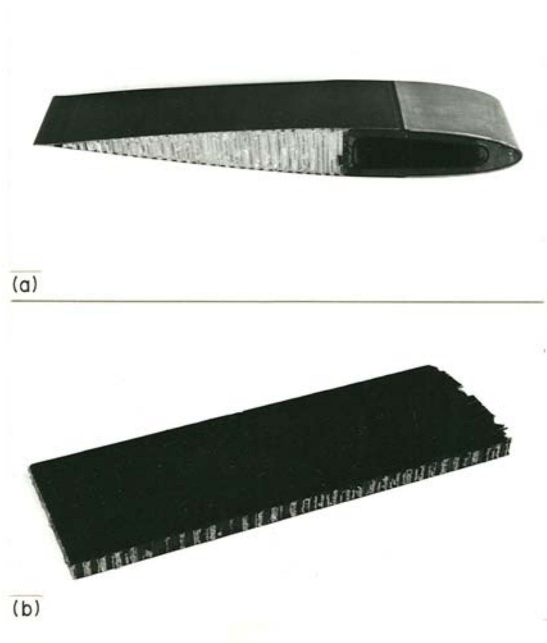
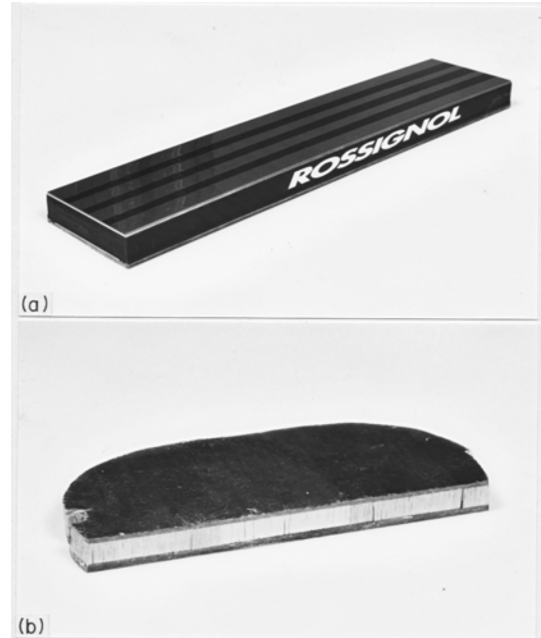
Lectures 18, 19 Sandwich Panel Notes, 3.054

Sandwich Panels

- Two stiff strong skins separated by a lightweight core
- Separation of skins by core increases moment of inertia, with little increase in weight
- Efficient for resisting bending and buckling
- Like an I beam: faces = flanges — carry normal stress
  core = web — carries shear stress
- Examples: engineering and nature
- Faces: composites, metals
  Cores: honeycombs, foams, balsa
  honeycombs: lighter then foam cores for required stiffness, strength
  foams: heavier, but can also provide thermal insulation
- Mechanical behavior depends on face and core properties and/or geometry
- Typically, panel must have some required stiffness and/or strength
- Often, want to minimize weight — optimization problem
  e.g. refrigerated vehicles; sporting equipment (sail boats, skis)





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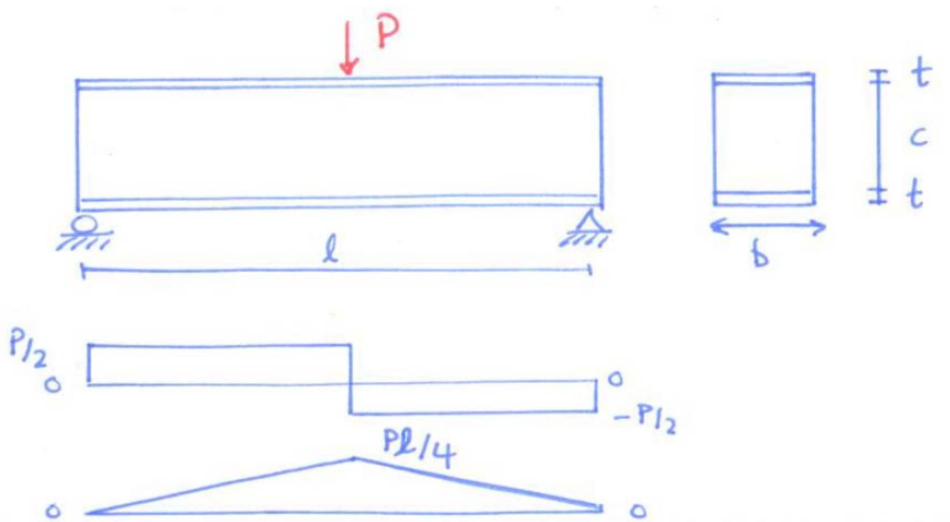
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Sandwich beam stiffness

- Analyze beams here (simpler than plates; same ideas apply)



V

B.M.

Face: $\rho_f,\ E_f,\ \sigma_{yf}$

Core: $\rho_c^*,\ E_c^*,\ \sigma_c^*$

(Solid: $\rho_s,\ E_s,\ \sigma_{ys}$)

Typically $E_c^* \ll E_f$

$\delta = \delta_b + \delta_s$: bending deflection δ_b and shear deflection (of core) δ_s
since $G_c^* \ll E_f$, core sheer deflections significant

$$\delta_b = \frac{P\,l^3}{B_1 (EI)_{eq}}$$

B_1=constant, depending on loading configuration
3 pt bend, $B_1 = 48$

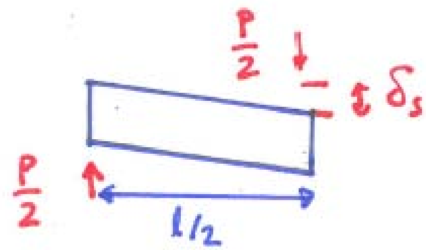
$$(EI)_{eq} = \left(\frac{E_f bt^3}{12} \times 2\right) + E_c\,\frac{bc^3}{12} + E_f\ bt\left(\frac{c_t t}{2}\right)^2 \times 2 \quad \text{parallel axis theorem}$$

$$= \frac{E_f bt^3}{6} + \frac{E_c bc^3}{12} + \frac{E_f bt}{2}\,(c+t)^2$$

Sandwich structures: typically $E_f \gg E_c^*$ and $c \gg t$

Approximate $(EI)_{eq} \approx \dfrac{E_f\, btc^2}{2}$

$\delta_s = ?$ core



$$\tau = G\gamma$$

$$\frac{P}{A} \propto G\,\frac{\delta_s}{l}$$

$$\delta_s = \frac{P\,l}{B_2\,(AG)_{eq}}$$

$$(AG)_{eq} = \frac{b(c+t)^2}{c}\,G_c \approx b_c\,G_c$$

$$\delta = \delta_b + \delta_s$$

$$\boxed{\delta = \frac{2Pl^3}{B_1 E_f\, b\, t\, c^2} + \frac{Pl}{B_2\, b\, c\, G_c^*}}$$

And also note:

$$G_c^* = C_2\, E_s(\rho^*/\rho_s)^2 \text{ (foam model)}$$

$$C_2 \approx 3/8$$

Minimum weight for a given stiffness

- Given face and core materials
    - beam length, width, loading geometry (e.g. 3 pt bend, B_1, B_2)
- Find: face and core thicknesses, t + c, and core density ρ_c^* to minimize weight $W = 2\,\rho_f\, g\, b\, t\, l + \rho_c^*\, b\, c\, l$
- Solve P/δ equation for ρ_c^* and substitute into weight equation
- Solve $\partial W/\partial c = 0$ and $\partial W/\partial t = 0$ to get t_{opt}, c_{opt}
- Substitute t_{opt}, c_{opt} into stiffness equation (P/δ) to get $\rho_{c\ opt}^*$
- Note that optimization possible by foam modeling $G_c = C_2\,(\rho^*/\rho_s)^2\, E_s$

$$\left(\frac{c}{l}\right)_{\text{opt}} = 4.3\left\{\frac{C_2\,B_2}{B_1^2}\left(\frac{\rho_f}{\rho_s}\right)^2\frac{E_s}{E_f^2}\left(\frac{P}{\delta\, b}\right)\right\}^{1/5}$$

$$\left(\frac{t}{l}\right)_{\text{opt}} = 0.32\left\{\frac{1}{B_1\,B_2^2\,C_2^2}\left(\frac{\rho_s}{\rho_f}\right)^4\frac{1}{E_f E_s^2}\left(\frac{P}{\delta\, b}\right)^3\right\}^{1/5}$$

$$\left(\frac{\rho_c^*}{\rho_s}\right)_{\text{opt}} = 0.59\left\{\frac{B_1}{B_2^3\,C_2^3}\left(\frac{\rho_s}{\rho_f}\right)\frac{E_f}{E_s^3}\left(\frac{P}{\delta\, b}\right)^2\right\}^{1/5}$$

Note: $\dfrac{W_{\text{faces}}}{W_{\text{core}}} = \dfrac{1}{4} \qquad \dfrac{\delta_b}{\delta} = \dfrac{1}{3} \qquad \dfrac{\delta_s}{\delta} = \dfrac{2}{3}$

Table 9.3 Optimum design of a sandwich panel subject to a stiffness constraint

Formulate objective function for the weight of the beam

$$W = 2\rho_f gblt + \rho_c^* gblc \qquad (Eqn.\ (9.7))$$

↓

Formulate the stiffness constraint

$$\delta/P = \frac{2l^3}{B_1 E_f btc^2} + \frac{l}{B_2 bcG_c^*} \qquad \text{(Eqn.(9.6))}$$

↓

Solve stiffness constraint for one variable (e.g. ρ_c^*)

$$\rho_c^* = \left\{\frac{B_1}{C_2 B_2}\frac{E_f}{E_s}\left(\frac{ltc}{B_1 btc^2 E_f(\delta/P) - 2l^3}\right)\right\}^{1/2} \rho_s \qquad \text{(Eqn.(9.8))}$$

and substitute this into the objective function

↓

Form $\partial W/\partial c = 0$ and $\partial W/\partial t = 0$

and solve to give

$c_{opt} = f(\delta/P$, *material properties, beam geometry*)

$t_{opt} = f(\delta/P$, *material properties, beam geometry*)

Substitute these into Eqn. (9.8) for core density to give optimum density, ρ_{copt}^*.

↓

Final Result:

Optimum		*As function of*
Core thickness, c_{opt}	}	Design stiffness, δ/P
Face thickness, t_{opt}	}	Material properties, $\rho_f, \rho_s, E_f, E_s, C_2$
Core density, ρ_{copt}^*	}	Loading geometry, B_1, B_2, l, b

Table 9.4 Optimization analysis for sandwich panels subject to a stiffness constraint

Geometry	W_f/W_c	δ_b/δ	δ_s/δ
Rectangular beam	1/4	1/3	2/3
Circular plate (distributed load over entire plate)	1/4	1/3	2/3
Circular plate (distributed load over radius r)	1/4	1/3	2/3

Comparison with experiments

- All faces with rigid PU foam core
- $G_c = 0.7\, E_s\, (\rho_c^*/\rho_s)^2$
- Beams designed to have same stiffness, P/δ, span l, width, b
- One set had $\rho_c^* = \rho_{c\ \mathrm{opt}}^*$, varied t, c
- One set had $t = t_{\mathrm{opt}}$, varied ρ_c^*, c
- One set had $c = c_{\mathrm{opt}}$, varied t, ρ_c^*
- Confirms minimum weight design; similar results with circular sandwich plates

Strength of sandwich beams

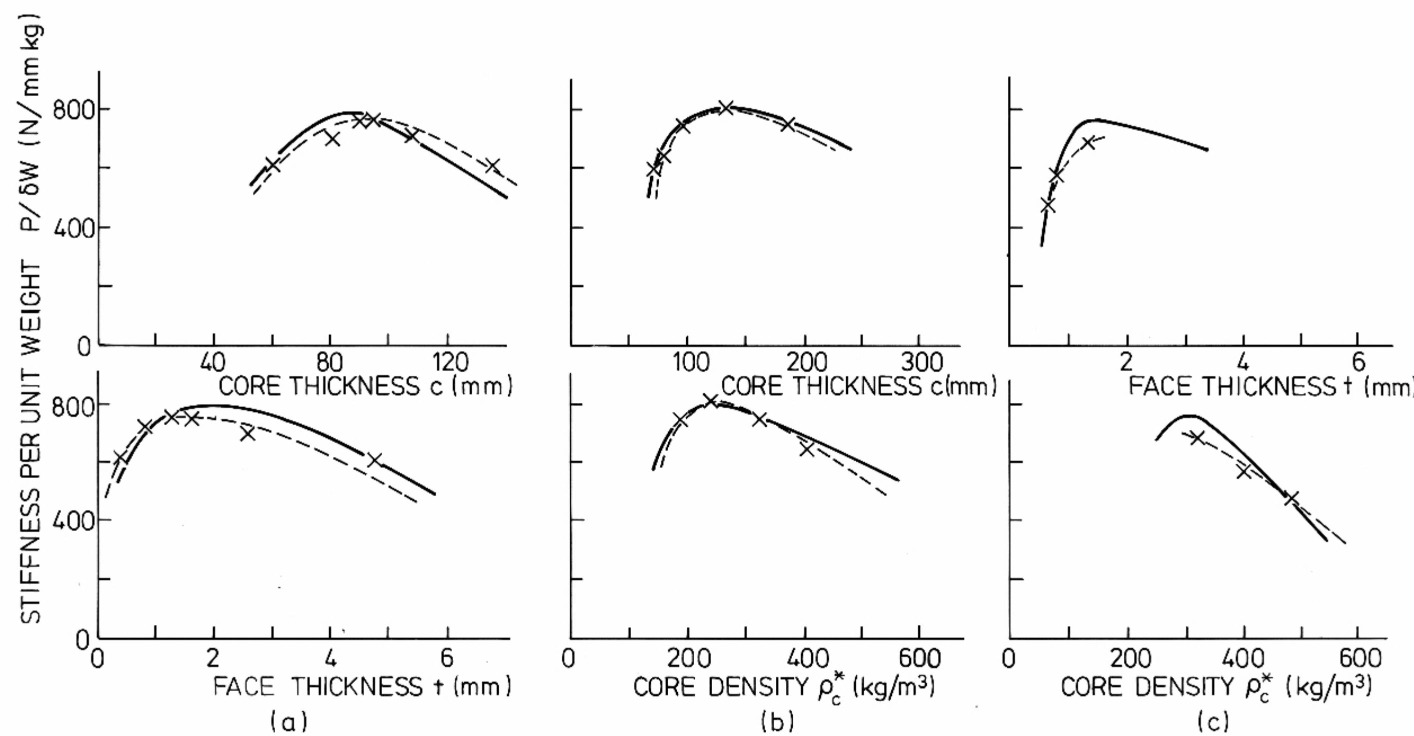
- Stresses in sandwich beams
  Normal stresses

$$\sigma_f = \frac{My}{(EI)_{eq}}\, E_f = M\, \frac{c}{2}\, \frac{2}{E_f\, b\, t\, c^2}\, E_f = \frac{M}{b\, t\, c}$$

$$\sigma_c = \frac{My}{(EI)_{eq}}\, E_c^* = M\, \frac{c}{2}\, \frac{2}{E_f\, b\, t\, c^2}\, E_c^* = \frac{M}{b\, t\, c}\, \frac{E_c^*}{E_f}$$

Since $E_c^* \ll E_f$ $\quad \sigma_c \ll \sigma_f \quad \Rightarrow$ faces carry almost all normal stress

Minimum Weight Design



Al faces; Rigid PU foam core

Figures 7, 8, 9: Gibson, L. J. "Optimization of Stiffness in Sandwich Beams with Rigid Foam Cores." *Material Science and Engineering* 67 (1984): 125-35. Courtesy of Elsevier. Used with permission.

- For beam loaded by a concentrated load, P (e.g. 3 pt bend)

$$M_{\max} = \frac{P\,l}{B_3} \text{ e.g. 3 pt bend } B_3 = 4\text{; cantilever } B_3 = 1$$

$$\boxed{\sigma_f = \frac{P\,l}{B_3\,btc}}$$

- Shear stresses vary parabolically through the cross-section, but if

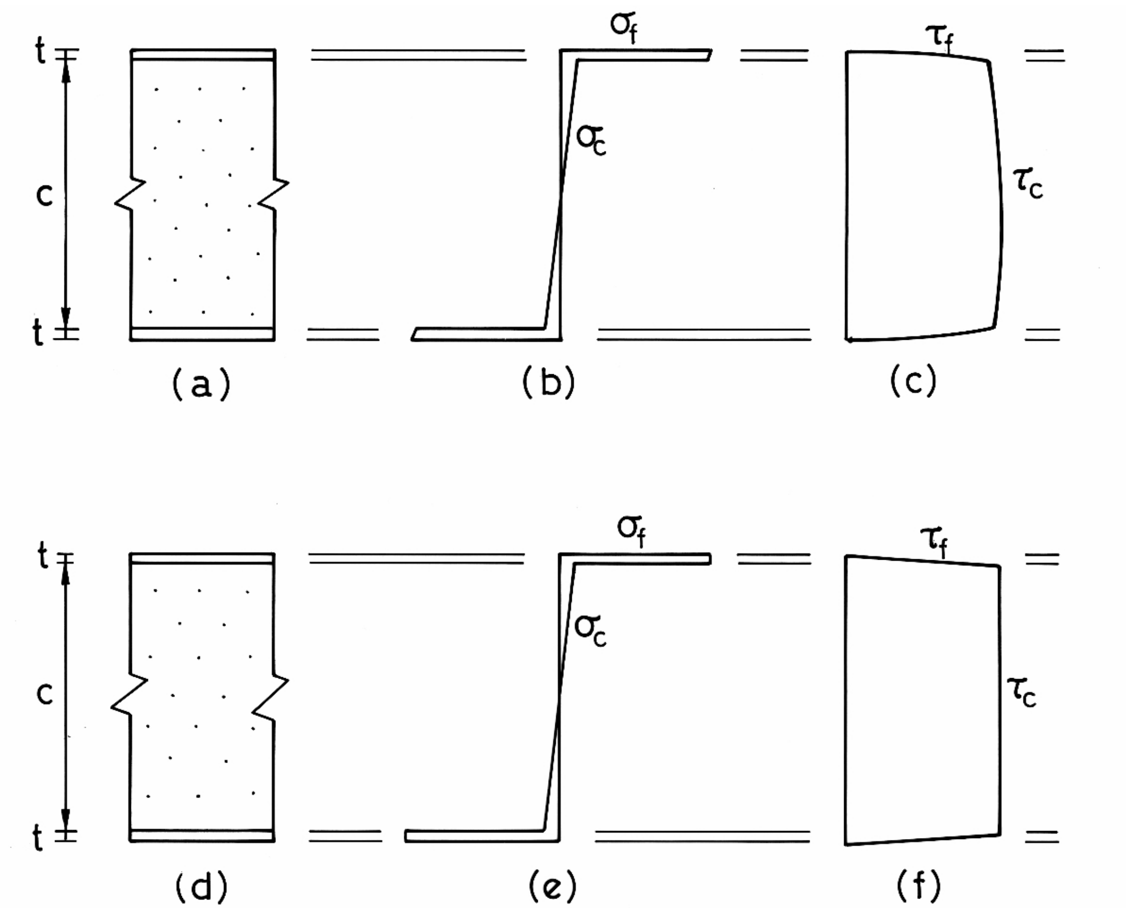
$E_f \gg E_c^*$ and $c \gg t$ $\qquad \tau_c = \frac{V}{b\,c}$ $\qquad$ V = shear force at section of interest

$$\boxed{\tau_c = \frac{P}{B_4\,bc}} \qquad V_{\max} = \frac{P}{B_4} \text{ e.g. 3 pt bend } B_4 = 2$$

Failure modes

face: can yield
compressible face can buckle locally – "wrinkling"
core: can fail in shear
also: can have debonding and indentation
we will assume perfect bond and load distributed sufficiently to avoid indentation

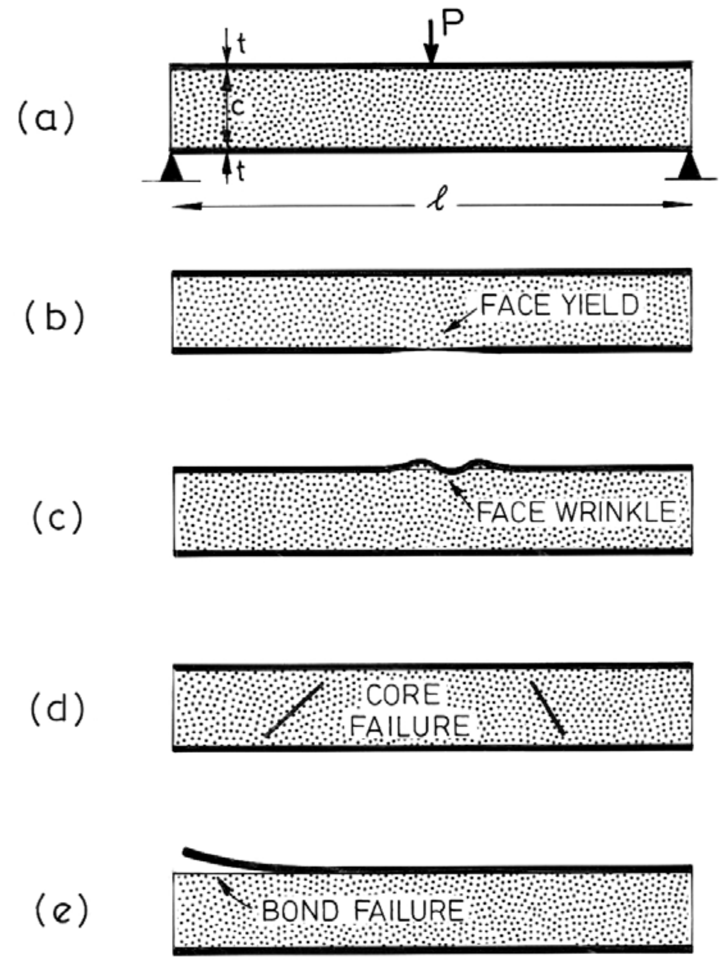
Stresses



Face: Normal stress
Core: Shear stress

Approximate stress distributions, for: $E_c << E_f$ and $t << c$

Failure Modes



(a) Face yielding

$$\sigma_f = \frac{P\,l}{B_s\,b\,t\,c} = \sigma_{yf}$$

(b) Face wrinkling: when normal stress in the face = local buckling stress

$$\sigma_{\text{buckling}} = 0.57\,E_f^{1/3}\,E_c^{*\,2/3} \qquad \text{buckling on an elastic foundation}$$

$$E_c^* = (\rho_c^*/\rho_s)^2\,E_s$$

$$\sigma_{\text{buckling}} = 0.57\,E_f^{1/3}\,E_s^{2/3}\,(\rho_c^*/\rho_s)^{4/3}$$

wrinkling occurs when $\sigma_f = \dfrac{P\,l}{B_s\,b\,t\,c} = 0.57\,E_f^{1/3}\,E_s^{2/3}\,(\rho_c^*/\rho_s)^{4/3}$

(c) Core shear failure

$$\tau_c = \tau_c^*$$

$$\frac{P}{B_4\,b\,c} = C_{11}\,(\rho_c^*/\rho_s)^{3/2}\,\sigma_{ys} \qquad C_{11} \approx 0.15$$

- Dominant failure load is the one that occurs at the lowest load
- How does the failure mode depend on the beam design?
- Look at transition from one failure mode to another
- At the transition — two failure modes occur at same load

face yielding: $P_{fy} = B_3\, b\, c(t/l)\, \sigma_{yf}$

face wrinkling: $P_{fw} = 0.57\, B_3\, b\, c\, (t/l)\, E_f^{1/3}\, E_s^{2/3}\, (\rho_c^*/\rho_s)^{4/3}$

core shear: $P_{cs} = C_{11}\, B_4\, b\, c\, \sigma_{ys}\, (\rho_c^*/\rho_s)^{3/2}$

- Face yielding and face wrinkling occur at some load if

$B_3\, b\, c\, (t/l)\, \sigma_{yf} = 0.57\, B_3\, b\, c\, (t/l)\, E_f^{1/3}\, E_s^{2/3}\, (\rho_c^*/\rho_s)^{4/3}$

or $(\rho_c^*/\rho_s) = \left(\dfrac{\sigma_{yf}}{0.57\, E_f^{1/3}\, E_s^{2/3}}\right)^{3/4}$

i.e. for given face and core materials, at constant (ρ_c^*/ρ_s)

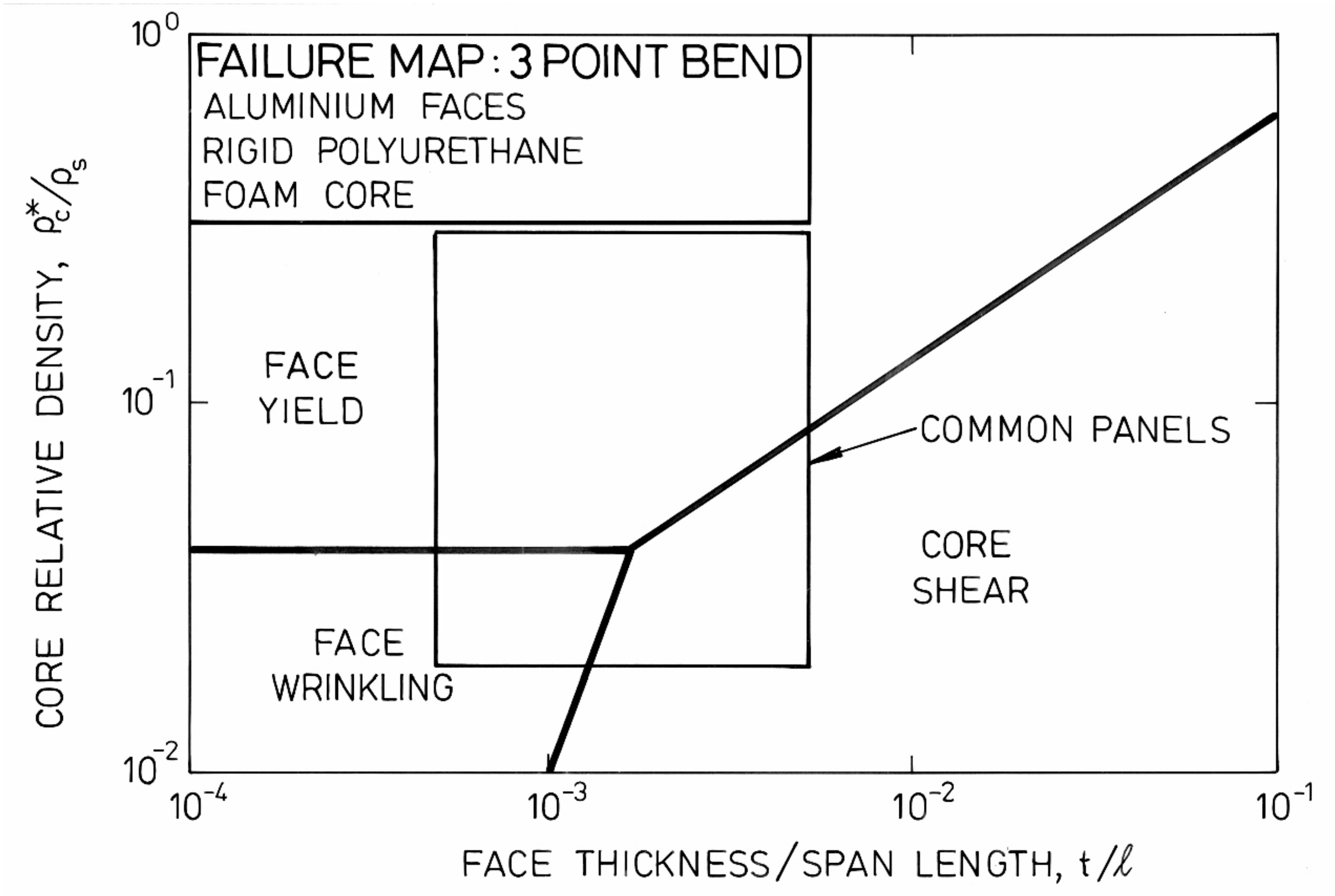
- Face yield — core shear $$\frac{t}{l} = \frac{C_{11}\, B_4}{B_3} \left(\frac{\rho_c^*}{\rho_s}\right)^{3/2} \left(\frac{\sigma_{ys}}{\sigma_{yf}}\right)$$
- Face wrinkling — core shear $$\frac{t}{l} = \frac{C_{11}\, B_4}{0.57\, B_3} \left(\frac{\sigma_{ys}}{E_f^{1/3}\, E_s^{2/3}}\right) \left(\frac{\rho_c^*}{\rho_s}\right)^{1/6}$$
- Note: transition equation only involve constants ($C_{11}\ B_3\ B_4$), material properties (E_f, E_s, σ_{ys}) and t/l, ρ_c^*/ρ_s; do not involve core thickness, c
- Can plot transition equation on plot with axes ρ_c^*/ρ_s and t/l
- Values of axes chosen to represent realistic values of

  ρ_c^*/ρ_s — typically 0.02 to 0.3

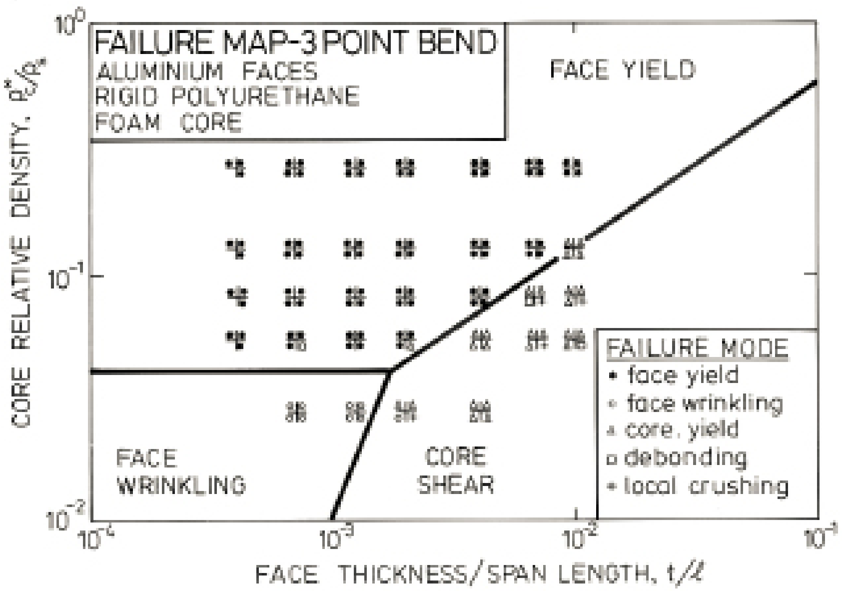
  t/l — typically $1/2000$ to $1/200 = 0.0005$ to 0.005

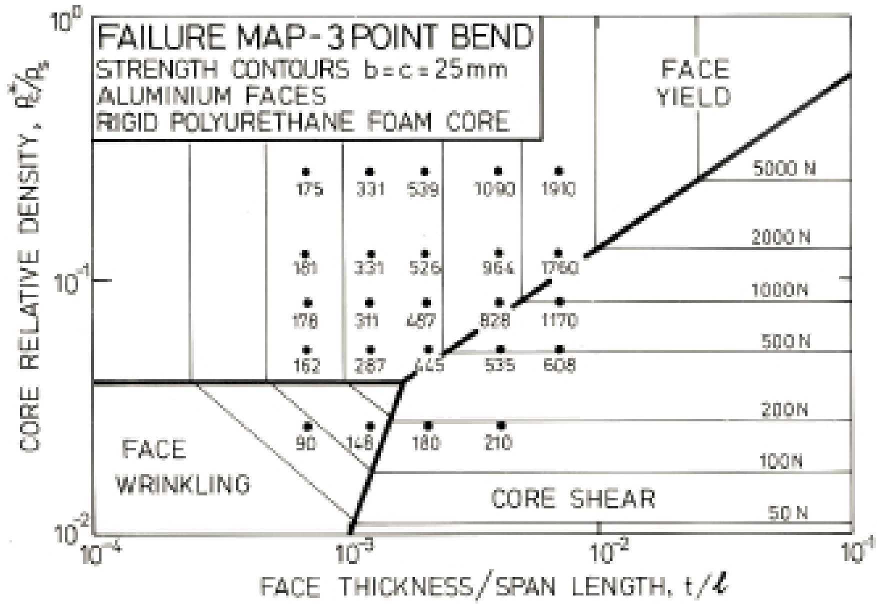
- Low values of t/l and $\rho_c^*/\rho_s \Rightarrow$ face wrinkling
  - t thin and core stiffness, which acts as elastic foundation, low
- Low values t/l, higher values $\rho_c^*/\rho_s \Rightarrow$ transition to face yielding
- Higher values of $t/l \Rightarrow$ transition to core failure

Failure Mode Map



Failure Map: Expts





Figures 12 and 13: Triantafillou, T. C., and L. J. Gibson. "Failure Mode Maps for Foam Core Sandwich Beams." *Materials Science and Engineering* 95 (1987): 37–53. Courtesy of Elsevier. Used with permission.

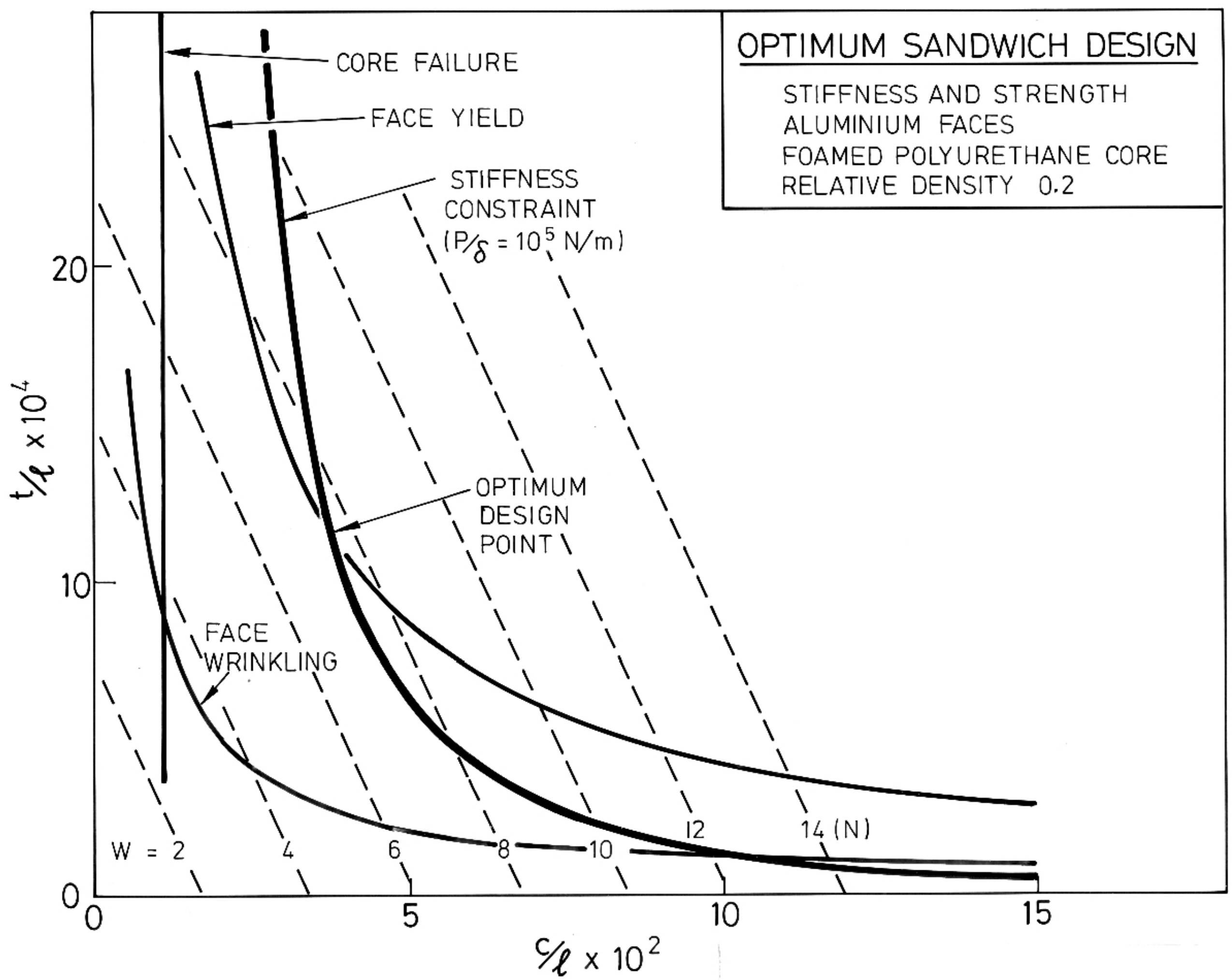
- Map shown in figure for three point bending ($B_2 = 4,\ B_4 = 2$)
- Changing loading configuration moves boundaries a little, but overall, picture similar
- Expts on sandwich beams with Al faces and rigid PU foam cores confirm equation
- If know b, c — can add contours of failure loads

Minimum weight design for stiffness and strength: t_{opt}, c_{opt}

Given:
stiffness P/δ
strength P_0
span l width D
loading configuration ($B_1\ B_2\ B_3\ B_4$)
face material ($\rho_f,\ \sigma_{yf},\ E_f$)
core material and density ($\rho_s,\ E_s,\ \sigma_{ys},\ \rho_c^*$)

Find: face and core thickness, t, c
to minimize weight

- Can obtain solution graphically, axes t/l and c/l
- Equation for stiffness constraint and each failure mode plotted
- Dashed lines — contours of weight
- Design-limiting constraints are stiffness and face yielding
- Optimum point — where they intersect
- Could add (ρ_c^*/ρ_s) as variable on third axis and create surfaces for stiffness and failure equation; find optimum in the same way
- Analytical solution possible but cumbersome
- Also, values of c/l obtained this way may be unreasonably large — then have to introduce an additional constraint on c/l (e.g. $c/l < 0.1$)



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Minimum weight design: materials

- What are best **materials** for face and core? (stiffness constraint)
- Go back to min. wt. design for stiffness
- Can substitute $(\rho_c^*)_{\text{opt}}$, t_{opt}, c_{opt} into weight equation to get min. wt.:

$$W = 3.18\, b\, l^2 \left[\frac{1}{B_1\, B_2^2\, C_2^2}\, \frac{\rho_f \rho_s^4}{E_f E_s^2} \left(\frac{P}{\delta\, b}\right)^3 \right]^{1/5}$$

- Faces: W minimized with materials that minimize ρ_f/E_f (or maximize E_f/ρ_f)
- Core: W minimized with materials that minimize ρ_s^4/E_s^2 (or maximize $E_s^{1/2}/\rho_s$)
- Note:
  - faces of sandwich loaded by normal stress, **axially**
    if have solid material loaded axially, want to maximize E/ρ
  - core loaded in shear and in the foam, cell edges **bend**
    if have solid material, loaded as beam in bending and want to minimize weight for a given stiffness, maximize $E^{1/2}/\rho$
- Sandwich panels may have face and core same material: e.g. Al faces Al foam core
  integral polymer face and core
  "structural polymer foams"
  - then want to maximize $E^{3/5}/\rho$

Case study: Downhill ski design

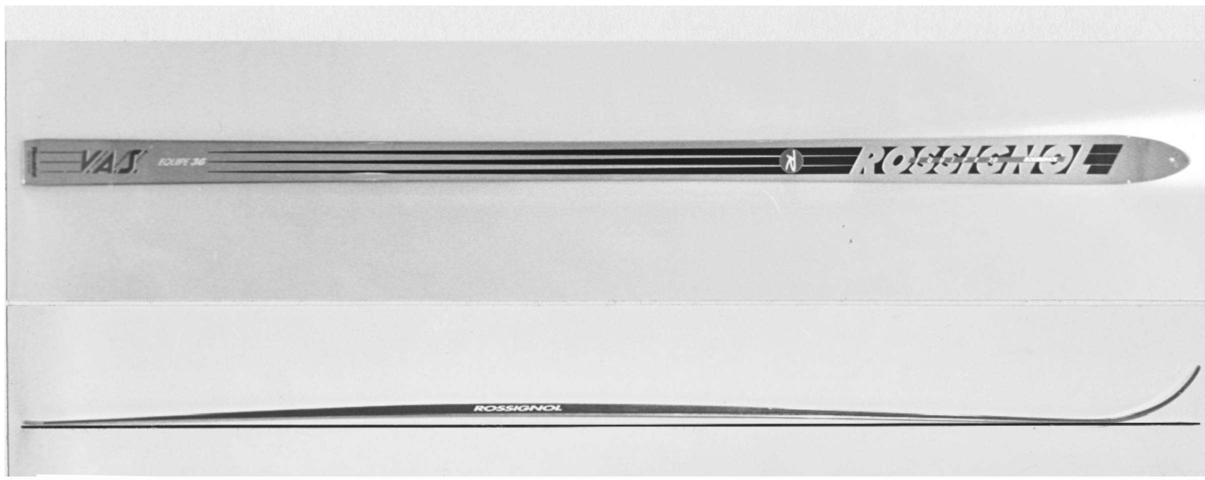
- Stiffness of ski gives skier right "feel"
- Too flexible — difficult to control

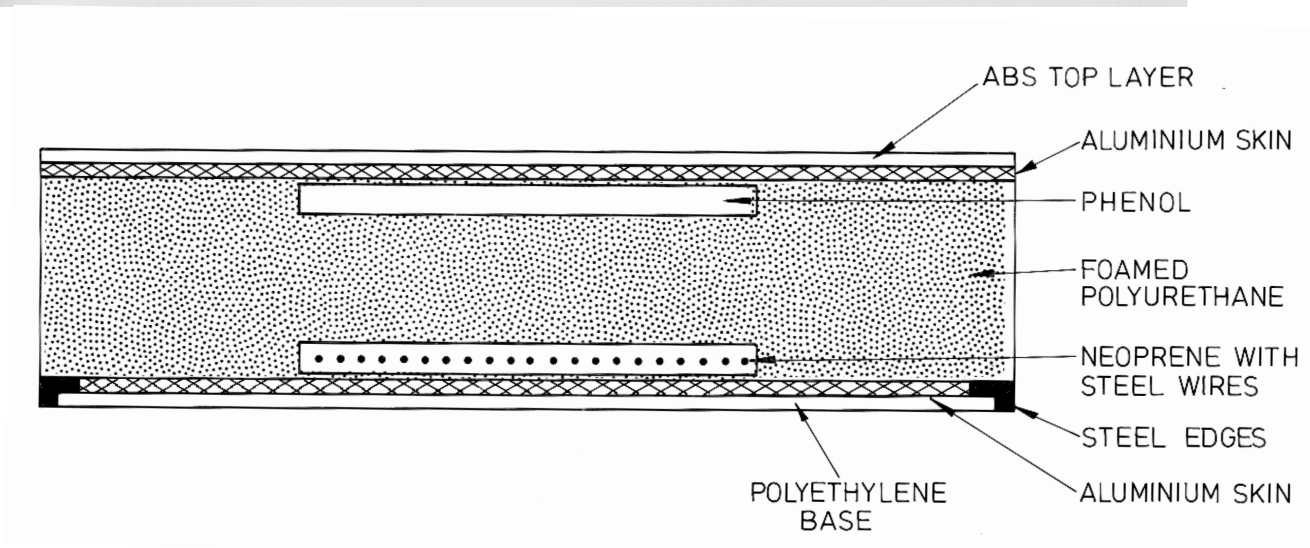
- Too stiff — skier suspended, as on a plank, between bumps
- Skis designed primarily for stiffness
- Originally skis made from a single piece of wood
- Next — laminated wood skis with denser wood (ash, hickory) on top of lighter wood core (pine, spruce)
- Modern skis – sandwich beams

  – faces — fiber composites or Al
  – core — honecombs, foams (e.g. rigid PU), balsa

  controls stiffness
- Additional materials
  - bottom-layer of polyethylene — reduces friction
  - short strip phenol — screw binding in
  - neoprene strip ~ 300 mm long — damping
  - steel edges — better control

Ski Case Study





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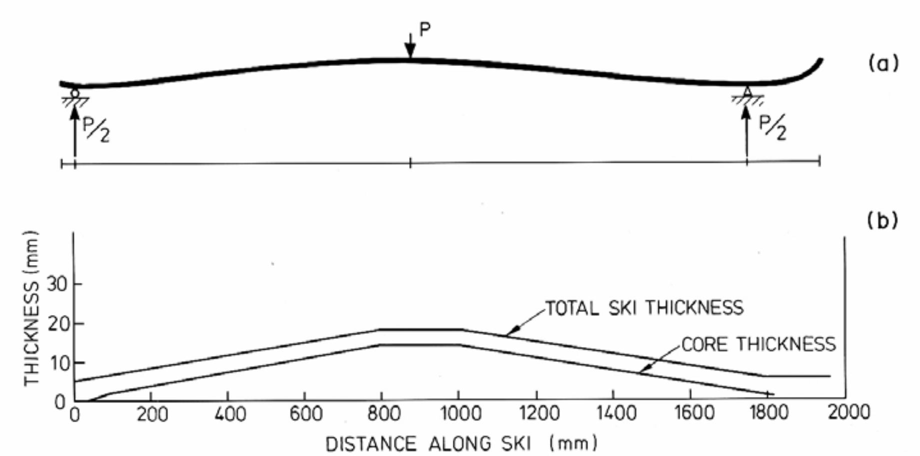
Ski case study

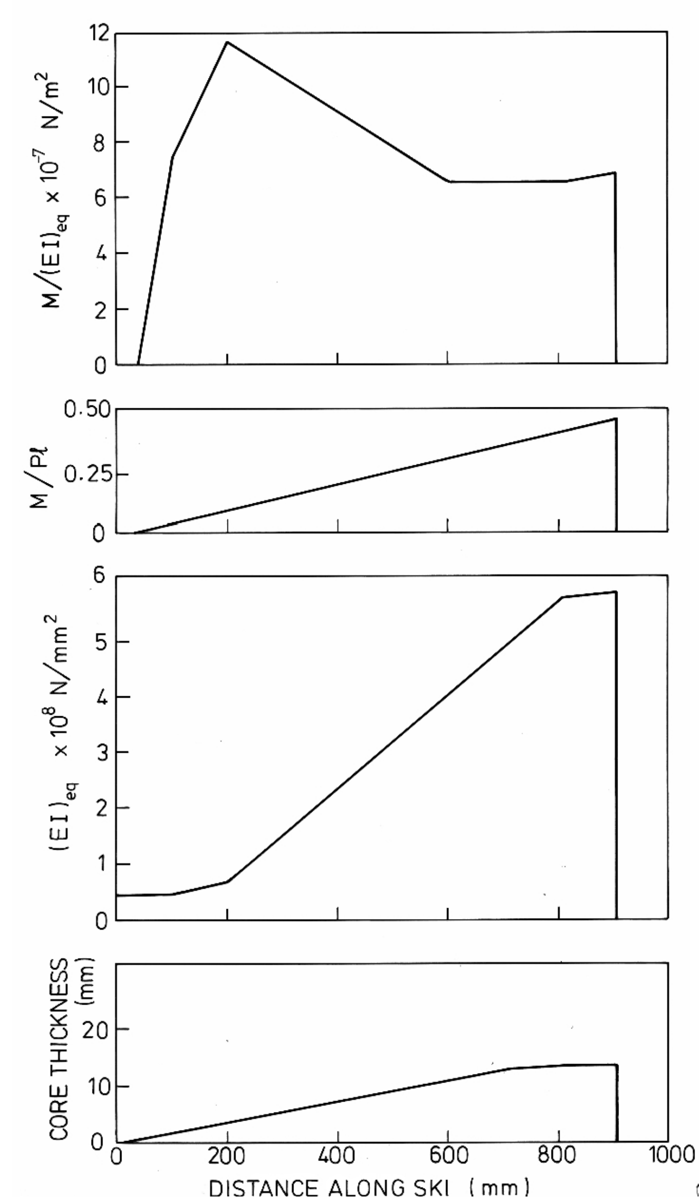
- Properties of face and core material

	Al	Solid PU	Foam PU
$\rho(\mathrm{Mg/m^3})$	2.7	1.2	0.53
E GPa	70	1.94	0.38
G GPa	–	–	0.14

- Ski geometry
    - Al faces constant thickness t
    - PU foam core — c varies along length, thickest at center, where moment highest
    - ski cambered
    - mass of ski = 1.3 kg (central 1.7 m, neglecting tip and tail)

Ski Case Study





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Bending stiffness

- Plot c vs. x, distance along ski
- Calculated $(\mathrm{EI})_{\mathrm{eq}}$ vs. x
- Calculated moment applied vs. x
- Get $\mathrm{M}/(\mathrm{EI})_{\mathrm{eq}}$ vs. x
- Can then find bending deflection, $\delta_b = 0.28$ P
- Shear deflection found from avg. equiv. shear rigidity

$$\delta_s = \frac{P\, l}{(AG)_{eq}} = 0.0045 \text{ P}$$

- $\delta = \delta_b + \delta_s = 0.29$ P $\quad P/\delta = 3.5$ N/mm $\quad$ measured $P/\delta = 3.5$ N/mm
- Note current design $\delta_s \ll \delta_b$; at optimum $\delta_s \sim 2\delta_b$ (constant c)
- Can ski be redesigned to give same stiffness, P/δ, at lower weight?
- If use optimization method described earlier (assuming c=constant along length)

c_{opt}=70 mm $\quad$ mass=0.31kg $\Rightarrow$ 75% reduction from current design

t_{opt}=0.095 mm

$p^*_{\mathrm{c\ opt}}$=29 kg/m^3 $\quad$ But this design impractical $\Rightarrow$ c too large, t too small

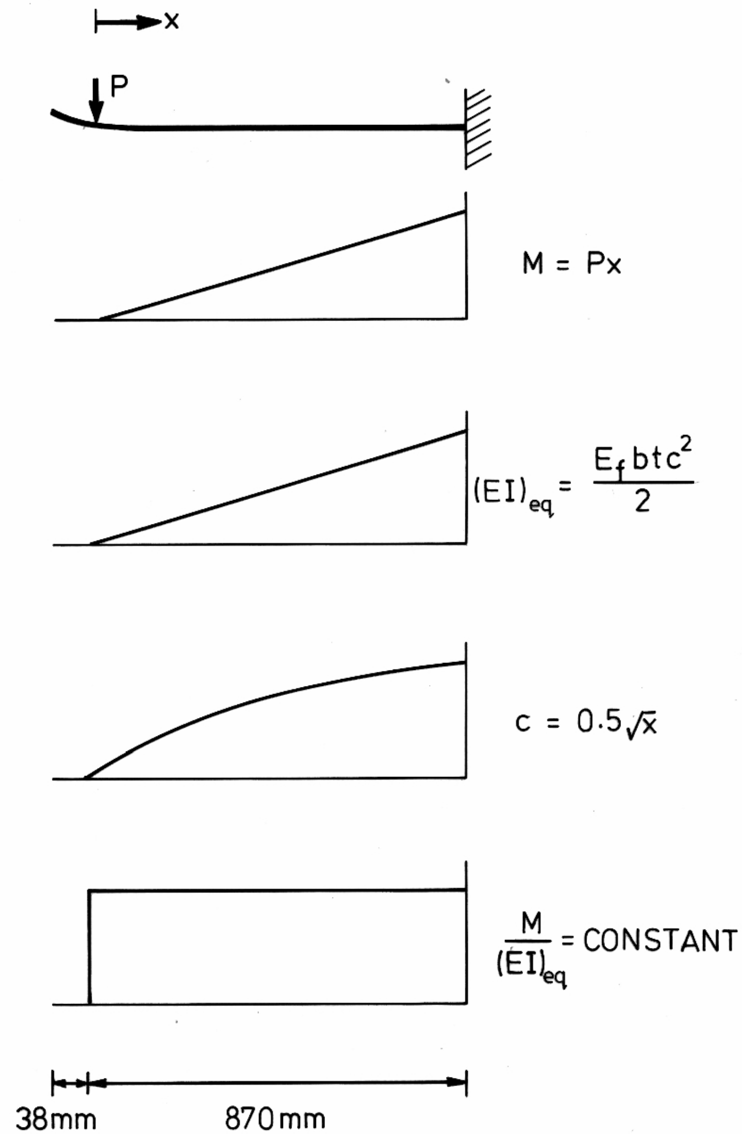
Alternative approach:

- Fix c = max value practical under binding and profile c to give constant $M/(EI)_{eq}$ along length of ski (use c_{max} = 15 mm)
- Find values of t, ρ_c^* to minimize weight for P/ρ=3.5 N/mm
- Moment M varies linearly along the length of the ski
- Want $(EI)_{eq}$ to vary linearly, too; $(EI)_{eq} = E_f\, b\, t\, c^2/2$
- Want $c \propto \sqrt{x}$, distance along length of ski
- Half length of ski is 870 mm and c_{max} =15 mm

$$c = 15 \left(\frac{x}{870}\right)^{1/2} = 0.51\, x^{1/2} \text{ (mm)}$$

- Can now do minimum weight analysis with

$$\delta = \frac{P\, l^3\, 2}{B_1\, E_f\, b\, t\, (c_{max} + t)^2} + \frac{P\, l}{B_2\, C_2\, b\, c_{max}(\rho_c^*/\rho_s)^2 E_s}$$



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- B_1 — corresponds to beams with constant M/EI
- B_2 — cantilever value ($B_2 = 1$) multiplied by average value of c divided by maximum value of c $B_2 = 2/3$
- Solve stiffness equation for ρ_c^*, substitute into weight equation and take $\dfrac{\partial \omega}{\partial t} = 0$
- Solve for t_{opt}, then $\rho^*_{\text{c opt}}$
- Find: c_{max}=15 mm $\rho^*_{\text{c opt}}$=1.63 kg/m^3
  $t_{opt} = 1.03$ mm mass = 0.88 kg $\Rightarrow$ 31% less than current design

Daedalus

- MIT designed and built human powered aircraft (1980s)
- Flew 72 miles in $\sim$ 4 hours from Crete to Santorini, 1988
- Kanellos Kanellopoulos — Greek bicycle champion pedaled and flew

mass	68.5#	=	31 kg	propeller: kevlar faces, PS foam core (11′ long)
length	29′	=	8.8 m	wiring and trailing edge strips kevlar faces / rohacell foam core
wingspin	112′	=	34 m	tail surface struts: carbon composite faces, balsa core

Daedalus

Courtesy of NASA. Image is in the public domain. NASA Dryden Flight Research Center Photo Collection.

Mass = 31 kg

Length = 8.8m

Wingspan = 34m

Propeller blades = 3.4m

Flew 72 miles, from Crete to Santorin, in just under 4 hours

Sandwich panels: propeller, wing and tail trailing edge strips, tail surface struts

Image: MIT Archives

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
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