
# Cell Membrane Stretching



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Closed cell foams - gas within cell may also contribute to $E^*$

- Cubic element of foam of volume $V_0$
- Under uniaxial stress, axial strain in direction of stress is $\epsilon$
- Deformed volume V is:

$$\frac{V}{V_0} = 1 - \epsilon(1 - 2\nu^*)$$

taking compressive strain as positive,

neglecting $\epsilon^2, \epsilon^3$ terms

$$\frac{V_g}{V_g^0} = \frac{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s}{1 - \rho^*/\rho_s}$$

$V_g$ = volume gas

$V_g^0$ = volume gas initially

- Boyle's law: $pV_g = p_0 V_g^0$

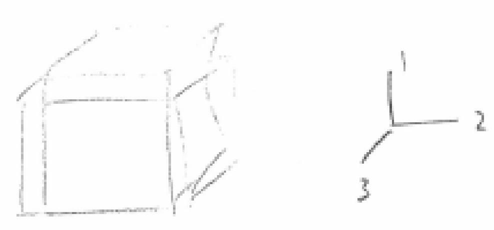
$p$ = pressure after strain $\epsilon$
$p_0$ = pressure initially

- Pressure that must be overcome is $p' = p - p_0$:

$$p' = \frac{p_0\, \epsilon\, (1 - 2\nu^*)}{1 - \epsilon\, (1 - 2\nu^*) - \rho^*/\rho_s}$$

- Contribution of gas compression to the modulus $E^*$:

$$E_g^* = \frac{dp'}{d\epsilon} = \frac{p_0(1 - 2\nu^*)}{1 - \rho^*/\rho_s}$$




$$V_0 = l_0^3 \qquad \epsilon_1 = \frac{l_1 - l_0}{l_0} \quad \rightarrow \quad l_1 = l_0 + \epsilon_1 l_0 = l_0(1+\epsilon_1)$$

$$V = l_1 l_2 l_3 \qquad \epsilon_2 = \frac{l_2 - l_0}{l_0} \quad \rightarrow \quad \begin{aligned} l_2 &= l_0 + \epsilon_2 l_0 \qquad \nu = -\frac{\epsilon_2}{\epsilon_1} \\ &= l_0 - \nu\epsilon_1 l_0 \quad \epsilon_2 = -\nu\epsilon_1 \\ &= l_0(1-\nu\epsilon_1) \end{aligned}$$

$$\epsilon_3 = l_0(1-\nu\epsilon_1)$$

$$\begin{aligned} V &= l_1 l_2 l_3 = l_0(1+\epsilon_1) l_0(1-\nu\epsilon_1) l_0(1-\nu\epsilon_1) = l_0^3(1+\epsilon_1)(1-\nu\epsilon_1)^2 \\ \frac{V}{V_0} &= \frac{l_0^3(1+\epsilon)(1-\nu\epsilon)^2}{l_0^3} = (1+\epsilon)(1-2\nu\epsilon+\nu^2\epsilon^2) \\ &= (1-2\nu\epsilon+\nu^2\epsilon^2) + \epsilon - 2\nu\epsilon^2 + \nu^2\epsilon^3 \\ &= 1-\epsilon+2\nu\epsilon \\ &= 1-\epsilon(1-2\nu) \end{aligned}$$

$$
\begin{aligned}
p' &= p - p_0 \\
p &= \frac{p_0 V_g'}{V_g} \\
p' &= p - p_0 = \frac{p_0 V_g'}{V_g} - p_0 = p_0\Big(\frac{V_g^0}{V_g} - 1\Big) \\
&= p_0\Big[\frac{1 - \rho^*/\rho_s}{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s} - 1\Big] \\
&= p_0\Big[\frac{1 - \rho^*/\rho_s(1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s)}{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s}\Big] \\
&= p_0\Big[\frac{\epsilon(1 - 2\nu^*)}{1 - \epsilon(1 - 2\nu^*) - \rho^*/\rho_s}\Big]
\end{aligned}
$$

## Closed cell foam

$$\boxed{\begin{array}{cccccc} \dfrac{E^*}{E_s} = & \phi^2\left(\dfrac{\rho^*}{\rho_s}\right)^2 & + & (1-\phi)\left(\dfrac{\rho^*}{\rho_s}\right) & + & \dfrac{p_0(1-2\nu^*)}{E_s(1-\rho^*/\rho_s)} \\ & \downarrow & & \downarrow & & \downarrow \\ & \text{edge bending} & & \text{face stretching} & & \text{gas compression} \end{array}}$$

- Note: if $p_0 = p_{\text{atm}} = 0.1$ MPa, gas compression term is negligible,
  except for closed-cell elastomeric foams
- Gas compression can be significant if $p_0 >> p_{\text{atm}}$; also modifies shape of stress plateau
  in elastomeric closed-cell foams

Shear modulus: edge bending, face stretching; shear $\Delta V = 0$ gas contribution is 0

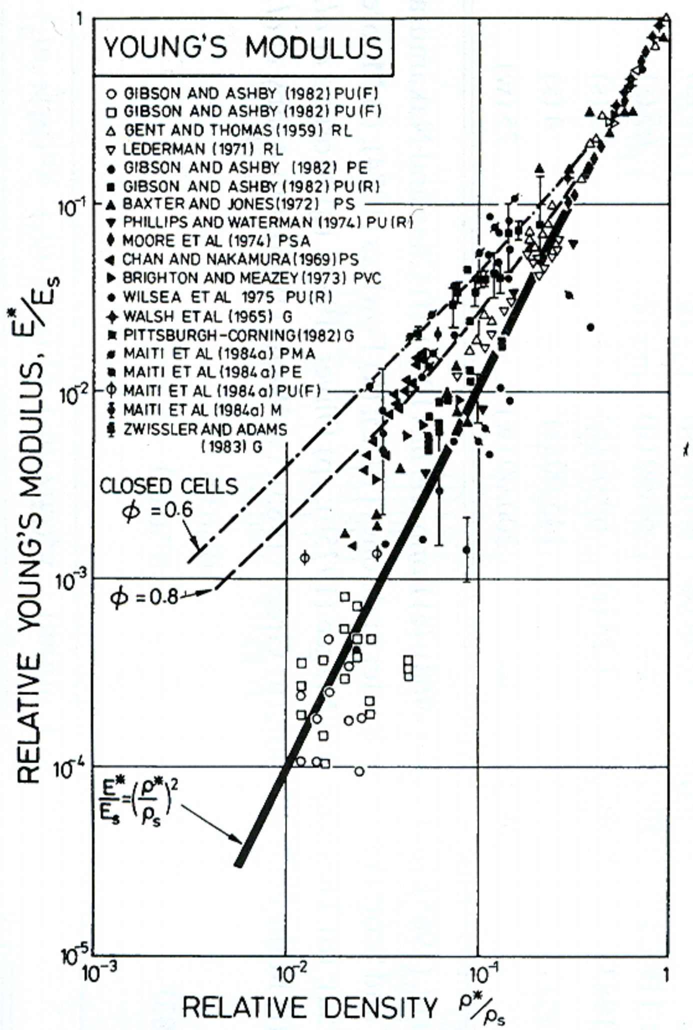
$$\frac{\epsilon^*}{E_s} = \frac{3}{8}\left[\phi^2\left(\frac{\rho^*}{\rho_s}\right)^2 + (1-\phi)\left(\frac{\rho^*}{\rho_s}\right)\right] \qquad \text{(isotropic foam)}$$

Poison's ratio = $f$ (cell geometry only) $\qquad \nu^* \approx 1/3$

**Comparison with data**

- Data for polymers, glasses, elastomers
- $E_s$, $\rho_s$ - Table 5.1 in the book
- Open cells — open symbols
- Closed cells — filled symbols

# Young's Modulus

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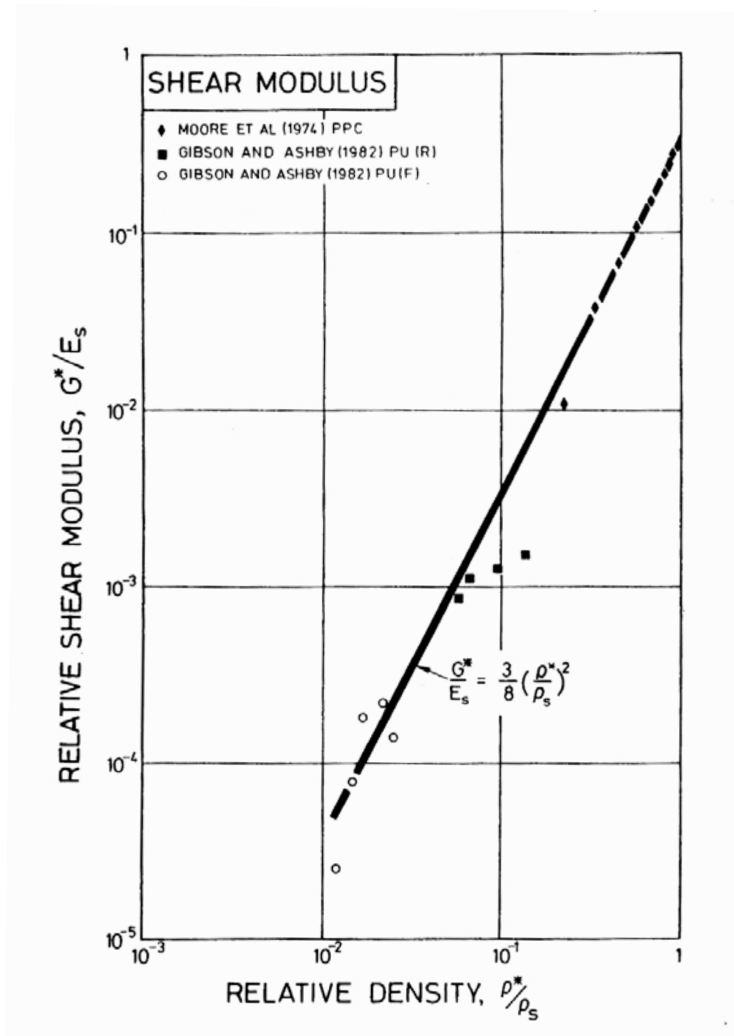
# Shear Modulus

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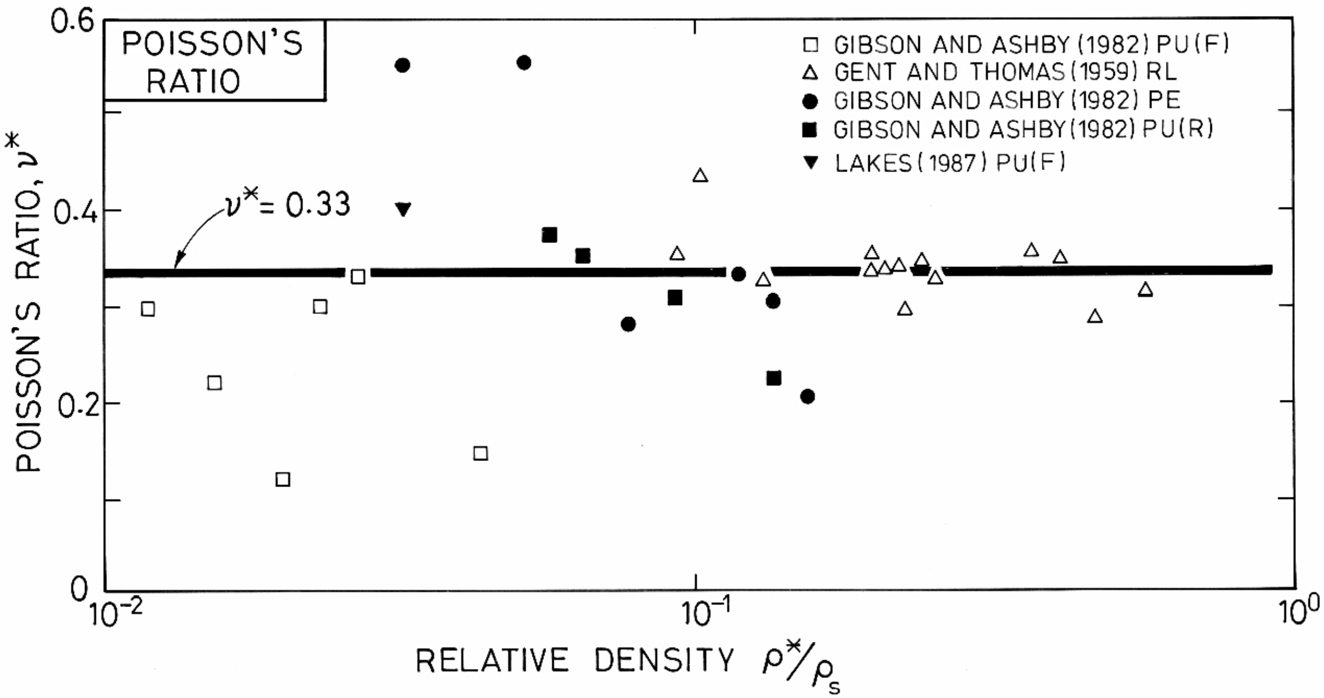
# Poisson's Ratio

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## Non-linear elasticity

### Open cells:

$$P_{\text{cr}} = \frac{n^2\pi^2 E_s I}{l^2}$$

$$\sigma^*_{\text{ei}} \propto \frac{P_{\text{cr}}}{l^2} \propto E_s\left(\frac{t}{l}\right)^4$$

$$\boxed{\sigma^*_{\text{el}} = C_4\, E_s\left(\frac{\rho^*}{\rho_s}\right)^2}$$

Data: $C_4 \approx 0.05$, corresponds to strain when buckling initiates, since $E^* = E_S\left(\frac{\rho^*}{\rho_s}\right)^2$

### Closed cells:

- $t_f$ often small compared to $t_e$ (surface tension in processing) - contribution small
- If $p_0 >> p_{\text{atm}}$, cell walls pre-tensioned, bucking stress has to overcome this

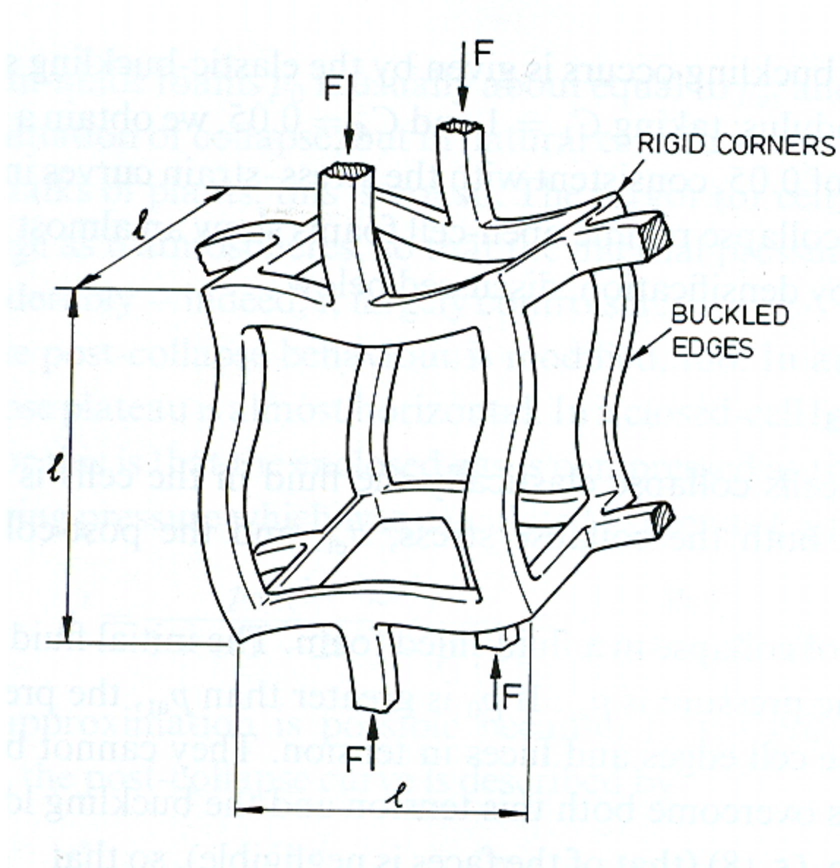
$$\sigma^*_{\text{el}} = 0.05 E_s\left(\tfrac{\rho^*}{\rho_s}\right)^2 + p_0 - p_{\text{atm}}$$

- Post-collapse behavior - stress plateau rises due to gas compression (if faces don't rupture) $\nu^* = 0$ in post-collapse regime

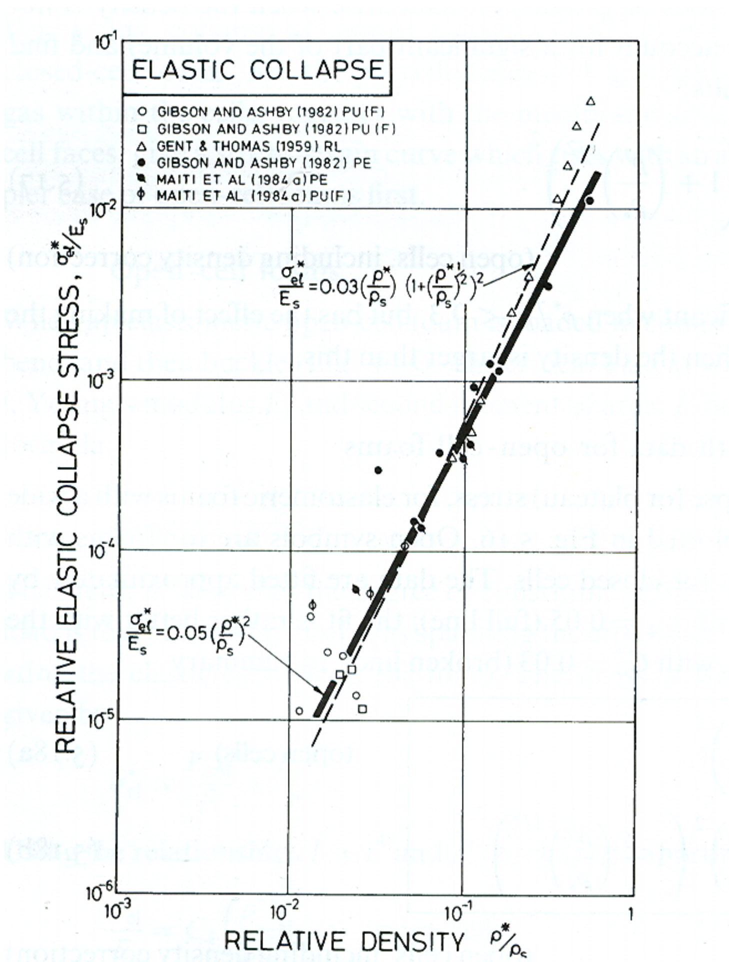
$$p' = \frac{p_0\,\epsilon(1-2\nu^*)}{1-\epsilon(1-2\nu^*) - \rho^*/\rho_s} = \frac{p_0\epsilon}{1-\epsilon-\rho^*/\rho_s}$$

$$\boxed{\sigma^*_{\text{post-collapse}} = 0.05\, E_s\left(\frac{\rho^*}{\rho_s}\right)^2 + \frac{p_0\epsilon}{1-\epsilon-\rho^*/\rho_s}}$$

# Elastic Collapse Stress



# Elastic Collapse Stress

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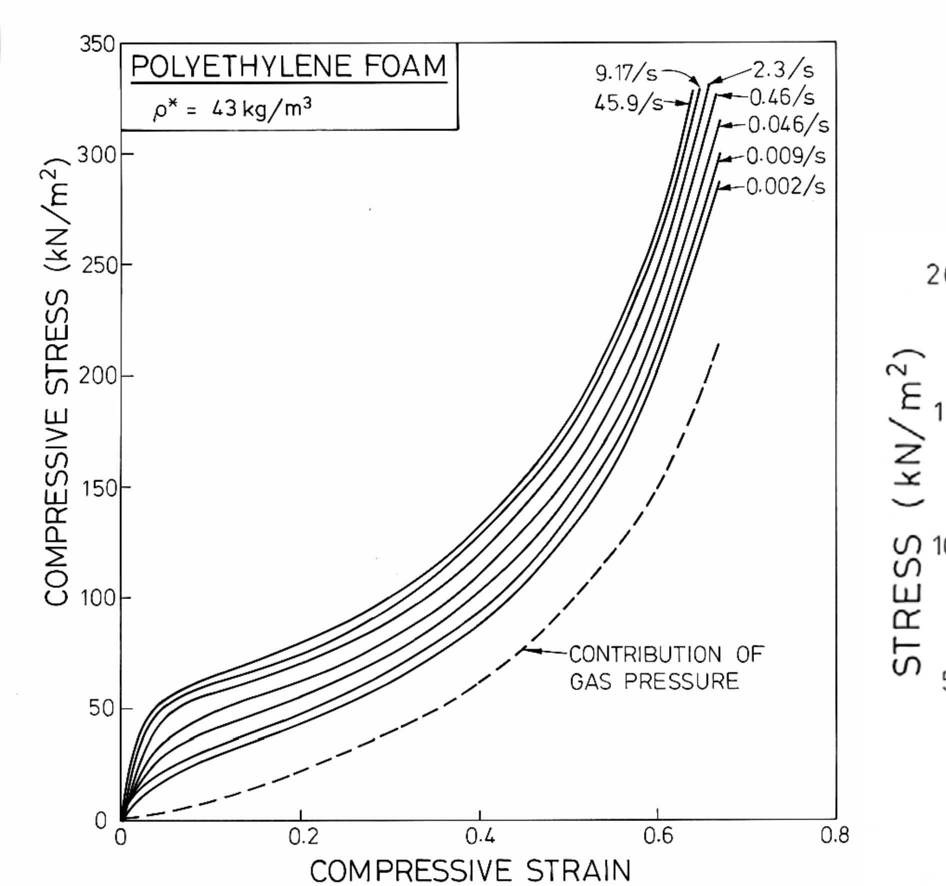
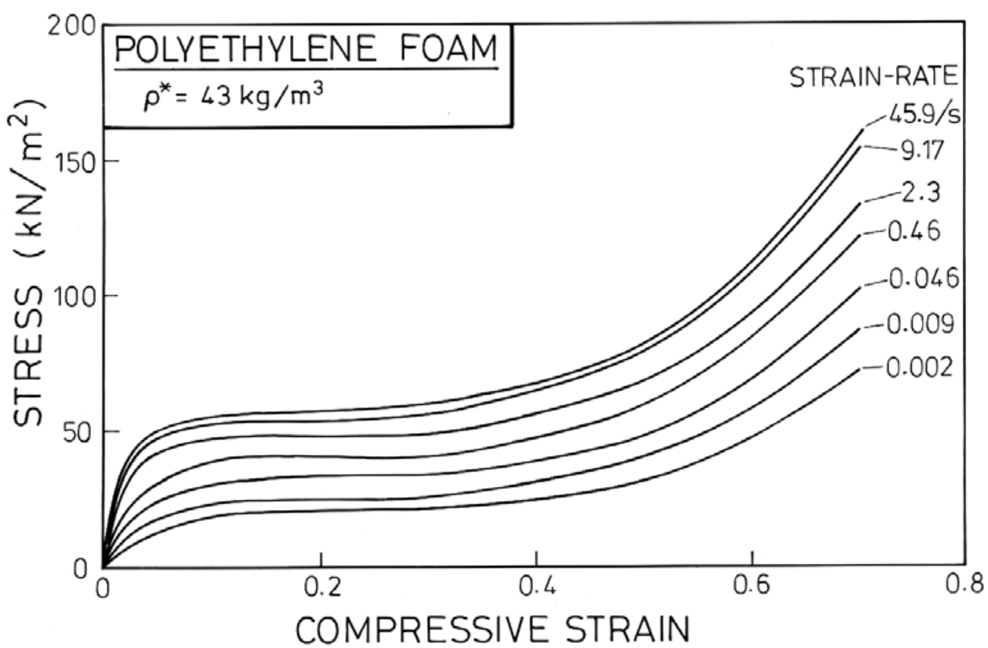
# Post-collapse stress strain curve

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# Plastic collapse

## Open cells:

- Failure when $M = M_p$
- $M_p \propto \sigma_{ys} t^3 \qquad M \propto \sigma^*_{pl} l^3$

  $\boxed{\sigma^*_{pl} = C_5 \sigma_{ys} (\rho^*/\rho_s)^{3/2}} \qquad C_5 \sim 0.3$ from data.
- Elastic collapse precedes plastic collapse if $\sigma^*_{el} < \sigma^*_{pl}$
- $$\begin{aligned} 0.05\, E_s (\rho^*/\rho_s)^2 &\leq 0.3\, \sigma_{ys} (\rho^*/\rho_s)^{3/2} \\ (\rho/\rho_s)_{\text{critical}} &\leq 36\, (\sigma_{ys}/E_s)^2 \end{aligned}$$

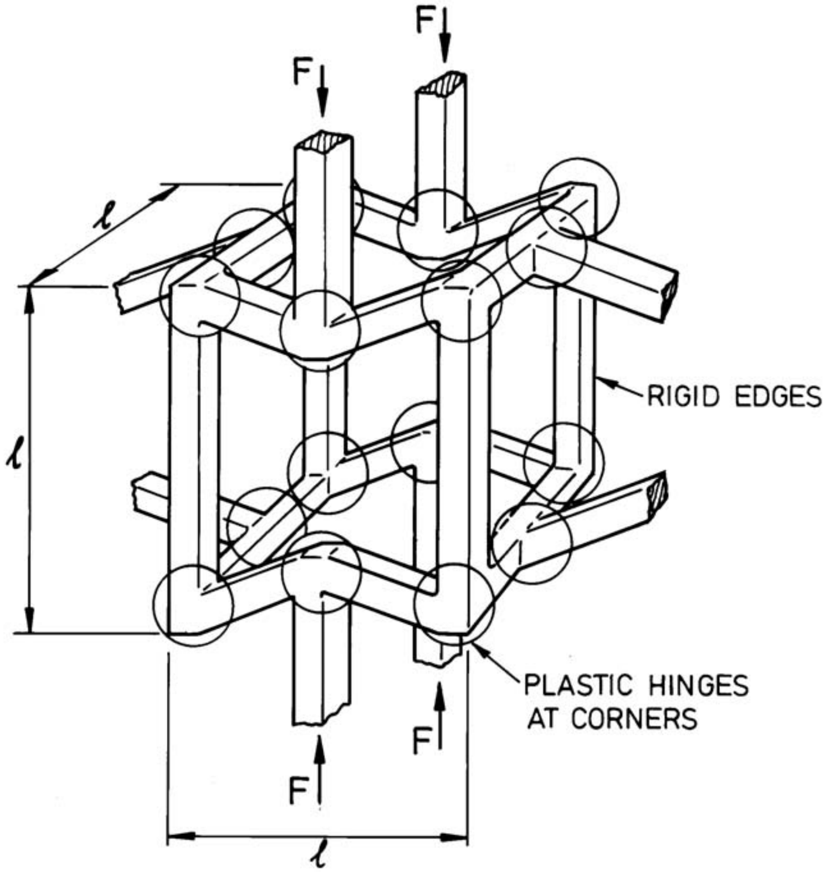
  rigid polymers $\quad (\rho^*/\rho_s)_{cr} < 0.04 \left(\frac{\sigma_{ys}}{E_s} \sim \frac{1}{30}\right)$

  metals $\quad (\rho^*/\rho_s)_{cr} < 10^{-5} \left(\frac{\sigma_{ys}}{E_s} \sim \frac{1}{1000}\right)$

# Closed cells:

- Including all terms: $$\sigma^*_{pl} = \underbrace{C_5\, \sigma_{ys} \left(\phi \frac{\rho^*}{\rho_s}\right)^{3/2}}_{\text{edge bending}} + \underbrace{C_5'\, \sigma_{ys} (1-\phi) \left(\frac{\rho^*}{\rho_s}\right)}_{\text{face stretching}} + \underbrace{p_0 - p_{\text{atm}}}_{\text{gas}}$$
- But in practice, faces often rupture around $\sigma^*_{pl}$ - often $\sigma^*_{pl} = 0.3\, (\rho^*/\rho_s)^{3/2}\, \sigma_{ys}$

# Plastic Collapse Stress

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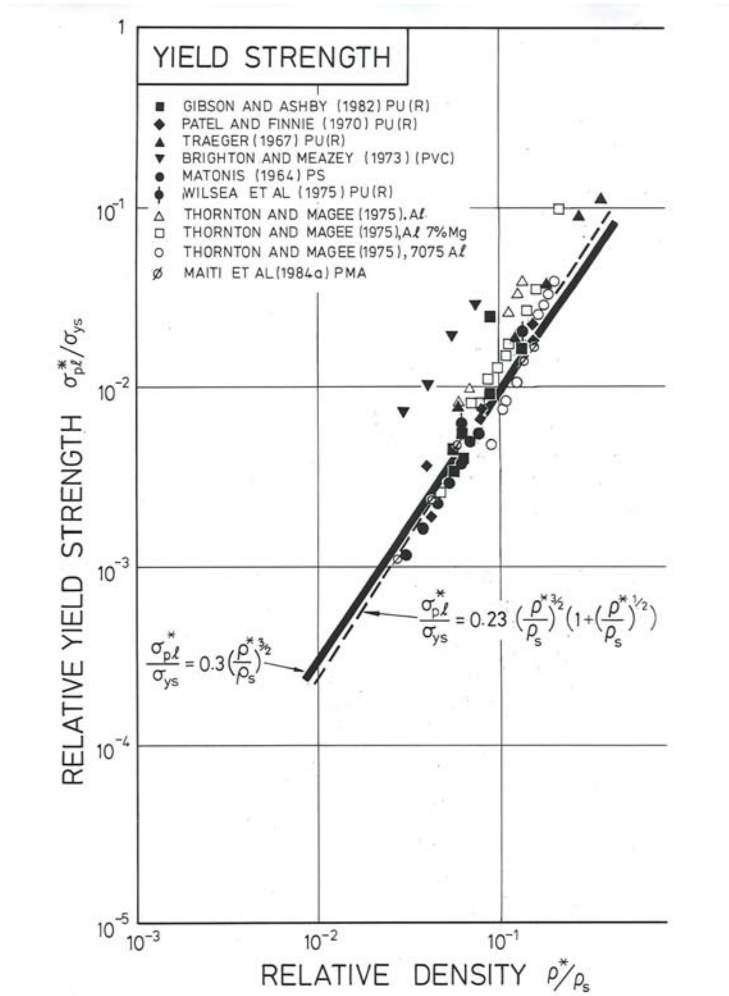
# Plastic Collapse Stress

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# Brittle crushing strength

## Open cells:

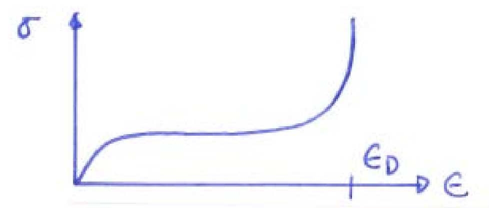
- Failure when $M = M_f$ $\quad M \propto \sigma^*_{\text{cr}}\, l^2 \quad M_f \propto \sigma_{\text{fs}}\, t^3$

$$\boxed{\sigma_{\text{cr}} = C_6\, \sigma_{\text{fs}}\, (\rho^*/\rho_s)^{3/2}} \qquad C_6 \approx 0.2$$

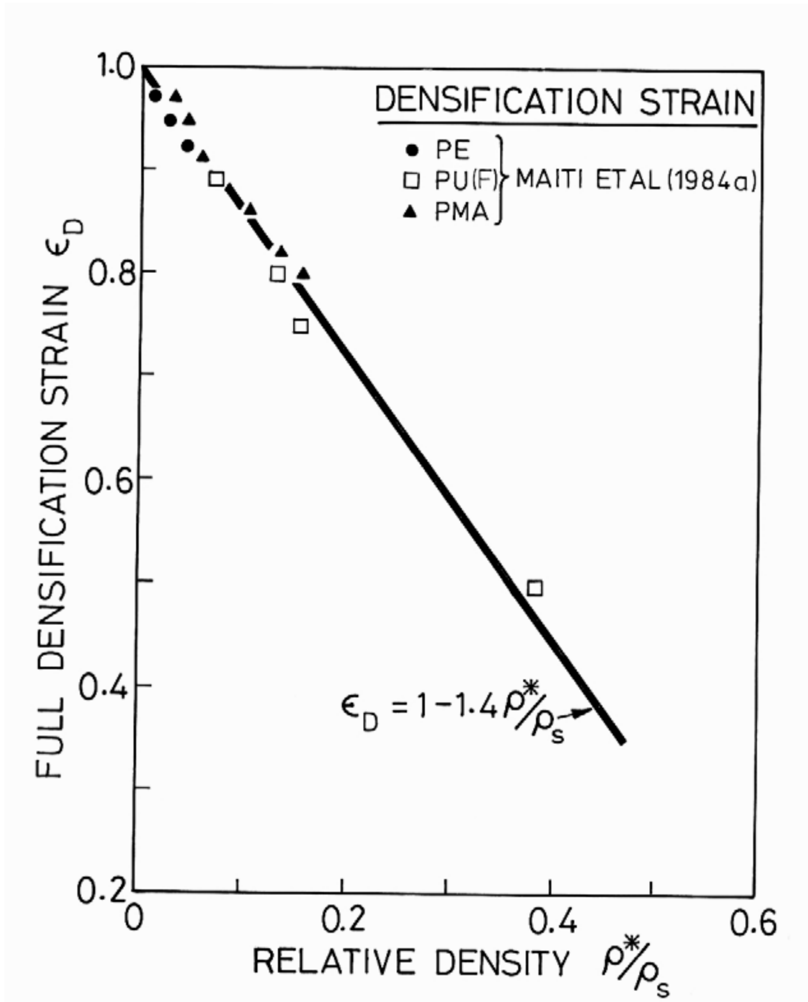
## Densification strain, $\epsilon_D$:

- At large comp. strain, cell walls begin to touch, $\sigma - \epsilon$ rises steeply
- $E^* \to E_s$; $\sigma - \epsilon$ curve looks vertical, at limiting strain
- Might expect $\epsilon_D = 1 - \rho^*/\rho_s$
- Walls jam together at slightly smaller strain than this:

$$\boxed{\epsilon_D = 1 - 1.4\, \rho^*/\rho_s}$$



# Densification Strain

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## Fracture toughness

Open cells: crack length $2a$, local stress $\sigma_l$, remote stress $\sigma^\infty$

$$\sigma_l = \frac{C\sigma^\infty\sqrt{\pi a}}{\sqrt{2\pi r}} \qquad \text{a distance } r \text{ from crack tip}$$

- Next unbroken cell wall a distance $r \approx l/2$, a head of crack tip subject to a force (integrating stress over next cell)

  $$F \propto \sigma_l\, l^2 \propto \sigma^\infty \sqrt{\tfrac{a}{l}}\, l^2$$

- Edges fail when applied moment, $M$ = fracture moment, $M_f$

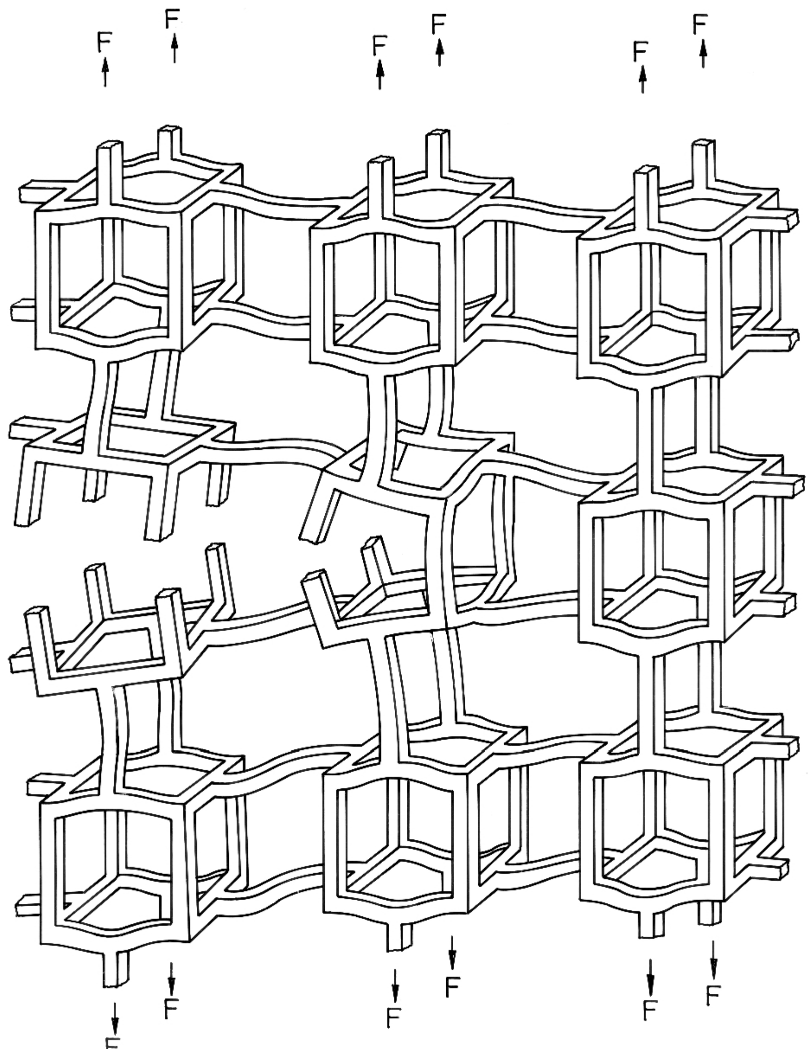
  $$M_f = \sigma_{\text{fs}}\, t^3$$

  $$M \propto F\, l \propto \sigma^\infty \left(\frac{a}{l}\right)^{1/2} l^3 \qquad M = M_f \quad \rightarrow \quad \sigma^\infty \left(\frac{a}{l}\right)^{1/2} l^3 \propto \sigma_{\text{fs}}\, t^3$$

  $$\sigma^\infty \propto \sigma_{\text{fs}} \left(\frac{l}{a}\right)^{1/2} \left(\frac{t}{l}\right)^3$$

$$\boxed{K_{IC}^* = \sigma^\infty \sqrt{\pi a} = C_8\, \sigma_{\text{fs}} \sqrt{\pi l}\left(\rho^*/\rho_s\right)^{3/2}} \qquad \text{Data: } C_8 \sim 0.65$$

# Fracture Toughness

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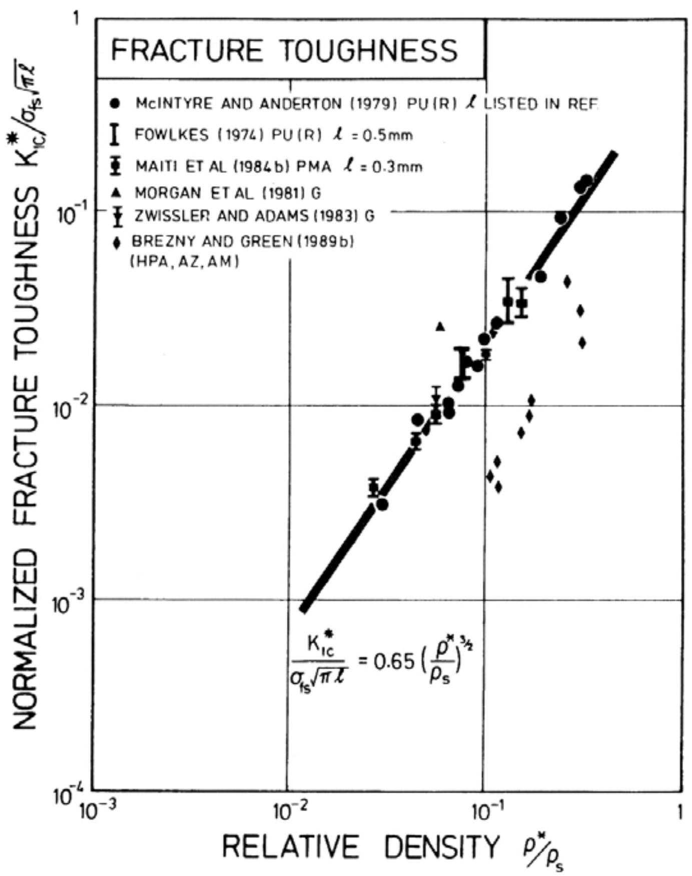
# Fracture Toughness

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