Lecture 17, Energy Absorption Notes, 3.054

Energy absorption in foams

- Impact protection must absorb the kinetic energy of the impact while keeping the peak stress below the threshold that causes injury or damage
- Direction of the impact may not be predictable
- Impact protection must itself be light e.g. helmet

- Capacity to undergo large deformation ($\epsilon \sim 0.8, 0.9$) at constant σ
- Absorb large energies with little increase in peak stress

- Foams — roughly isotropic — can absorb energy from any direction - light and cheap
- For a given peak stress, foam will always absorb more energy than solid it is made from
- Strain rates: Instron typically $\dot{\epsilon} \sim 10^{-8}$ to 10^{-2}/s

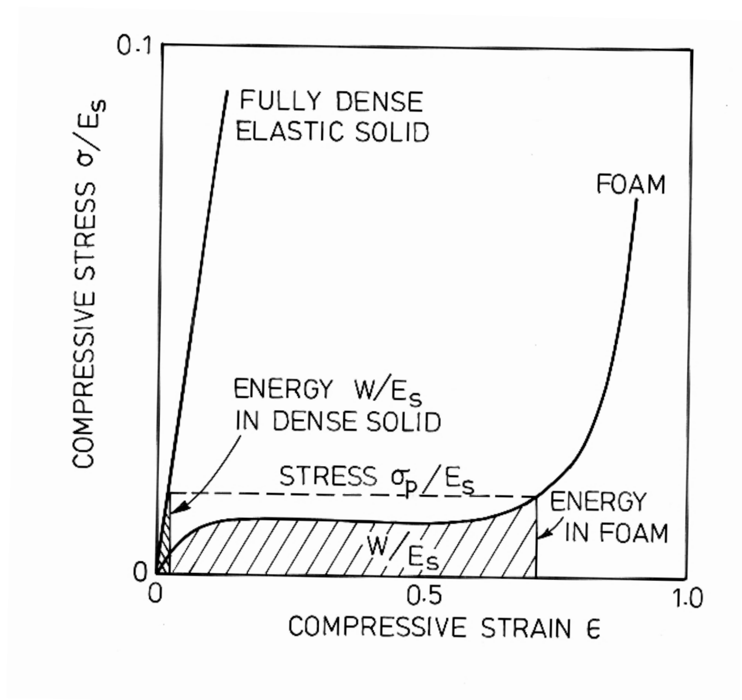
impact e.g. drop from height of 1 m, if thickness of foam=100mm

$$v_{\text{impact}} = \sqrt{2gh} = \sqrt{2(9.8)(1)} = 4.4\,\text{m/s}; \quad \dot{\epsilon}\frac{4.4\text{m/sec}}{0.1\,m} = 44/s$$

• servo controlled Instrons, drop hammer tests — up to $\dot{\epsilon} = 100/s$

blast: $\dot{\epsilon} = 10^3 - 10^4/s$ — inertial effects impt (we won't consider this)

Energy Absorption



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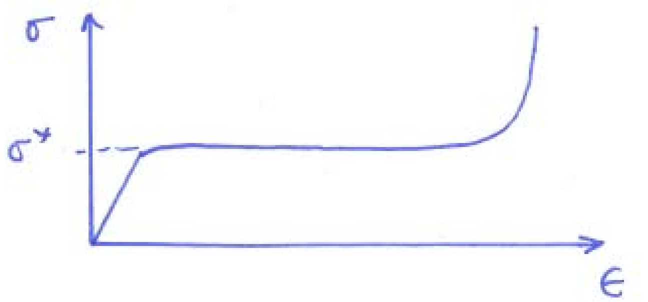
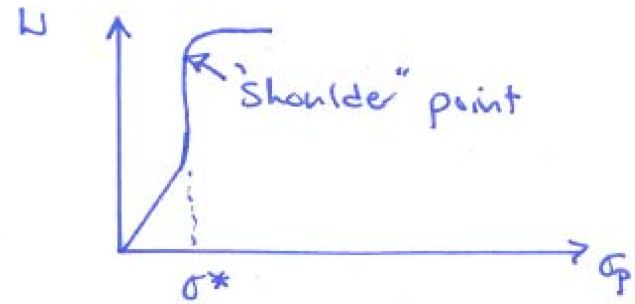
Energy absorption mechanisms

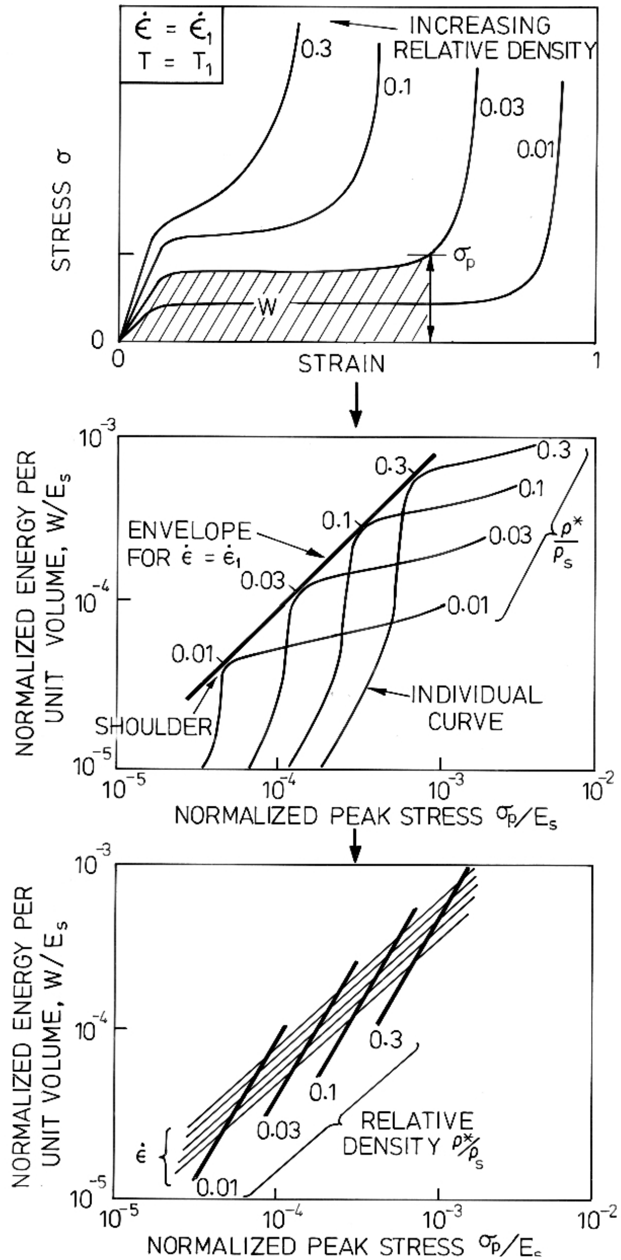
- Elastomeric foams — elastic buckling of cells
  — elastic deformation recovered $\rightarrow$ rebound
  — also have damping - energy dissipated as heat
- Plastic foams, brittle foams — energy dissipated as plastic work or work of fracture
  — no rebound
- Natural cellular materials — may have fiber composite cell walls
  — dissipate energy by fiber pullout and fracture
- Fluid within cells
  - open cell foams — fluid flow dissipation only important if fluid is viscous, cells are small or rates are high
  - closed cell foams — compression of cell fluid
    — energy recovered as unkading

Energy absorption diagrams





- At stress plateau, energy W increases with little increase in peak stress, σ_p
- As foam densifies, $W \sim$ constant and σ_p increases sharply
- Ideally, want to be at "shoulder" point
- More generally — see Figure
- Test series of one type of foam of different ρ^*/ρ_s at constant $\dot{\epsilon}$ and temperature, T
- Plot W/E_s vs. σ_p/E_s for each curve (E_s at standard $\dot{\epsilon}$ and T)
- Heavy line joins the shoulder points for each curve
- Mark ρ^*/ρ_s for each foam on that line
- Repeat for varying $\dot{\epsilon}$ $\rightarrow$ join lines for constant ρ^*/ρ_s
- Build up family of optimum energy absorption curves
- Can treat different temperatures, T, in same way



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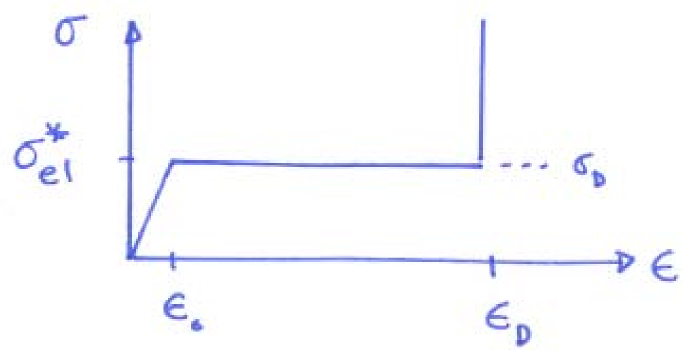
Notes:

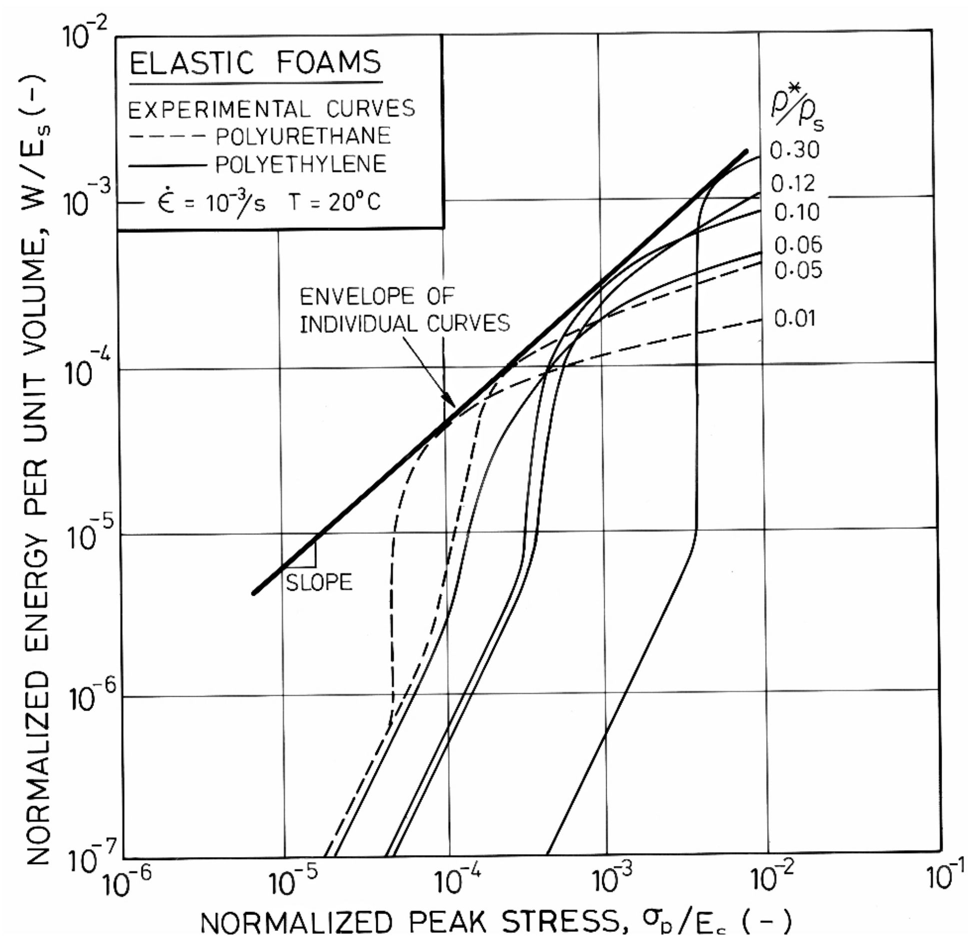
- Elastomeric foams can all be plotted on one curve since $E^* \propto E_s$ and $\sigma^*_{el} \propto E_s$ (normalize W/E_s and σ_p/E_s)
- Figure: polyurethane and polyethylene
- polymethacrylimid: $\sigma^*_{pl} \Rightarrow$ typical of foams with plastic collapse stress with $\sigma_{ys}/E_s = 1/30$
- Can generate energy absorption diagrams from data, or use models for foam properties

Modelling energy absorption diagrams

Open cell elastomeric foams



Elastomeric Foams



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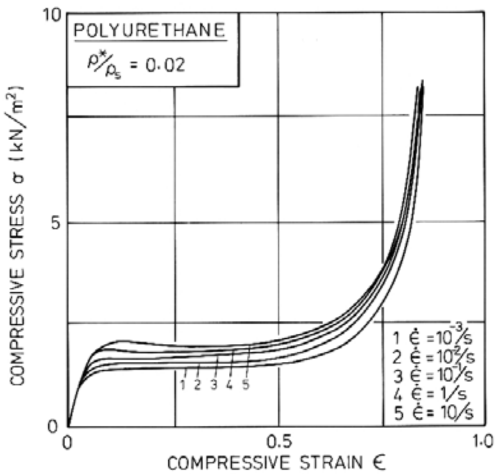
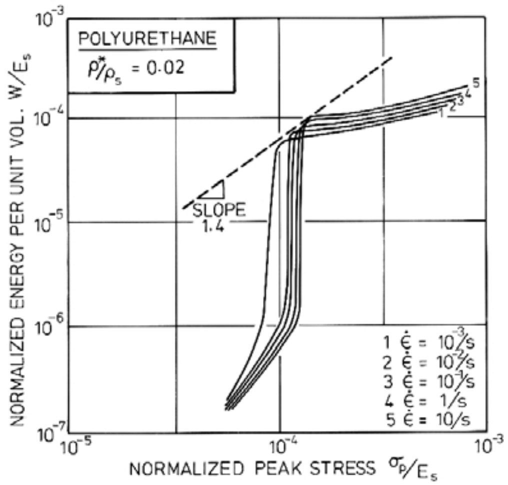
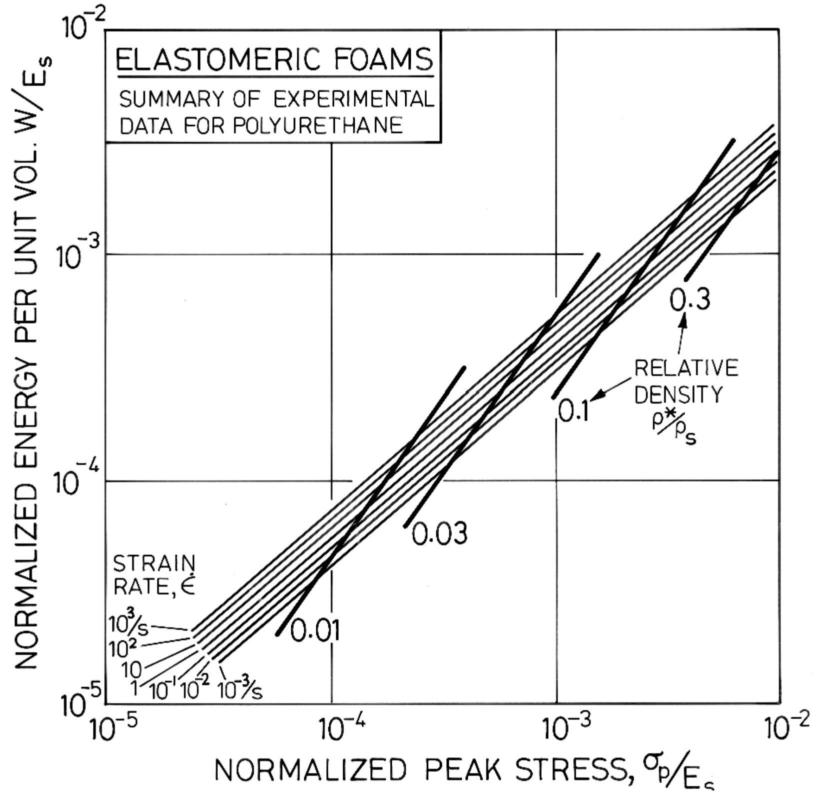
Flexible Polyurethane







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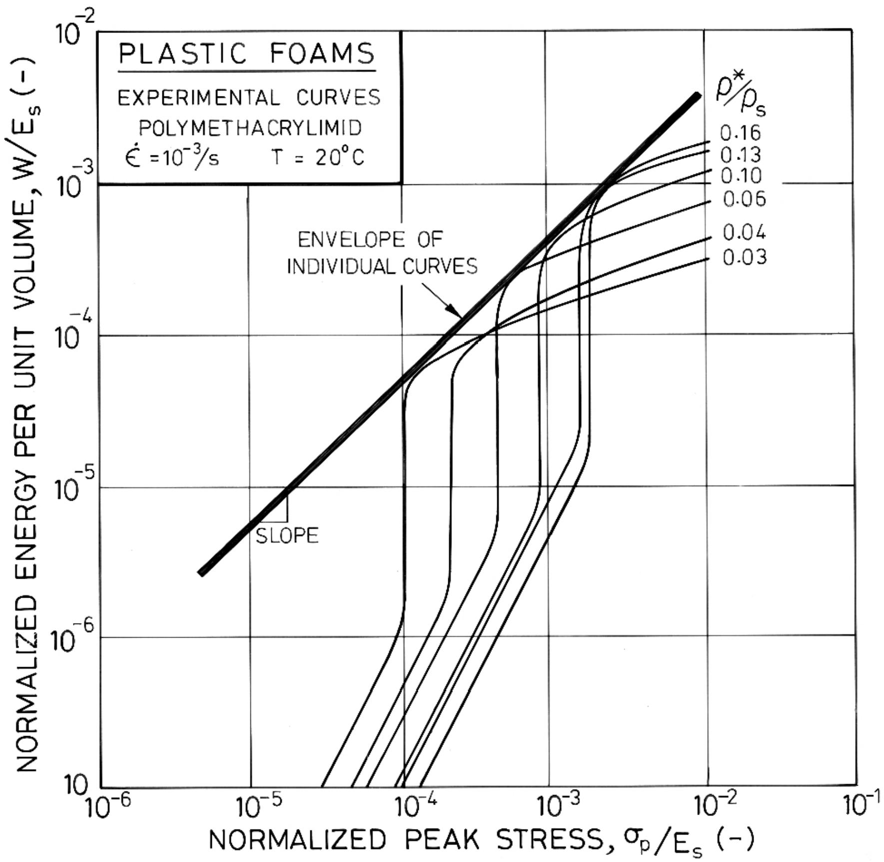
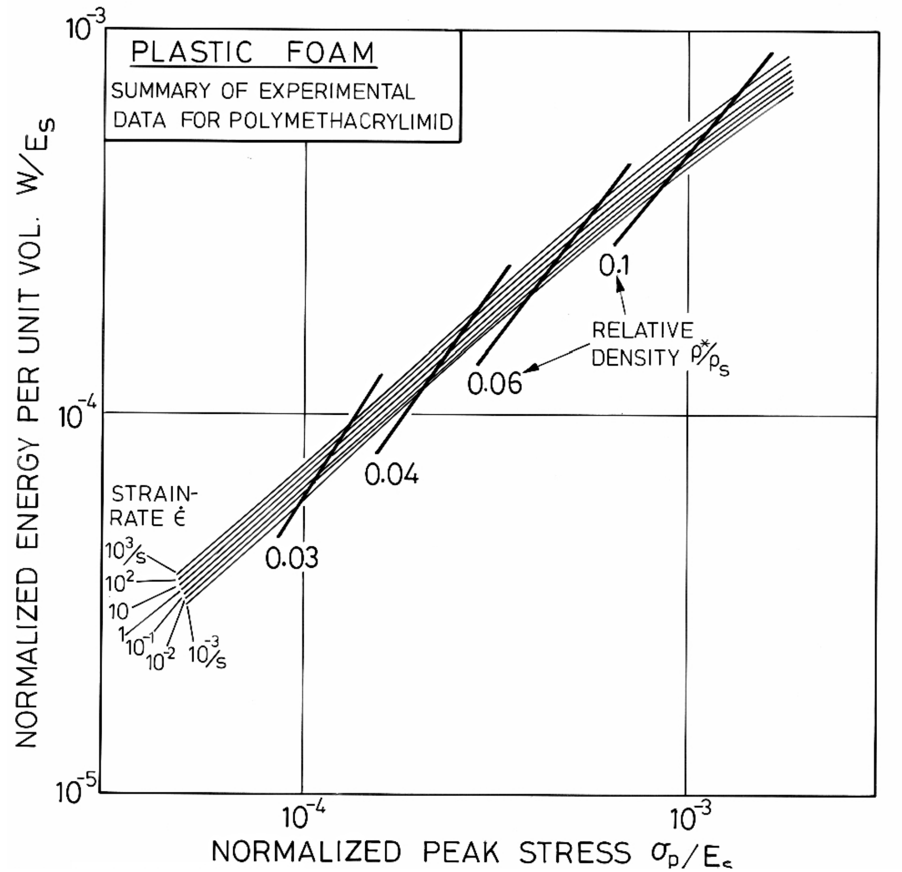
Polymethacrylimid





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(a) Linear elastic region $\epsilon < \epsilon_0$

$$W = \frac{1}{2}\frac{\sigma_p^2}{E^*} \qquad \frac{W}{E_s} = \frac{1}{2}\left(\frac{\sigma_p^2}{E_s}\right)^2 \frac{1}{(\rho^*/\rho_s)^2}$$

(b) Stress plateau $\epsilon_0 < \epsilon < \epsilon_D$

$$dW = \sigma_{el}^* \, d\epsilon \qquad \frac{W}{E_s} = 0.05\,(\rho^*/\rho_s)^2\,(\epsilon - \epsilon_0)$$

- family of vertical lines on figure
- plateau ends at densification strain ϵ_D
- then W/E_s vs. σ_p/E_s becomes horizontal

(c) At end of stress plateau $\epsilon \sim \epsilon_D$

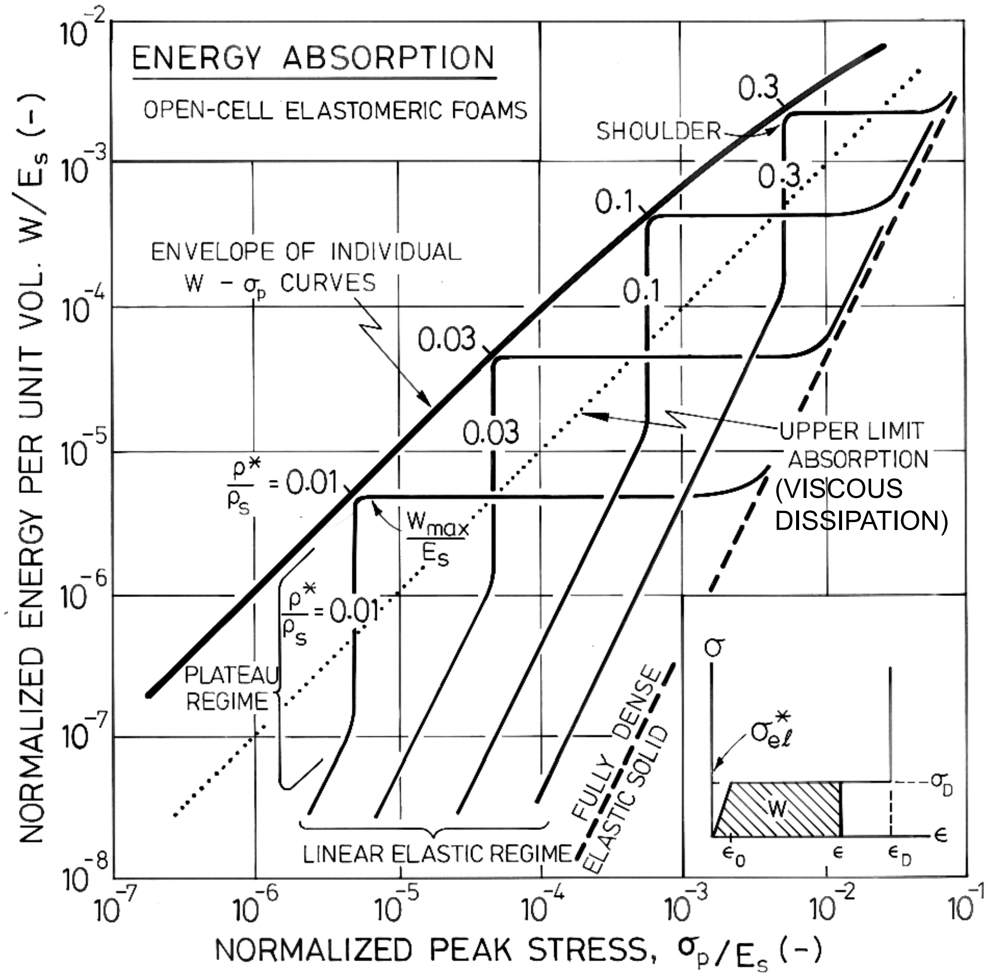
- maximum energy absorbed just before reach ϵ_D (shoulder point)

$$\frac{W_{\max}}{E_s} - = 0.05\,(\rho^*/\rho_s)^2(1 - 1.4\,\rho^*/\rho_s) \text{ (assuming } \epsilon_0 << \epsilon_D \text{ and neglecting } \epsilon_0)$$

- optimum choice of foam is one with shoulder point that lies at $\sigma_p = \sigma_D$
- envelope of shoulder points, for optimum foams, at:

$$\sigma_p = \sigma_D = 0.05\,E_s(\rho^*/\rho_s)^2 \qquad \rho^*/\rho_s = \left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}$$

Open-cell Elastomeric Foams: Modelling



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Substituting into equation for $W_{\max}/E_s$:

$$\frac{W_{\max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 1.4\left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}\right] \qquad\qquad \frac{W_{\max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 6.26\left(\frac{\sigma_p}{E_s}\right)^{1/2}\right]$$

- Line of slope 1 at low stresses, falling to 7/8 at higher σ

(d) Densification

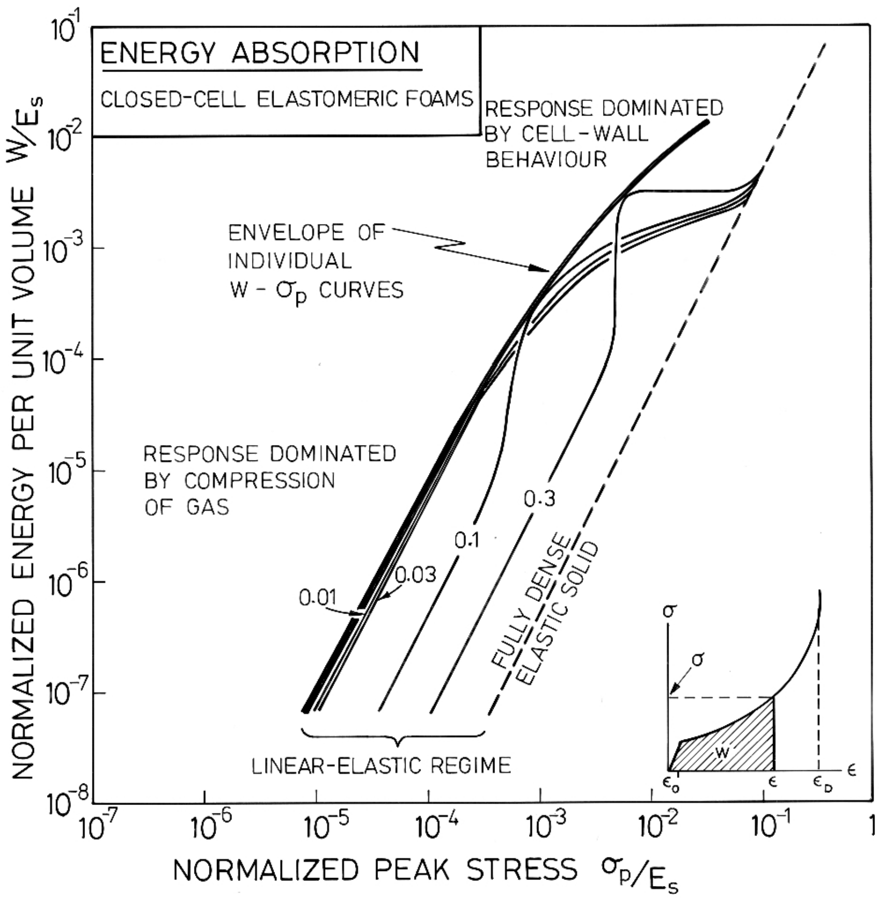
- when foam fully densified and compressed to a solid, then energy absorption curve joins that for the fully dense elastomer

$$\frac{W}{E_s} = \frac{1}{2}\frac{\sigma_p^2}{E_s}$$

Note:

- Model curves have same shape as expts.
- Model shows W/E_s depends on σ_p/E_s and ρ^*/ρ_s only — one diagram for all elastomer foams
- For a given W/E_s, σ_p/E_s for the foam less than that of the fully dense solid, by a factor of 10^{-3} to 10^{-1}

Closed-cell Elastomeric Foams: Modelling



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Modeling: open-cell foams that yield

- Analysis similar to elastomeric foams, with σ_{pl} replacing σ_{el}
- Note that some closed cell foams that yield, face contribution to E^* σ_{pl} negligible
- Neglect fluid contribution

$$\sigma^*_{pl} = 0.3\,\sigma_{ys}(\rho^*/\rho_s)^{3/2}$$

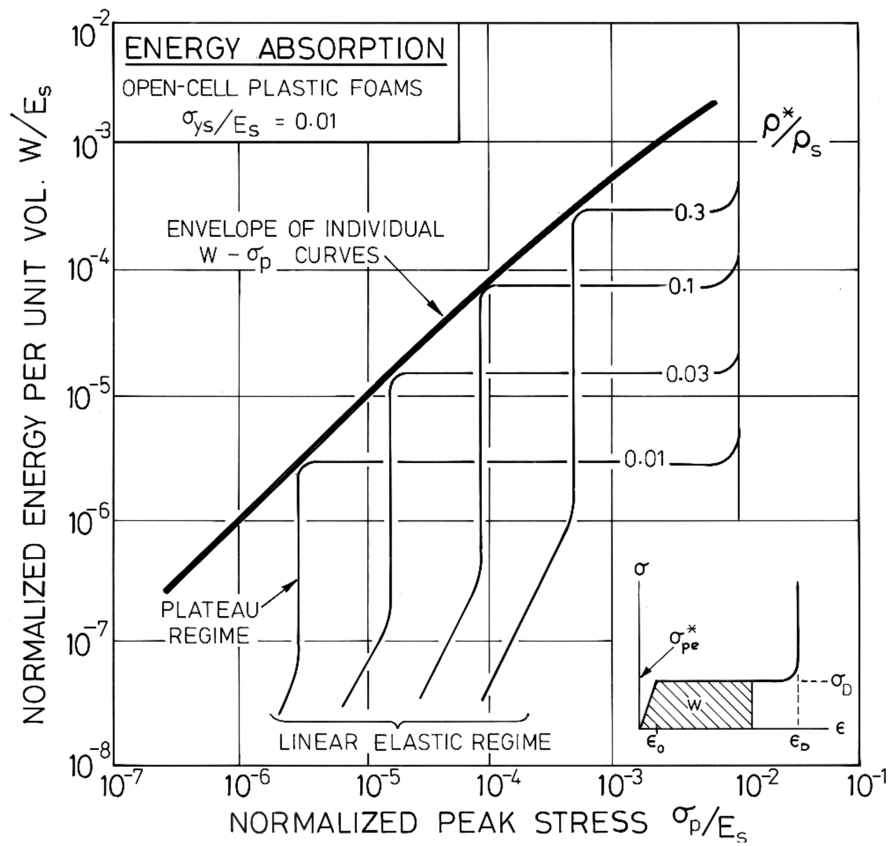
(a) Linear elastic regime: same as elastomeric foam: $\dfrac{W}{E_s} = \dfrac{1}{2}\left(\dfrac{\sigma_p}{E_s}\right)^2 \dfrac{1}{(\rho^*/\rho_s)^2}$

(b) Stress plateau: $\dfrac{W}{E_s} = 0.3\ \dfrac{\sigma_{ys}}{E_s}\left(\dfrac{\rho^*}{\rho_s}\right)^{3/2}(\epsilon - \epsilon_0)$

(c) End of stress plateau: $\dfrac{W_{\max}}{E_s} \approx 0.3\ \dfrac{\sigma_{ys}}{E_s}\left(\dfrac{\rho^*}{\rho_s}\right)^{3/2}(1 - 1.4\,\rho^*/\rho_s)$

- optimum choice of foam — absorbs maximum energy without σ_p rising sharply at ϵ_D

Plastic Foams: Modelling



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- Curve of optimum energy absorption (heavy line on figure) is envelope that touches $W - \sigma_p$ curve at shoulder points
- For σ_p, $\dfrac{\rho^*}{\rho_s} = \left(\dfrac{3.3\,\sigma_p}{\sigma_{ys}}\right)^{2/3}$
- Substituting in $W_{\max}/E_s$ equation: $\dfrac{W_{\max}}{E_s} = \dfrac{\sigma_D}{E_s}\left\{1 - 3.1\left(\dfrac{\sigma_D}{\sigma_{ys}}\right)^{2/3}\right\}$
- Model curves explain general features of experimental curves
- Modeling — curves less general than for elastomers
  — this cure for a particular value of $\sigma_{ys}/E_s = 1/100$
  (typical value for polymers)

Design and selection of foams for impact protection

- Typically know object to be protected and some details about it

mass, m	max allowable acceleration, a
contact area, A	(e.g. head injury - 100g)
max drop height, h	peak stress allowable, σ_p
(or energy to be absorbed, u)	

- Variables: foam material, density, thickness

Example 1

Given: mass, m=0.5 kg
contact area, A=0.01 m^2
drop height, h=1 m
max deceleration, a=10g
foam: flexible polyurethane $E_s = 50$ MPa

Find: optimum foam density
optimum foam thickness

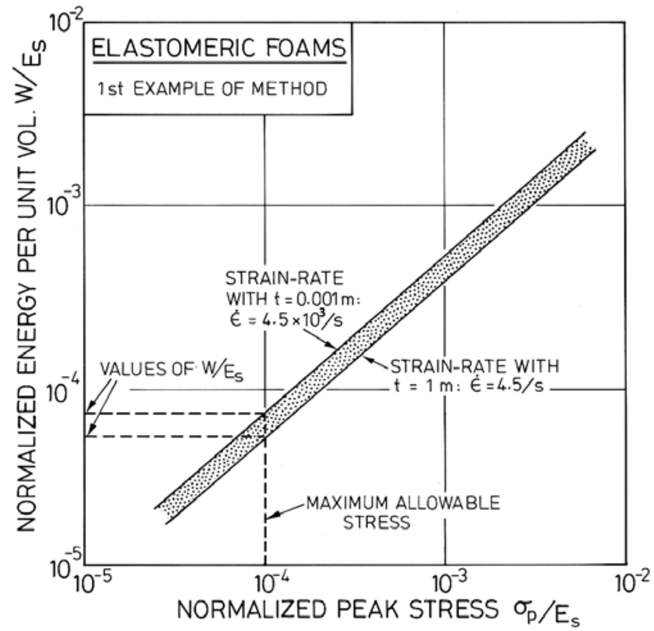
Example 1:
Find Foam Density and Thickness

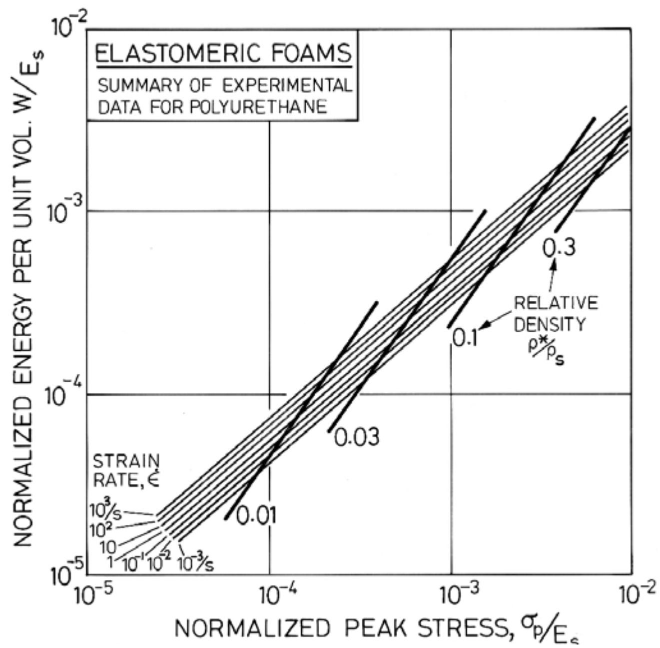
Table 8.2 Example 1: selection of foams

Specification of the problem

Mass of the package object, $m = 0.5\,\mathrm{kg}$
Area of contact between foam and object, $A = 0.01\,\mathrm{m}^2$
Velocity of package on impact (drop height $h = 1\,\mathrm{m}$), $v = 4.5\,\mathrm{m/s}$
Energy to be absorbed, $U = mv^2/2 = 5\,\mathrm{J}$
Maximum allowable package force (based on deceleration of $10g$), $F = ma = 50\,\mathrm{N}$
Maximum allowable peak stress, $\sigma_p = F/A = 5\,\mathrm{kN/m^2}$
Solid modulus in foam (flexible polyurethane), $E_s = 50\,\mathrm{MN/m^2}$
Maximum allowable normalized peak stress, $\sigma_p/E_s = 10^{-4}$

Iterative procedure		
1st Iteration	$t_1 \gg t$	$t_1 \ll t$
Initial choice of t_1	1 m	0.001 m
Resulting strain-rate, $\dot{\epsilon} = v/t_1$	$4.5\,\mathrm{s}^{-1}$	$4.5 \times 10^3\,\mathrm{s}^{-1}$
Resulting (W/E_s) at $\sigma_p/E_s = 10^{-4}$	5.25×10^{-5}	7.4×10^{-5}
Energy absorbed per unit volume, W	$2620\,\mathrm{J/m^3}$	$3700\,\mathrm{J/m^3}$
2nd Iteration		
Revised t_2 (from $U = WAt$)	0.19 m	0.14 m
Revised $\dot{\epsilon} = v/t_2$	$24\,\mathrm{s}^{-1}$	$32\,\mathrm{s}^{-1}$
Revised (W/E_s)	6.6×10^{-5}	6.7×10^{-5}
Revised W	$3300\,\mathrm{J/m^3}$	$3350\,\mathrm{J/m^3}$
3rd Iteration		
Revised t_3 (from $U = WAt$)	0.15 m	0.15 m
Optimum density, ρ^*/ρ_s (Fig. 8.8)	A little below 0.01	





- Energy to be absorbed, u=mgh=(0.5 kg)(10 m/s ^{2})(1m)=5 J
- Maximum allowable force on package = F=ma=(0.5 kg)(10g)=50 N
- Peak stress, σ_p=F/A=50 N / 0.01 m^2=5 kN/m^2
- Normalized peak stress, σ_p/E_s=5 kPa / 50 MPa = 10^{-4}
- Draw vertical line on energy absorption diagram at $\sigma_p/E_s = 10^{-4}$
- Need to know $\dot{\epsilon} \approx v/t$ velocity $v = \sqrt{2gh} = 4.5$ m/s
- Iterative approach — choose arbitrary thickness, t

- e.g. $t_1 = 1$ m
  $\dot{\epsilon} = 4.5$/s
  $W/E_s = 5.25 \times 10^{-5}$
  $W = 2620$ J/m^3
  (u=WAt$_1$) $t_2 = WA/u = 0.19$ m
  $\dot{\epsilon}_2 = 24$/s
  $W/E_s = 6.6 \times 10^{-5}$
  $W = 3300$ J/m^3

$t_1 = 0.001$ m
$\dot{\epsilon} = 4.5 \times 10^3$/s
$W/E_s = 7.4 \times 10^{-5}$
$W = 3700$ J/m^3
$t_2 = 0.14$ m
$\dot{\epsilon}_2 = 32$/s
$W/E_s = 6.7 \times 10^{-5}$
$W = 3350$ J/m^3

Third iteration: t_3=0.15 m
(both W)
Optimum density (Fig)
$\rho^*/\rho_s \sim 0.01$
Note: t converges quickly even from very different initial guesses for t

Example 2

Given	m = 2.5 kg	Find	foam material
	A = 0.025 m^2		foam density
	t = 20 mm		
	h = 1 m		
	a = 100 g		

Calculate W, σ_p, $\dot{\epsilon}$

$$W = \frac{mgh}{At} = \frac{(2.5\text{ kg})(10\text{ m/s}^2)(1\text{ m})}{0.025\text{ m}^2(0.02\text{ m})} = 5 \times 10^{-4}\text{ J/m}^3$$

$$\sigma_p = \frac{F_{\max}}{A} = \frac{ma}{A} = \frac{(2.5\text{ kg})(100)(10\text{ m/s}^2)}{0.025\text{ m}^2} = 10^5\text{ N/m}^2$$

$$\dot{\epsilon} = \frac{v}{t} = \frac{\sqrt{2gh}}{t} = \frac{\sqrt{2(10\text{ m/s}^2)(1\text{ m})}}{0.02\text{ m}} = \frac{4.5\text{ m/s}}{0.02\text{ m}} = 225\text{ /s}$$

Select arbitrary value of E_s=100 MPa

Plot $\quad W/E_s = 5 \times 10^{-4}$ $\quad$ point A

$\sigma_p/E_s = 10^{-3}$

Example 2:
Find Foam Material and Density

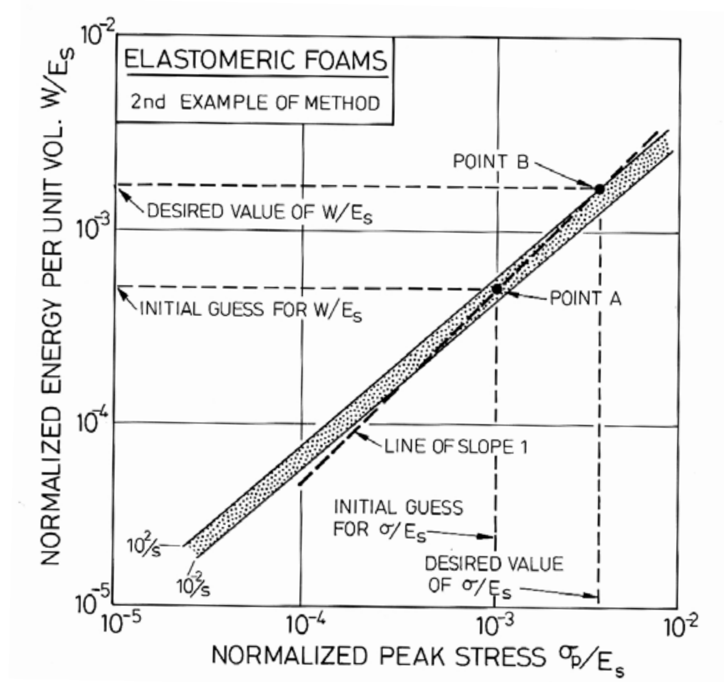
Table 8.3 Example 2: selection of foams

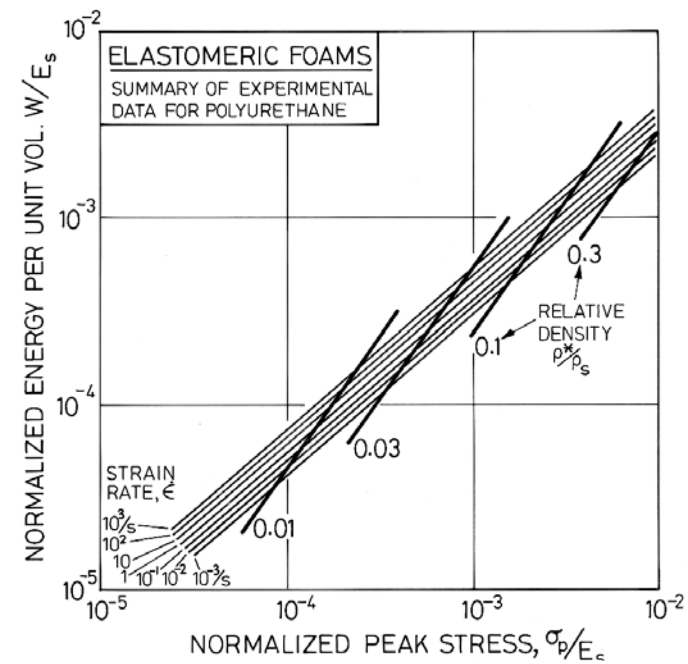
Specification of the problem

Mass of the package object, $m = 2.5\,\text{kg}$
Area of contact between foam and object, $A = 0.025\,\text{m}^2$
Thickness of foam, $t = 20\,\text{mm}$
Drop height, $h = 1\,\text{m}$
Velocity of impact $v = (2gh)^{1/2} = 4.5\,\text{m/s}$
Strain-rate $\dot{\epsilon} = v/t = 225/\text{s}$
Energy to be absorbed $U = mgh = 25\,\text{J}$
Energy to be absorbed per unit volume of foam $W = U/At = 5 \times 10^4\,\text{J/m}^3$
Maximum allowable force (based on decleration of $100g$) $= 2500\,\text{N}$
Maximum allowable peak stress $\sigma_p = F/A = 10^5\,\text{N/m}^2$

Trial design point A, using $E_s = 100\,\text{MN/m}^2$
Normalized energy $W/E_s = 5 \times 10^{-4}$
Normalized peak stress $\sigma_p/E_s = 10^{-3}$

Final design point B, read from diagram
Normalized energy $W/E_s = 1.8 \times 10^{-3}$
Normalized stress $\sigma_p/E_s = 3.7 \times 10^{-3}$
Resulting derived value of $E_s = 28\,\text{MN/m}^2$
Desired foam density ≈ 0.1



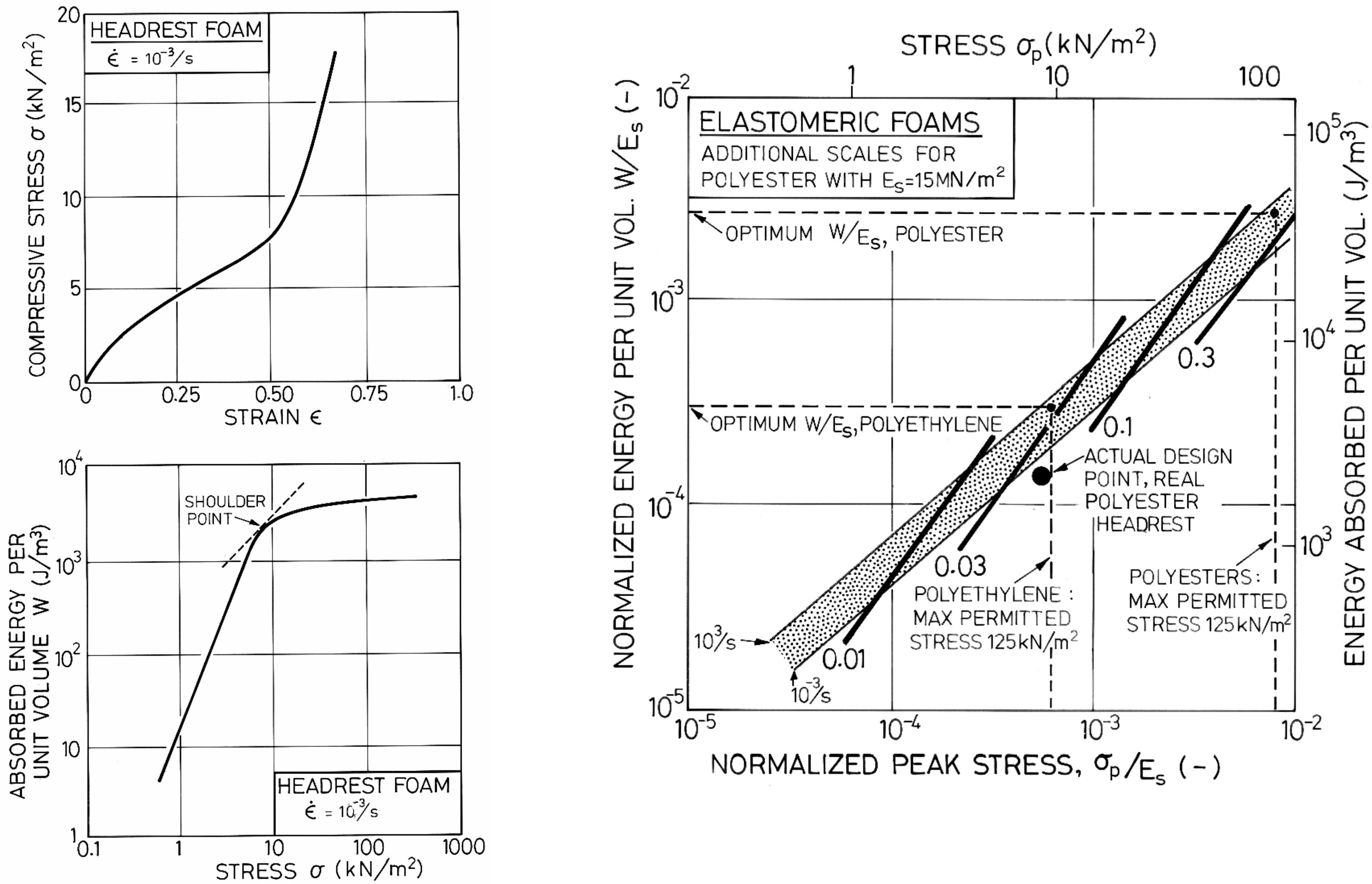


- Construct a line of slope 1 through this point (broken line)
- Moving along this line simply changes E_s
- Select the point where the broken line intersects the appropriate $\dot{\epsilon} \sim 10^2$/s (point B)
- Read off values of $W/E_s = 1.8 \times 10^{-3}$
  $\sigma_p/E_s = 37 \times 10^{-3}$
- Resulting value of $E_s = 28$ MPa $\Rightarrow$ low modulus, flexible polyurethane
- Replotting on more detailed figure: $\rho^*/\rho_s = 0.1$
- If point A above all energy contours and lone of slope 1 does not intersect them, specification cannot be achieved, A or t has to increase
- If point A below all contours, then A and t larger than need to be — can be reduced

Case study: design of car head rest

- Head rest should absorb kinetic energy of head while keeping force less than that which would cause injury
- Example in book:
  mass of head = 25 kg
  max. deceleration = a = 50 g = 500 m/s^2
  area of contact, A = 0.01 m^2
  thickness of padding t = 0.17 m
  max. allowable force F = ma = 1250 N
  max. allowable stress σ_p = F/A = 125 kN/m^2
  energy to be absorbed/vol, W = $\frac{1/2\, mv^2}{At}$ = 735 v^2 J/m^3
  peak strain rate $\dot{\epsilon}$=v/t [s^{-1}]
  current material — flexible polyester foam $\rho^*/\rho_s = 0.06$
  from plot: for σ_p = 125 kN/m^2 W = 5 × 10^3 J/m^3
  maximum collision velocity = v = $\sqrt{\frac{W}{735}} = \sqrt{\frac{5\times10^3}{735}}$ = 2.6 m/s = 5.8 mph

Car Head Rest Design



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Alternative design #1

- Consider energy absorption diagram for elastomeric foams
- Add sales for polyester (using E_s=15 MPa)
- For $\sigma_p = 125$ kN/m^2 could use polyester foam ρ^*/ρ_s=0.2

  then $W/E_s = 2.6 \times 10^{-3}$ and v = 7.3 m/s =16 mph

Alternative design #2

- Use different material e.g. low density open cell polyethylene $E_s = 200$ MPa
- $\sigma_p/E_s = \frac{0.125}{200} = 6.3 \times 10^{-4}$
- At $\dot{\epsilon} = v/t \approx 100$/s (estimated)

  $W/E_s = 3.2 \times 10^{-4}$ (from fig)

  $W = (3.2 \times 10^{-4})(200MPa) = 6.4 \times 10^4$ J/m^4

  $v = \sqrt{\frac{W}{735}} = \sqrt{\frac{6.4\times10^4}{735}} = 9.3$ m/s = 21 mph

- Reading from figure 1: $\rho^*/\rho_s = 0.03$

Case study: foams for bicycle helmets

US: 600-700 bicycle deaths/year
$>$ 90% not wearing a helmet
$\sim$ 50,000 cyclists injured (2009)

(US Nat. Hwy Traffic Safety Admin Bicycle Helmet Safety Inst.)

- Helmets consist of solid outer shell and foam liner (e.g. expanded PS)
- Liner thickness typically 20 mm
- Wish to absorb as much energy as possible while keeping peak acceleration less than that to cause head injury
- Foam liner
  - Redistributes load over larger area, reducing stress on head
  - Peak stress on head limited by plateau stress of foam (as long as don't reach densification)
  - Max. tolerable acceleration = 300 g (if for a few milliseconds)
  - Mass of head $\approx$ 3 kg

    $$\mathrm{F_{max}} = \mathrm{ma} = (3 \text{ kg})(300)(10 \text{ m/s}^2)\text{=9 kN}$$

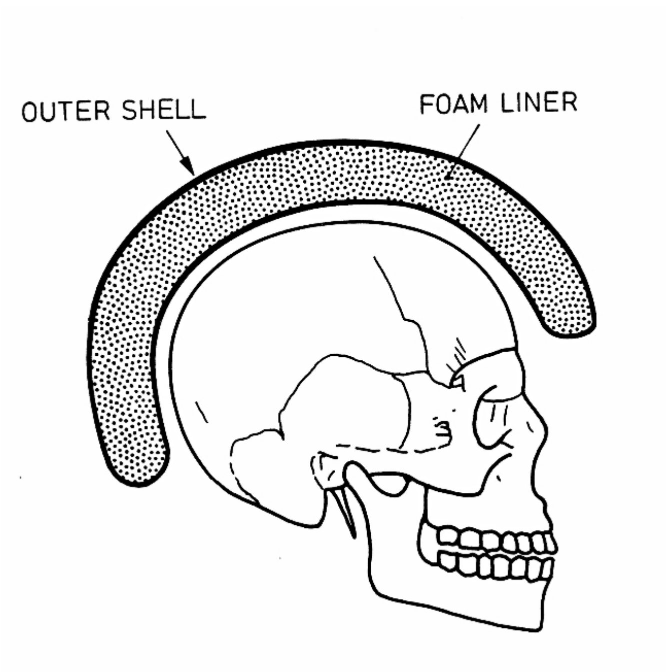
  - As foam crushes, it distributes load over area $\sim A \sim 0.01\mathrm{m}^2$ (may be high)

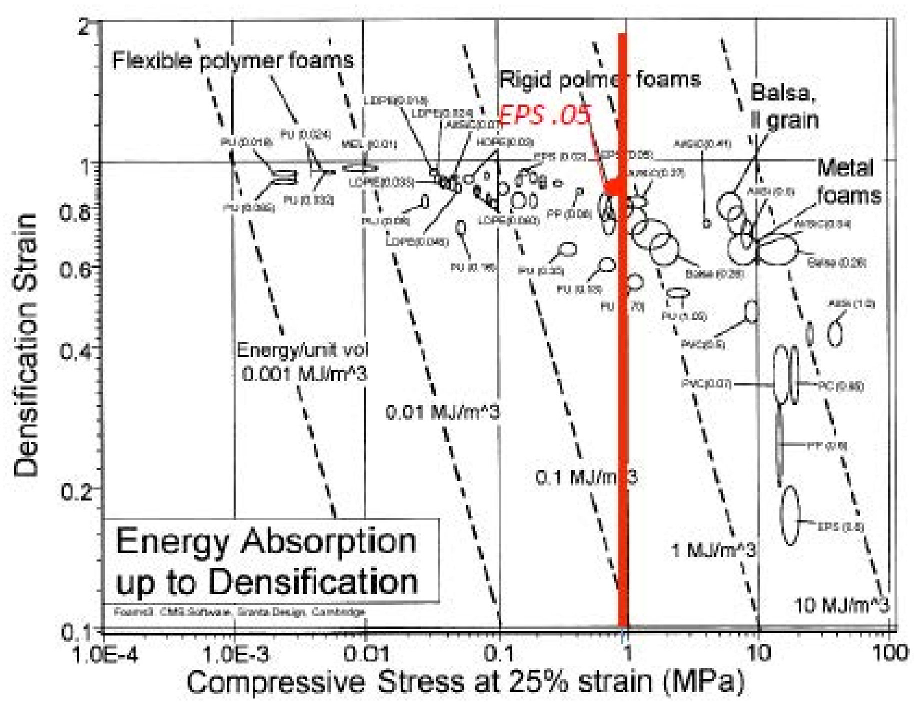
    $$\sigma_p = \frac{9\mathrm{kN}}{0.01\mathrm{m}^2} = 0.9 \text{ MPa}$$

Figure $\Rightarrow$ EPS $\rho^* = 0.05$ Mg/m^3
absorbs W = 0.8 MJ / m^3

- Diagram allows easy identification of possible candidate materials
- More complete analysis can then be done
- Energy absorbed U=0.8×10^6 J/m^3 $\times$ 0.01 m^2 $\times$ 0.02 m = 160 J (u=WAt)
- $1/2\, mv^2 = U$; $v_{\max} = \sqrt{\dfrac{2U}{m}} = \sqrt{\dfrac{2(160)\mathrm{kg}}{3\ \mathrm{kg}}\dfrac{\mathrm{m}^2}{\mathrm{s}^2}}$=10 m/s $\approx$ 22 mph

Case Study:
Foams for Bicycle Helmets





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