# Lecture 6, Wood notes, 3.054

## Honeycomb-like materials in nature: wood

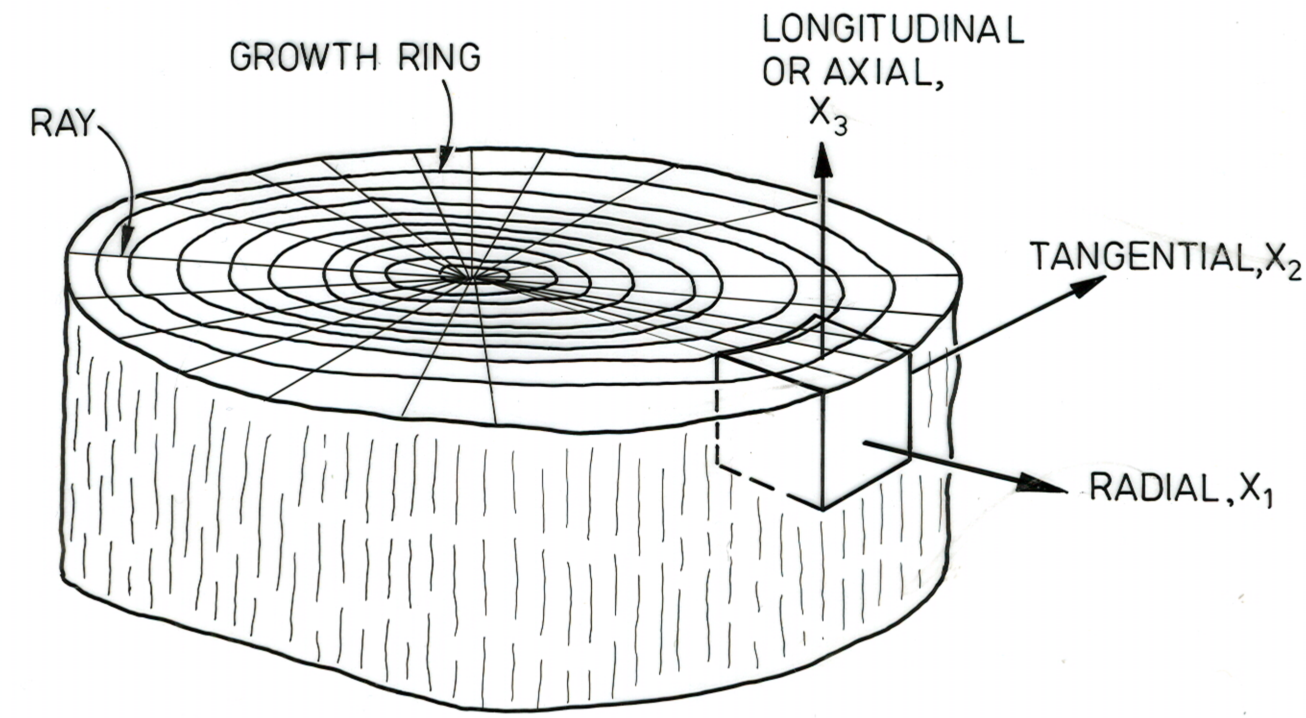
- "Materials" derives from Latin "materies, materia", means wood or trunk of a tree
- Old Irish - names of first letters of the alphabet refer to woods

| | | | |
|---|---|---|---|
| A | alem | = | elm |
| B | beith | = | birch |
| C | coll | = | hazel |
| D | dair | = | oak |

## Wood - structure

- Orthotropic (if neglect curvature of growth rings)
- $\rho^*/\rho_s$ ranges from 0.05 (balsa) to 0.80 (lignum vitae)
- Trees have cambial layer, beneath bark
- Cell division at cambial layer:
  - New cells on outer part of cambial layer $\rightarrow$ bark
  - New cells on inner part of cambial layer $\rightarrow$ wood

# Wood structure




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

- Living plant cells — plasma membrane and protoplast
- Living cells secrete plant cell wall — analogous to extra cellular matrix in animal tissues
- In trees, cells lay down cell wall over a few weeks, then die
- Always retain a cambial layer of cells

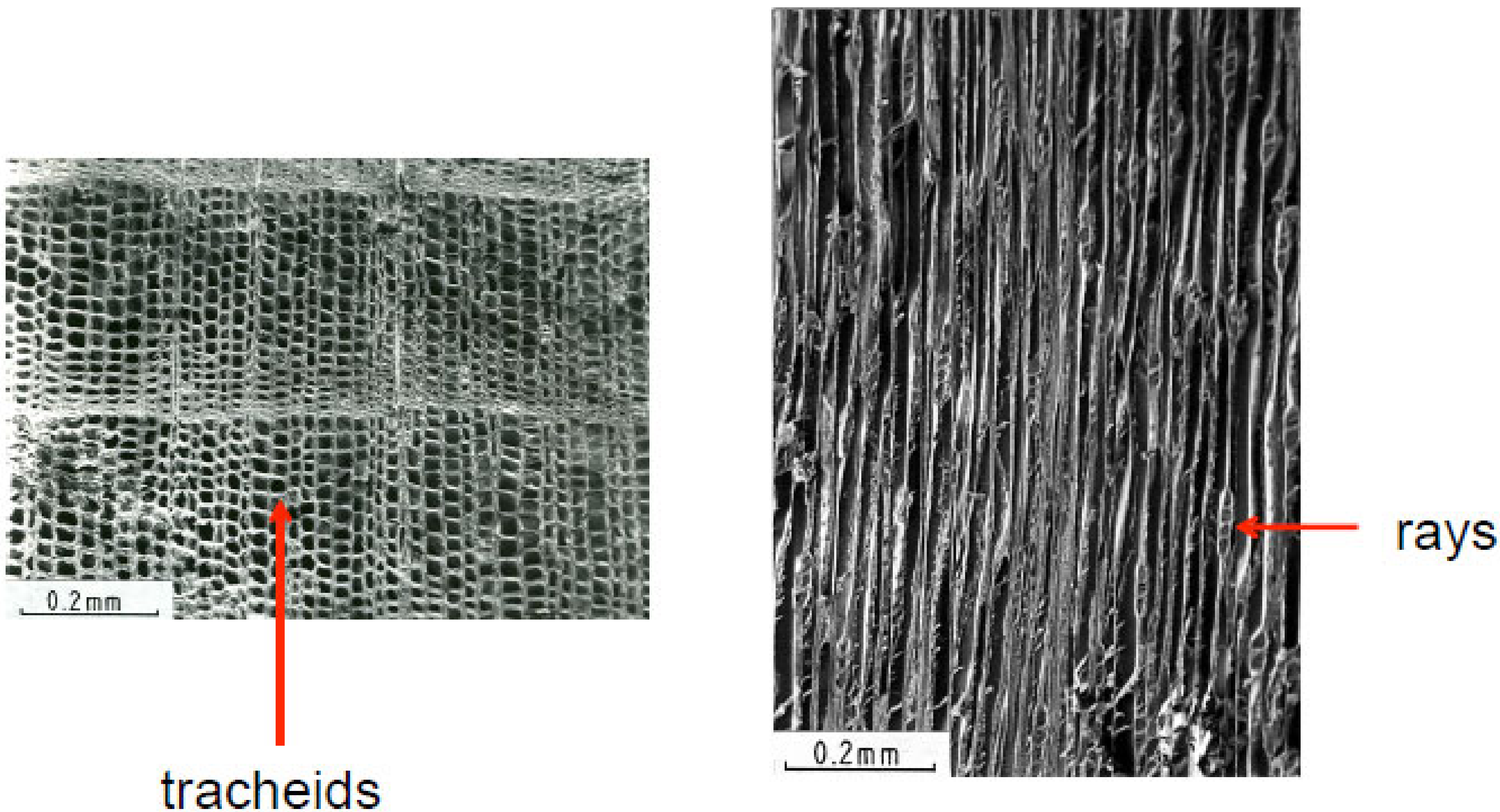
## Cellular structure: softwoods

- Tracheids
  - Bulk of cells (90%), provide structural support
  - Have holes in cell wall for fluid transport (pits)
  - $\sim$ 2.5–7.0 mm long; 20–80$\mu m$ across; $t$ = 2–7$\mu m$
- Rays
  - Radial arrays of smaller parenchyma cells that store sugars

## Cellular structure: hardwoods

- Fibers provide structural support; 35–70% of cells
- Vessels — sap channels — conduction of fluids; 6–55% of cells
- Rays — store sugars; 10–30% of cells

# Softwood: Cedar

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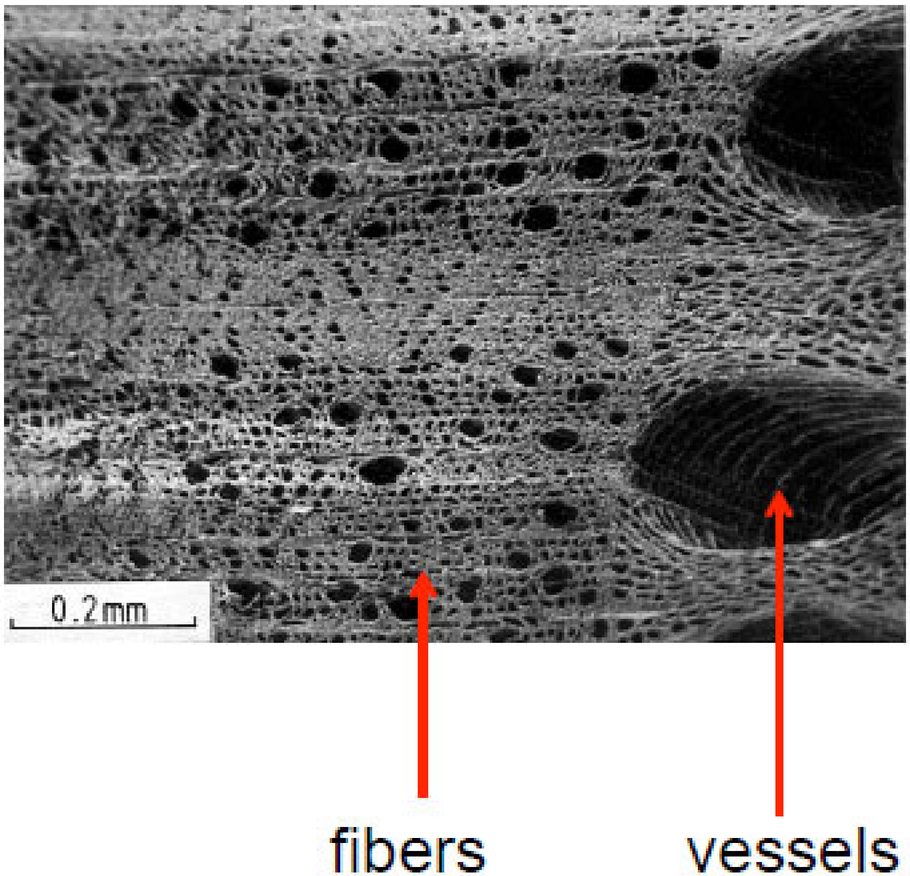
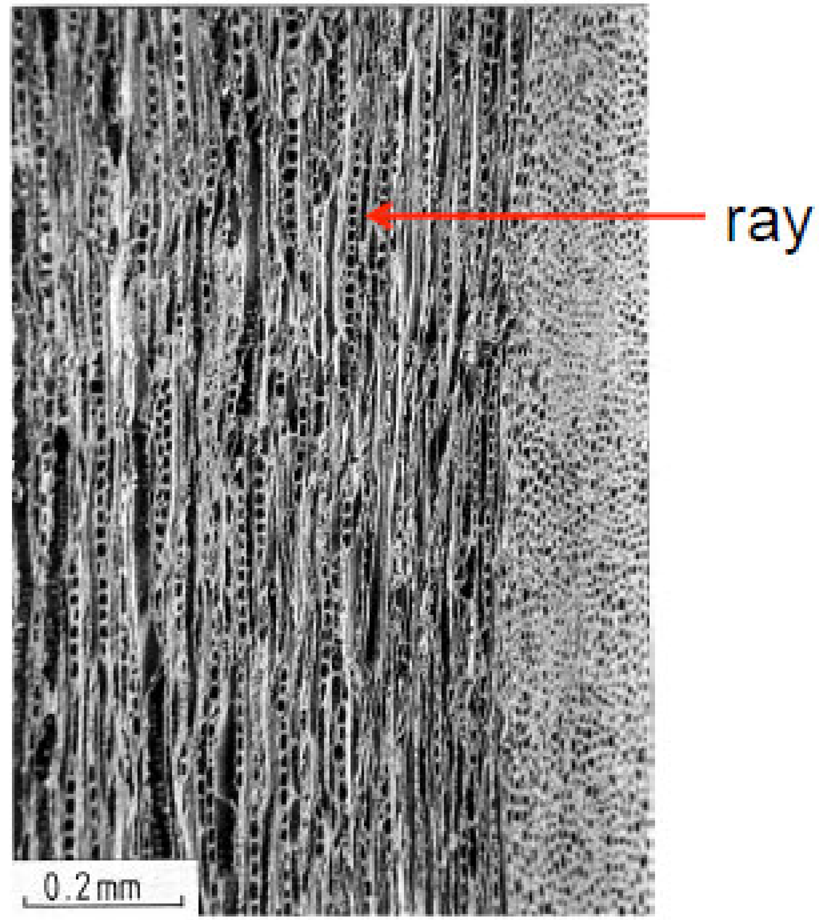
# Hardwood: Oak




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

## Structure: cell wall

- Fiber-reinforced composite
- Cellulose fibers in matrix of lignin / hemicellulose
- Four layers, each with fibers at different orientation
- Between two cells: middle lamella

## Cell wall properties

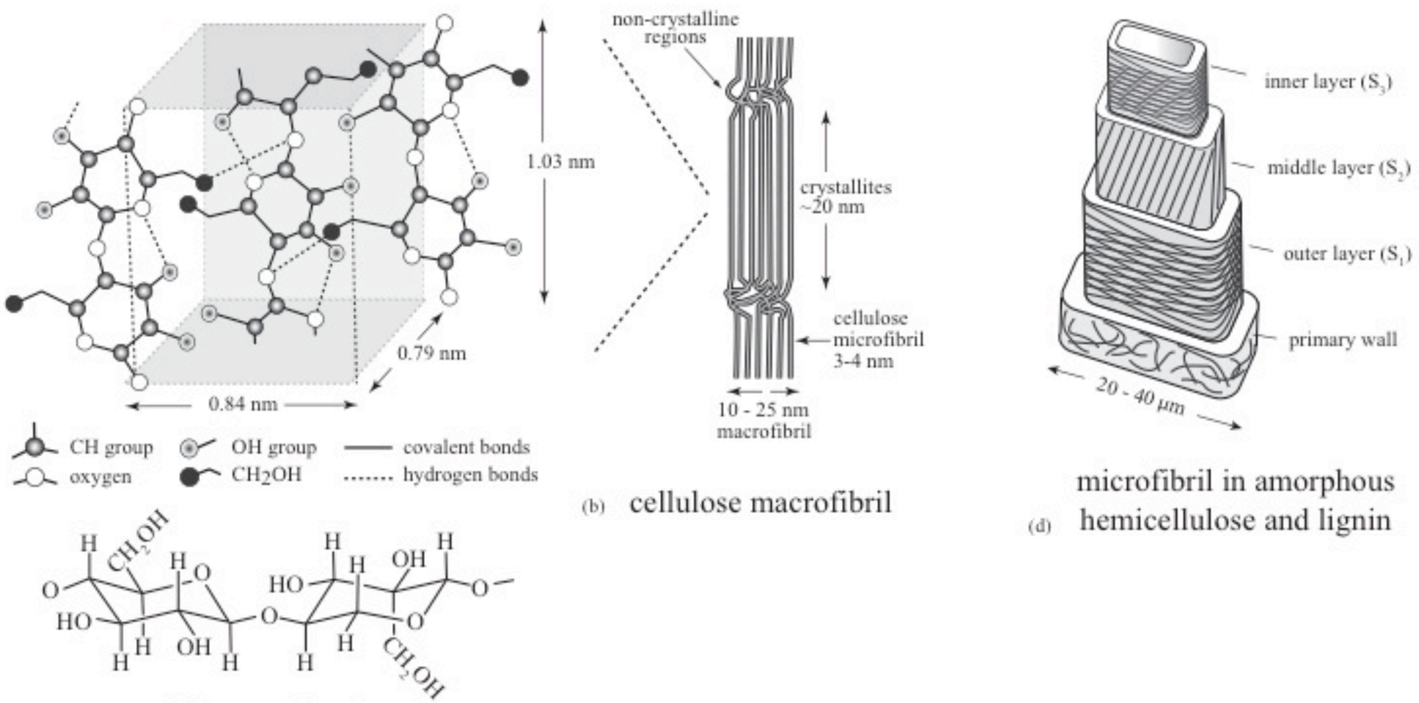
- Similar in different species of wood

$$
\begin{aligned}
\rho_s &= 1500\,\mathrm{kg/m^3} \\
E_{SA} &= 35\,\mathrm{GPa} \\
E_{ST} &= 10\,\mathrm{GPa} \\
\sigma_{ysA} &= 350\,\mathrm{MPa} \\
\sigma_{ysT} &= 135\,\mathrm{MPa}
\end{aligned}
$$

(Note cellulose: $E \sim 140\,\mathrm{GPa}$
$\sigma_y \sim 750\,\mathrm{MPa}$)

A = axial direction
T = transverse direction

# Wood Structure




(a) cellulose molecule

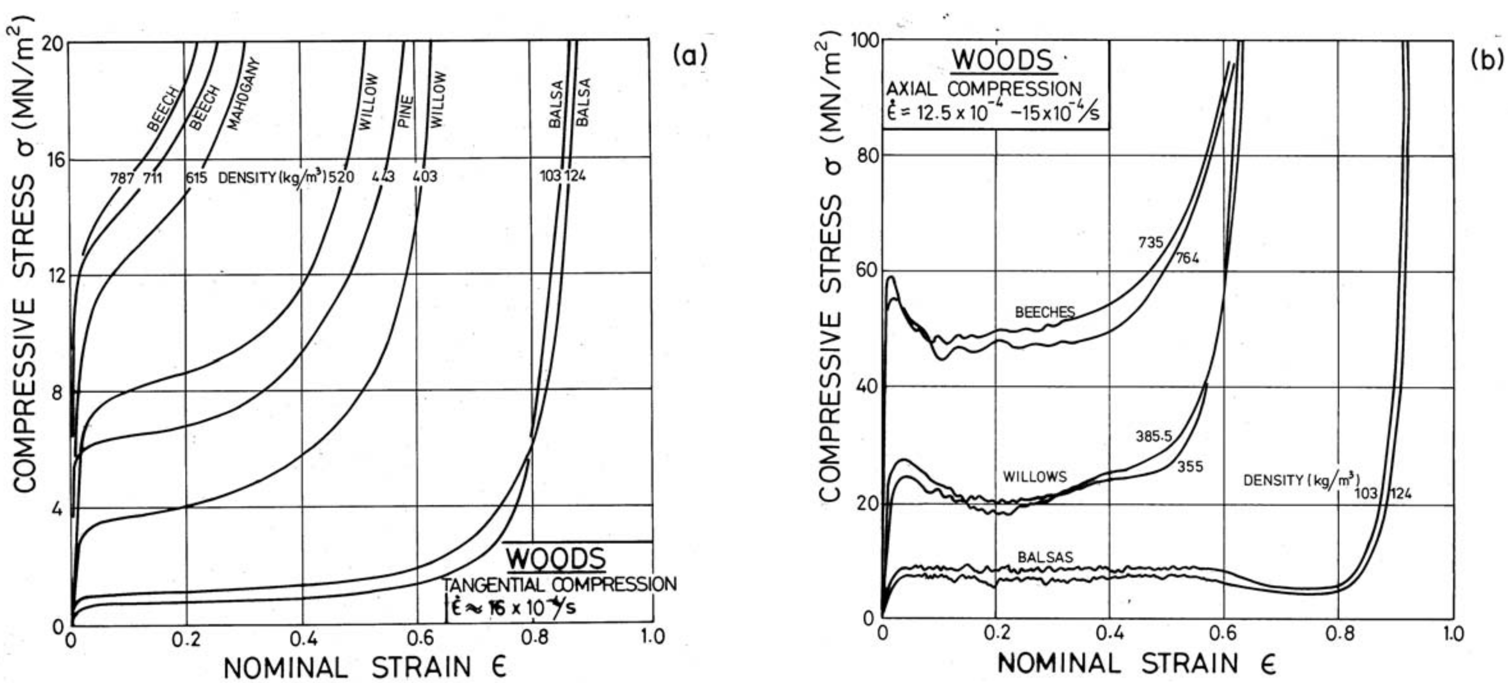
(b) cellulose macrofibril

(d) microfibril in amorphous hemicellulose and lignin

## Stress-strain curves

- $\sigma - \epsilon$ curves resemble those for honeycombs
- Mechanisms of deformation most easily identified on low density balsa
- Curves and images for balsa
- Tangential loading: formation of plastic hinges in bent cell walls
- Radial loading:
  - Rays act as reinforcing
  - Plastic yielding in cell walls
  - Starts at platens and moves inwards
- Axial loading:
  - Axial deformation of cell walls
  - Then break end caps
  - Serration corresponds to each layer of end caps breaking
  - Failure by plastic buckling, formation of kink bands also observed
- Denser species:
  - Douglas fir — tangential, radial compression
  - Norway spruce — axial compression

# Stress strain curves




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

# Balsa

Figure removed due to copyright restrictions. See Figure 3: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

# Balsa: Tangential

Figure removed due to copyright restrictions. See Figure 4: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

Figure removed due to copyright restrictions. See Figure 7: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

# Balsa: Radial

Figure removed due to copyright restrictions. See Figure 5: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

# Balsa: Axial

Figure removed due to copyright restrictions. See Figure 6: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

# Douglas Fir: Tangential Comp

Figure removed due to copyright restrictions. See Bodig, J., and B. A. Jayne. *Mechanics of Wood and Wood Composites*. Van Nostrand Reinhold, 1982.

# Douglas fir: Radial comp.

Figure removed due to copyright restrictions. See Bodig, J., and B. A. Jayne. *Mechanics of Wood and Wood Composites*. Van Nostrand Reinhold, 1982.

# Norway spruce: Axial comp

Images removed due to copyright restrictions. See Figure 5.14: Dinwoodie, J. M. *Timber: Its Nature and Behaviour.* Van Nostrand Reinhold, 1981.

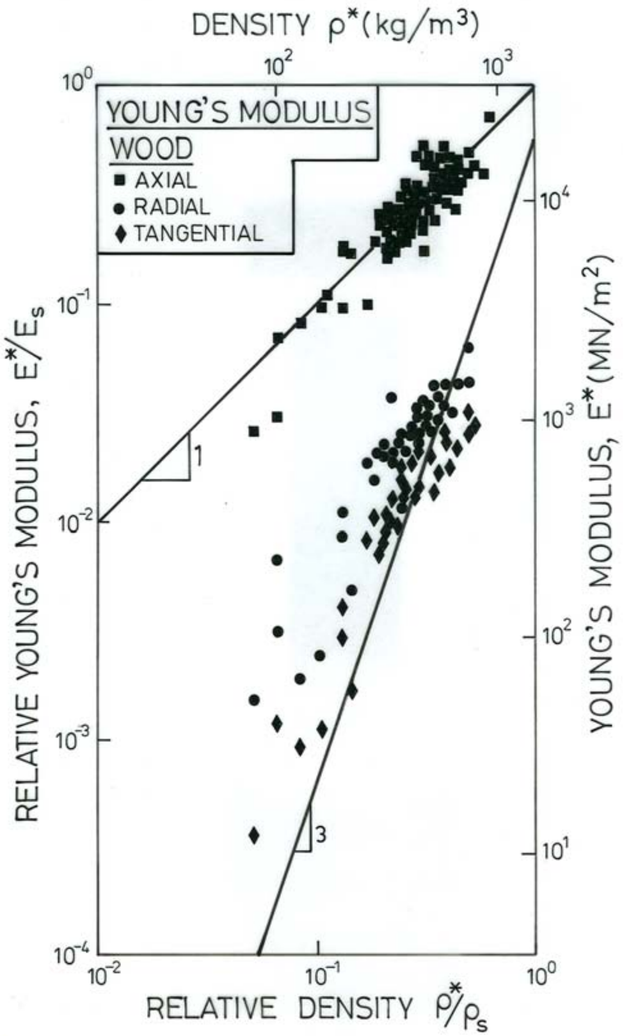
## Data for wood

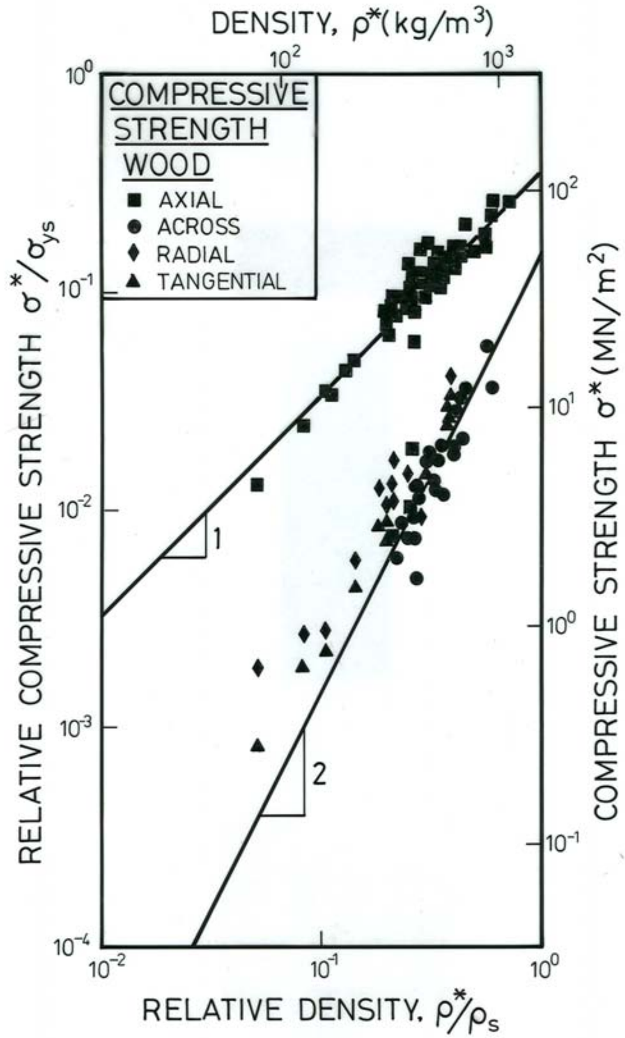
$$
\begin{array}{llll}
E^*/E_s & \propto & \rho^*/\rho_s & \text{(axial)} \\
E^*/E_s & \propto & (\rho^*/\rho_s)^3 & \text{tangential; radial somewhat stiffer)} \\
\sigma^*/\sigma_{ys} & \propto & (\rho^*/\rho_s) & \text{(axial)} \\
\sigma^*/\sigma_{ys} & \propto & (\rho^*/\rho_s)^2 & \text{(tangential/radial)}
\end{array}
$$

$$
\begin{array}{lll}
\nu^*_{RT} \sim 0.5\text{–}0.8 & \nu^*_{RA} \sim 0.02\text{–}0.07 & \nu^*_{AR} \sim 0.25\text{–}0.5 \\
\nu^*_{TR} \sim 0.2\text{–}0.6 & \nu^*_{TA} \sim 0.01\text{–}0.04 & \nu^*_{AT} \sim 0.35\text{–}0.5
\end{array}
$$

## Modeling wood properties

- Very simplified model — first order
- Does not attempt to capture finer details (eg., softwoods vs. hardwoods)
- Cell wall has been modeled as fiber composite; it is itself anisotropic
- We normalize all properties with respect to $E_s$, $\sigma_{ys}$ *axial*
- Constant of proportionality also reflects cell wall anisotropy




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

# Model for wood microstructure

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## Linear elastic moduli

- Tangential loading — model as honeycomb — cell wall bending
  $E_T^*/E_s \sim (\rho^*/\rho_s)^3$
  - rays, end caps end to stiffen wood — data lie slightly above $(\rho^*/\rho_s)^3$
- Radial loading — rays act as reinforcing plates and are higher density than fibers

  $V_R =$ volume fraction of rays

  $R = (\rho^*/\rho_s)_{\text{rays}}/(\rho^*/\rho_s)_{\text{fibers}} \approx 1.1$ to 2

  $E_R^* = V_R\, R^3 E_T^* + (1 - V_R) E_T^* \approx 1.5 E_T^*$

  $E_R^*$ slightly larger than $E_T^*$; $\sim (\rho^*/\rho_s)^3$

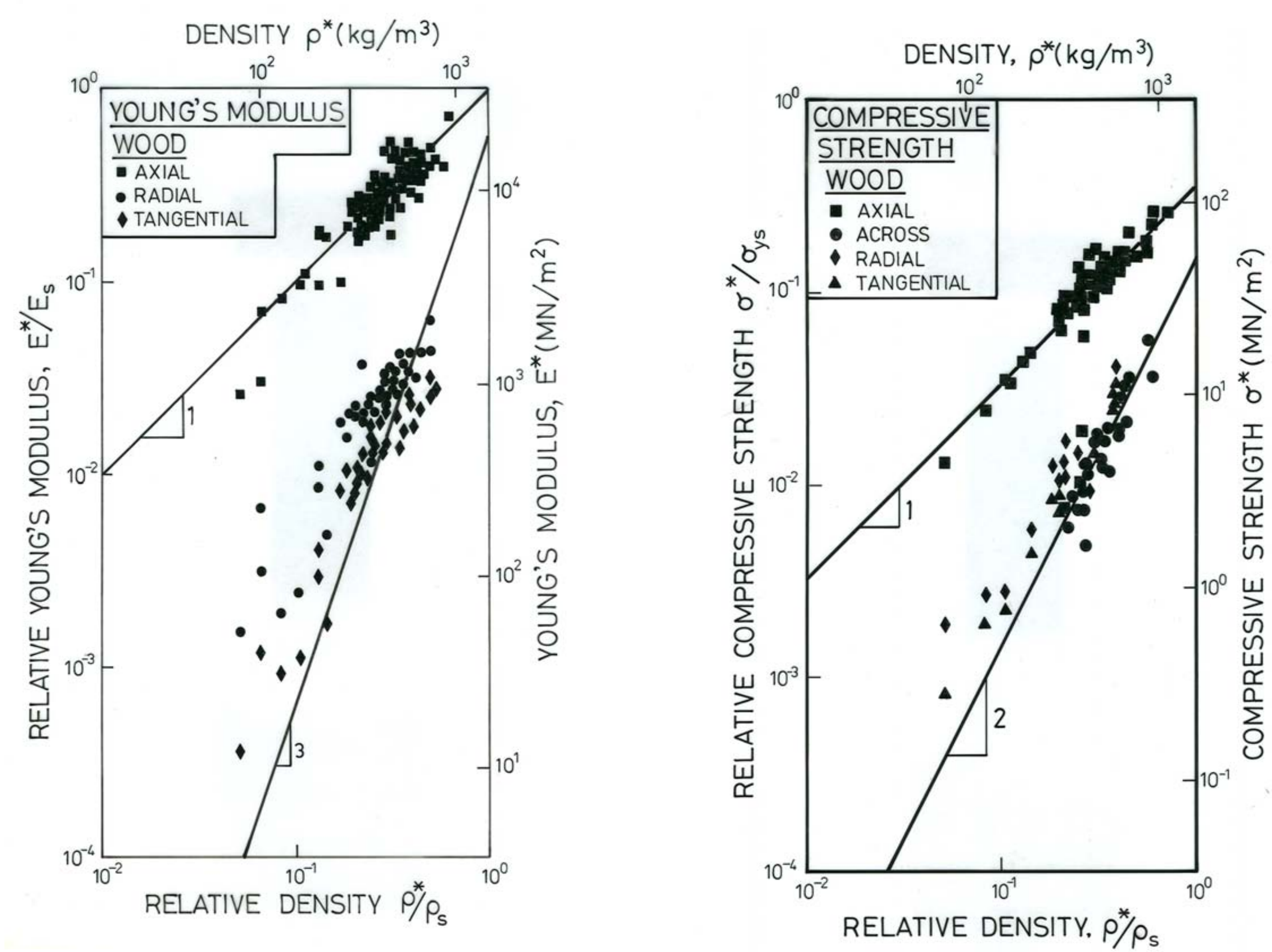
- Axial loading
  - Axial deformation in cell walls
    $E_A^*/E_s \sim (\rho^*/\rho_s)$
- Explains, to first order:
  - Density dependence
  - Anisotropy

## Modeling Poisson's Ratios

| | **Model** | |
|---|---|---|
| $\nu^*_{RT} = 0.5$–$0.8$ | 1 | constraining effect |
| $\nu^*_{TR} = 0.2$–$0.6$ | 1 | of rays and end caps |
| | | |
| $\nu^*_{RA} = 0.02$–$0.07$ | 0 | |
| $\nu^*_{TA} = 0.01$–$0.04$ | 0 | |
| | | |
| $\nu^*_{AR} = 0.25$–$0.5$ | $\nu_s$ | data close to $0.4 \sim \nu_s$ |
| $\nu^*_{AT} = 0.35$–$0.5$ | $\nu_s$ | |

## Modeling - compressive strength

- Tangential loading — bending, plastic hinges $\sigma^*_T/\sigma_{ys} \propto (\rho^*/\rho_s)^2$
- Radial loading:
  - $\sigma^*_R = V_R\, R^2\, \sigma^*_T + (1 - V_R)\, \sigma^*_T$
  - balsa: $V_R \sim 0.14$ $\quad R \sim 2$ $\quad \sigma^*_R = 1.4\sigma^*_T$
  - Higher density woods — R smaller
  - $\sigma^*_R$ slightly larger than $\sigma^*_T$; both $\propto\ (\rho^*/\rho_s)^2$
- Axial loading
  - Initial failure by axial yield (then end cap fracture, or buckling)
  - $\sigma^*_A/\sigma_{ys}\ \propto\ \rho^*/\rho_s$




Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

## Modeling: cell wall plus cellular structure

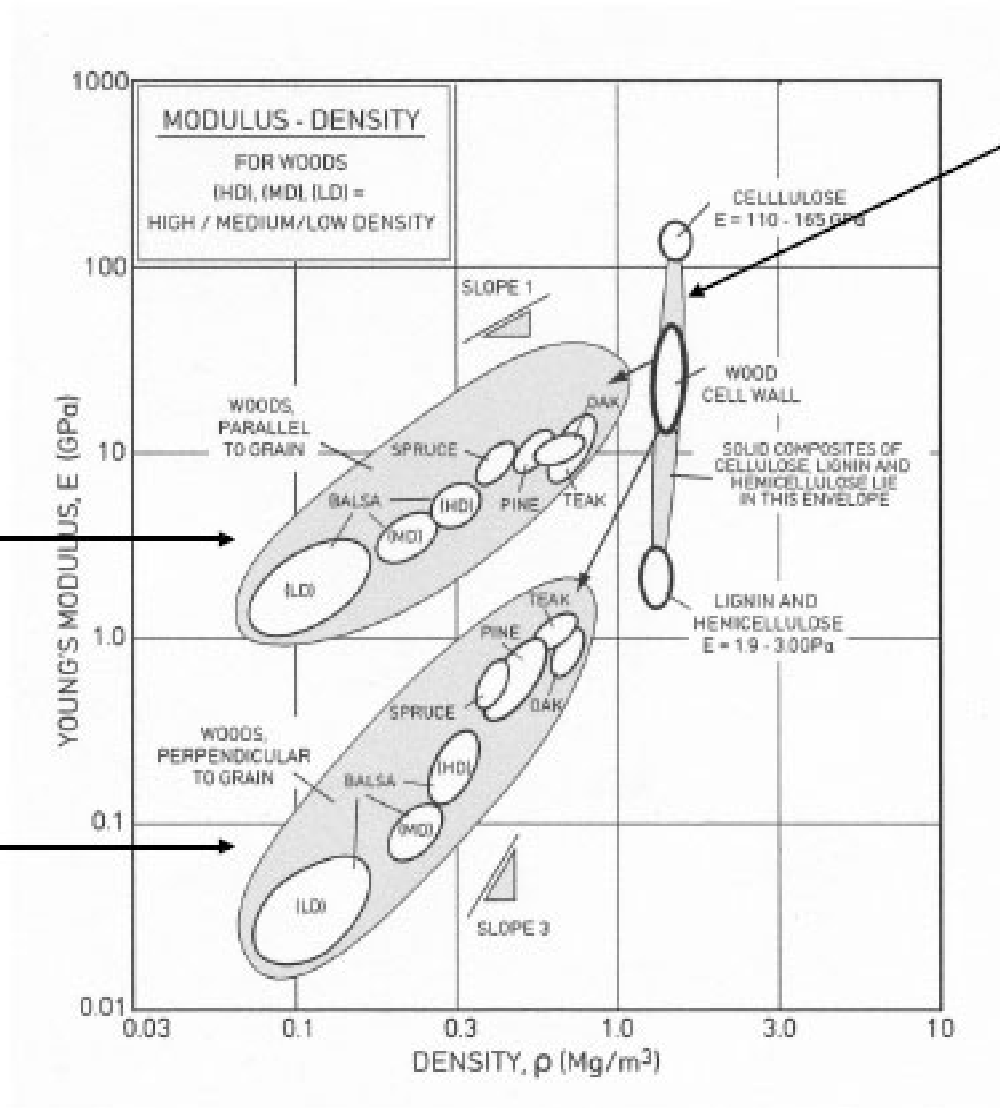
- Cell wall can be modeled as a fiber composite
    - Celloluse $E \sim 140$ GPa;
    - Ligning/hemicellulose $E \sim 2$ GPa
    - Composite upper and lower bounds give envelope at right of figure
    - Measured values for $E_{S\text{ Axial}} = 35$ GPa; $E_{S\text{ Transverse}} - 10$ GPa

- Can also show cellular solids model on some plot
- Overall, plot shows how wood hierarchical structure, density variation give wood moduli that vary by a factor of 1000
- Can make similar plot for strength

# Wood: Honeycomb Models

$$\frac{E^*}{E_{s\,along}} = \frac{\rho^*}{\rho_s}$$

$$\frac{E^*}{E_{s\,across}} = \left(\frac{\rho^*}{\rho_s}\right)^3$$




Cell wall: Fiber Composite Model

Gibson, L. J., and M. F. Ashby. *Cellular Materials in Nature and Medicine.* 2nd ed. Cambridge University Press, © 2010. Figure courtesy of Lorna Gibson and Cambridge University Press.

# Wood: Honeycomb Models

Diagram removed due to copyright restrictions. See Figure 5b: Gibson, L. J. "The Hierarchical Structure and Mechanics of Plant Materials." *Journal of the Royal Society Interface* 9 (2012): 2749-66.

# Material selection

- For a beam of a given stiffness, $\rho/\delta$, length, $l$, square cross-section with edge length, $t$, what material minimizes the mass, $m$, of the beam?

  $m = \rho t^2 l$

  $\delta = \dfrac{Pl^2}{CEI} \qquad \dfrac{P}{\delta} = \dfrac{CEt^4}{l^3} \qquad t^2 = \left[\left(\dfrac{P}{\delta}\right)\dfrac{l^3}{CE}\right]^{1/2}$

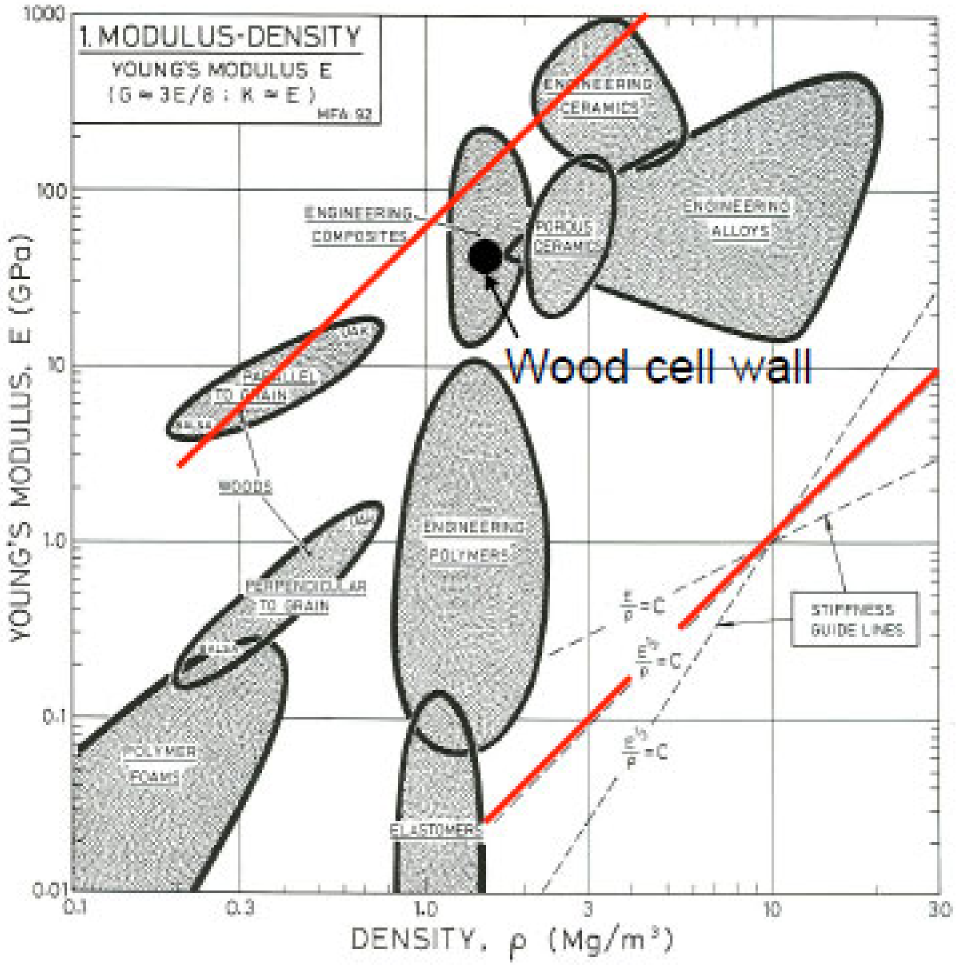
  $m = \rho\left[\left(\dfrac{P}{\delta}\right)\dfrac{l^3}{CE}\right]^{1/2} l$

  to minimize mass, choose material with minimum $\rho/E^{1/2}$ or maximize $E^{1/2}/\rho$

- Material selection chart: plot $\log E$ vs $\log\rho$
- Line of constant $E^{1/2}/\rho$ shown in red on plot
- Materials with largest values of $E^{1/2}/\rho$ at upper left of the plot
- Woods have similar values of $E^{1/2}/\rho$ as engineering composites
- Note that tree trunks, branches, loaded primarily in bending
- Also note, from models, $\dfrac{(E^*)^{1/2}}{\rho^*} = \dfrac{E_s^{1/2}}{\rho_s} \cdot \dfrac{\rho_s}{\rho}^{1/2}$

  $\rightarrow$ performance index for wood higher than that for the solid cell wall
- Similarly for strength in bending

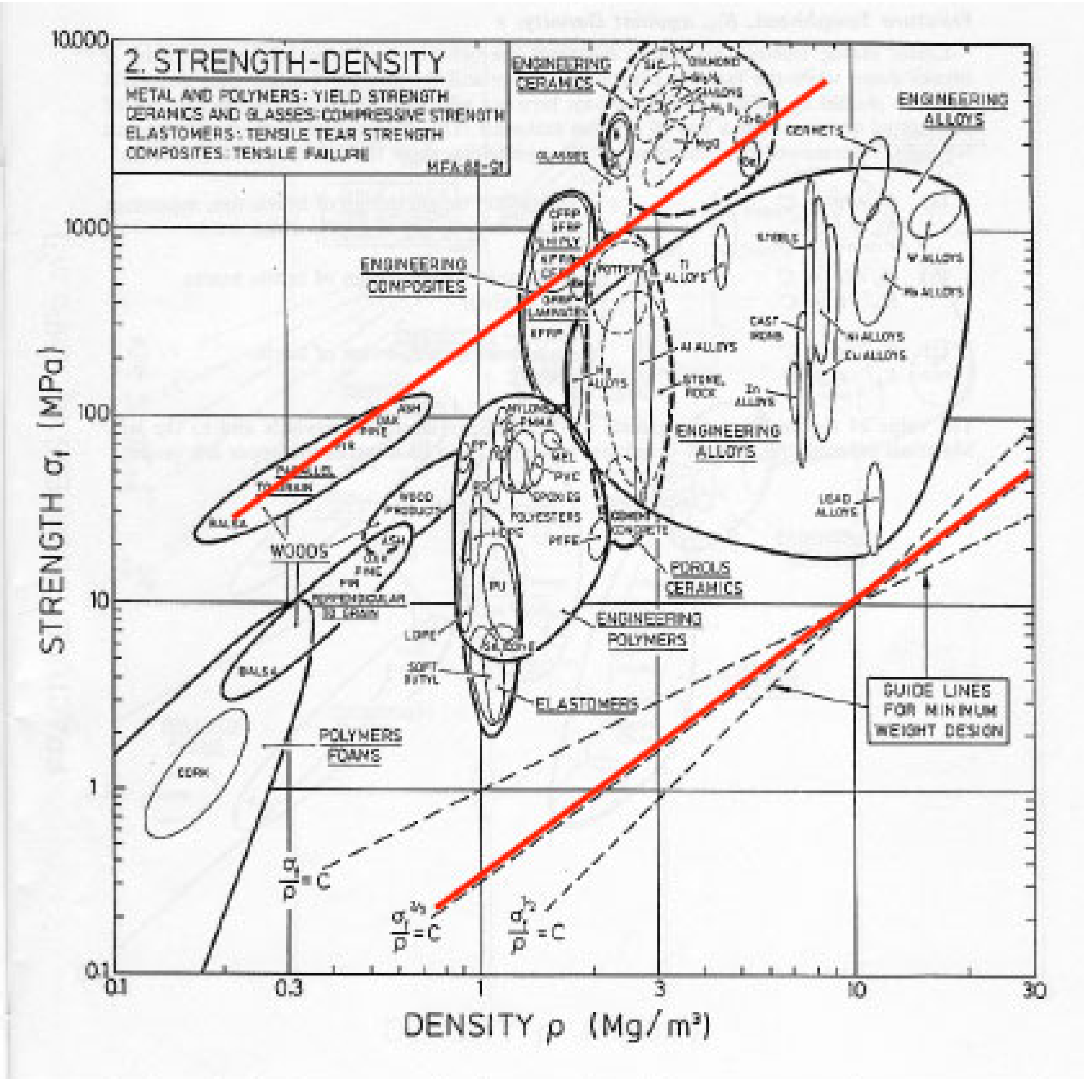
# Wood in Bending: $E^{1/2}/\rho$




$$\frac{\left(E^*\right)^{1/2}}{\rho^*} = \frac{\left(E_s\right)^{1/2}}{\rho_s}\left(\frac{\rho_s}{\rho^*}\right)^{1/2}$$

Stiffness performance index for wood in bending is similar to that for best engineering composites

# Wood in Bending: $\sigma_f^{2/3}/\rho$




$$\frac{\left(\sigma_f^*\right)^{2/3}}{\rho^*} = \frac{\left(\sigma_{ys}\right)^{2/3}}{\rho_s}\left(\frac{\rho_s}{\rho^*}\right)^{1/3}$$

Strength performance index for wood in bending is similar to that for best engng composites

# Wood Use in Design

## Historical example: seventeenth century wooden ships

- Colonial times, importance of navies to colonial powers
- Used particular species for different parts of ship, based on their properties
- Oak — used for much of the hull, ribs, knees, planking $\rightarrow$ dense wood; stiff and strong
  - "Straight oak" — straight pieces, cut from trunk
  - "Compass oak" — carved pieced from trunk and branch, so that grain runs along carved, cut piece — maximum $E$, $\sigma^*$; used for knees, wing transom — curved pieces of ship hull
- Eastern white pine
  - British Royal Navy used for masts, imported from New England
  - England had run out of tall straight trees for masts
  - Strategic resource — ship speed, size — depended on size of mast and sail area
  - Eastern white pine known fro straight, tall trunks; some over 100 feet tall
- Lignum vitae
  - Densest wood; acts as own lubricant
  - Used in block and tackle
  - Also used in clock gears
  - John Harrison's chronometer — Story of Longitude, Dava Sobel
  - H4 1759 lost 5 seconds in 81 days at sea

Figure removed due to copyright restrictions. See *The international book of wood*. Bramwell, M, ed. Artists House, 1982. pp 186-87.

## Modern example: glue-laminated timber

- Glue long pieces of wood, typically 1-2” thick, together
- Select strips to avoid defects (e.g., knots)
- Glue-lam has better mechanical properties than sawn lumber
- Also, can make curved members by using curved molds and clamps during bonding process
  - Grain runs along the curve
  - Architecturally attractive
  - Exploits high stiffness and strength of wood along the grain

Image of graceful glued-laminated timber arch bridge removed due to copyright restrictions. See Figure 13: *Engineered Wood Products: A Guide for Specifiers, Designers and Users*. Smulski, S., ed. PFS Research Foundation, 1997.

Engineered Wood Products: A Guide for Specifiers, Designers and Users, S. Smulski Ed. PFS Research Foundation, 1997