Energy absorption in foams

- impact protection must absorb the kinetic energy of the impact while keeping the peak stress below the threshold that causes injury or damage

- direction of impact may not be predictable

- impact protection must itself be light eg. helmet

Foam

- capacity to undergo large def^n ($\epsilon \sim 0.8, 0.9$) at constant σ

- absorb large energies with little increase in peak stress

- foams - roughly isotropic - can absorb energy from any direction
  - light + cheap

- for a given peak stress, foam will always absorb more energy than solid it is made from

- strain rates: Instron typically $\dot{\epsilon} \sim 10^{-8}$ to 10^{-2} /s

   impact eg. drop from height of 1m, if thickness of foam = 100 mm

   $$V_{impact} = \sqrt{2gh} = \sqrt{2(9.8)(1)} = 4.4 \, m/s \; ; \; \dot{\epsilon} = \frac{4.4 \, m/sec}{0.1 \, m} = 44/s$$

   - Servo Controlled Instrons, drop hammer tests - up to $\dot{\epsilon} = 100/s$
   blast: $\dot{\epsilon} = 10^3 - 10^4$ /s - inertial effects must (we won't consider this)

Energy Absorption
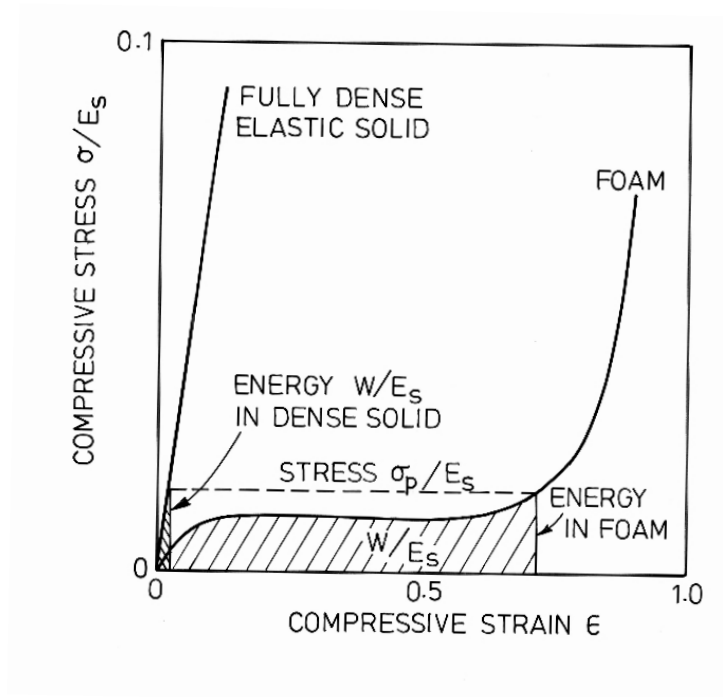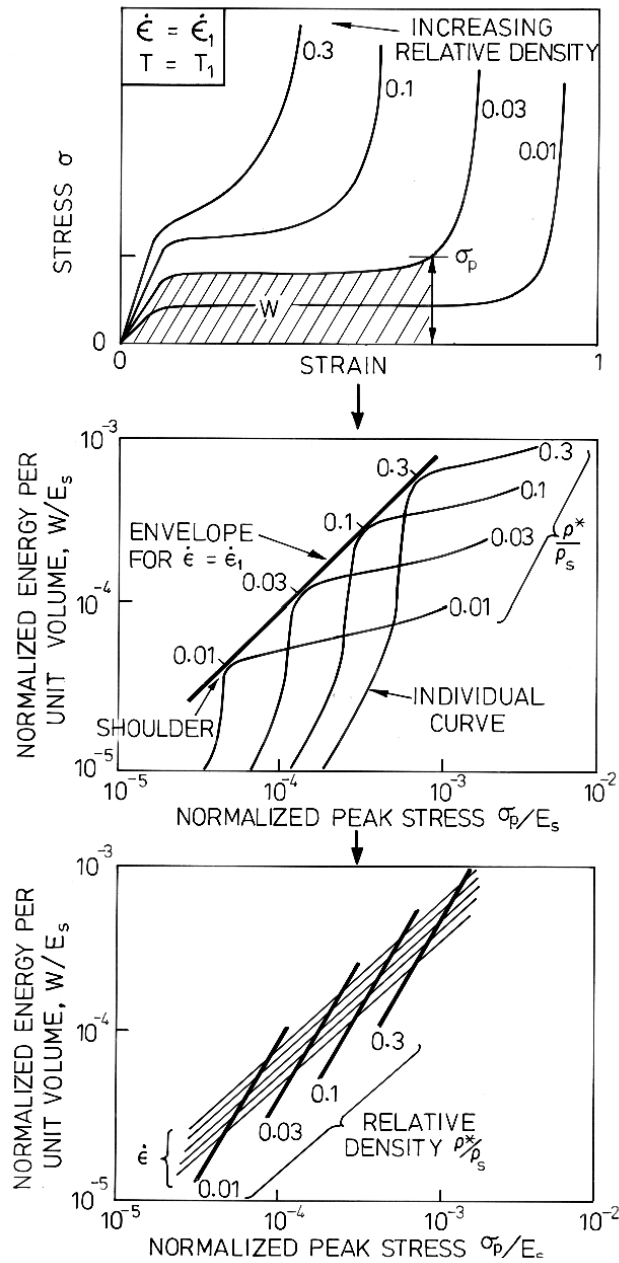
Energy absorption mechanisms

- elastomeric foams — elastic buckling of cells
    - elastic defⁿ recovered ⇒ rebound
    - also have damping — energy dissipated as heat
- plastic foams, brittle foams — energy dissipated as plastic work or work of fracture
    - no rebound
- natural cellular materials — may have fiber composite cell walls
    - dissipate energy by fiber pullout + fracture

- fluid within cells
    - open cell foams — fluid flow dissipation only impt. if fluid is viscous, cells are small or rates are high
    - closed cell foams — compression of cell fluid
        - energy recovered on unloading

Energy absorption diagrams
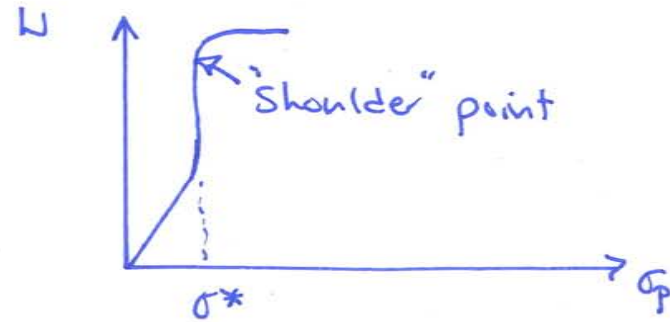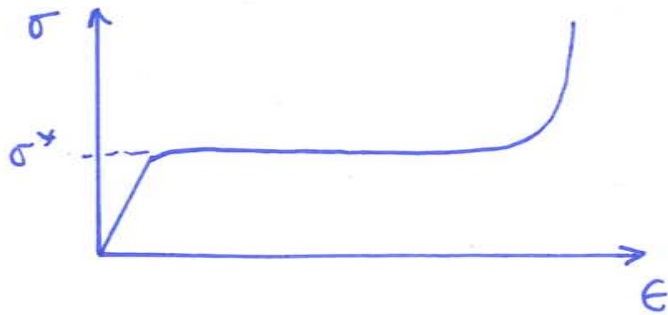
- at stress plateau, energy W increases with little increase in peak stress, σ_p
- as foam densifies, $W \sim$ constant & σ_p increases sharply
- ideally, want to be at "shoulder" point

- more generally - see fig
- test series of one type of foam of different ρ^*/ρ_s at constant $\dot{\varepsilon}$ + Temp, T.
- plot W/E_s vs σ_p/E_s for each curve (E_s at standard $\dot{\varepsilon}$ + T)
- heavy line joins the shoulder points for each curve
- mark ρ^*/ρ_s for each foam on that line
- repeat for varying $\dot{\varepsilon}$ $\Rightarrow$ join lines for constant ρ^*/ρ_s
- build up family of optimum energy absorption curves
- can treat different temperatures, T, in same way

Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

Notes:

- elastomeric foams can all be plotted on one curve since

  $E^* \propto E_s$ and $\sigma_{el}^* \propto E_s$ (normalize W/E_s & σ_p/E_s)

- figure: polyurethane + polyethylene

- polymethacrylimid: σ_{pl}^* ⇒ typical of foams with plastic collapse stress

  with $\sigma_{ys}/E_s = 1/30$

- can generate energy absorption diagrams from data, or use models for foam properties

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Modelling energy absorption diagrams
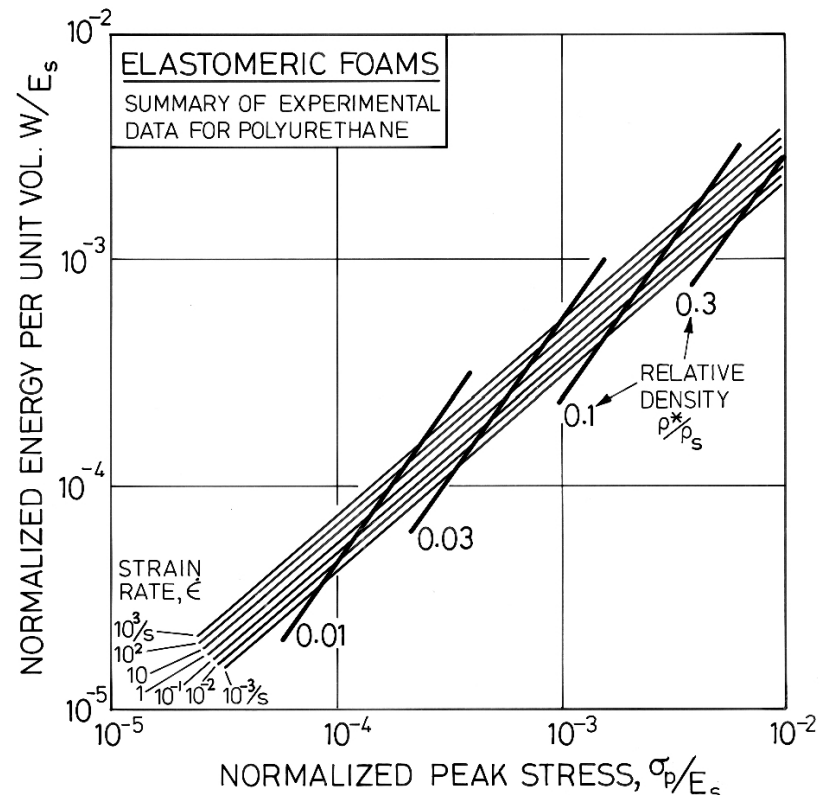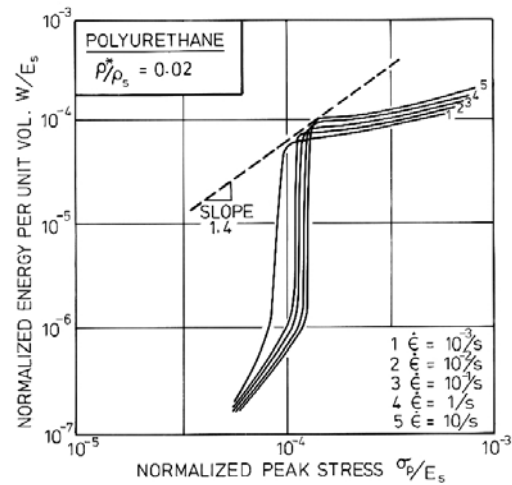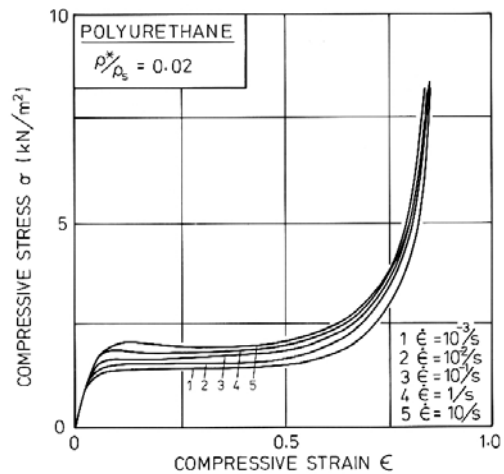
Open cell elastomeric foams
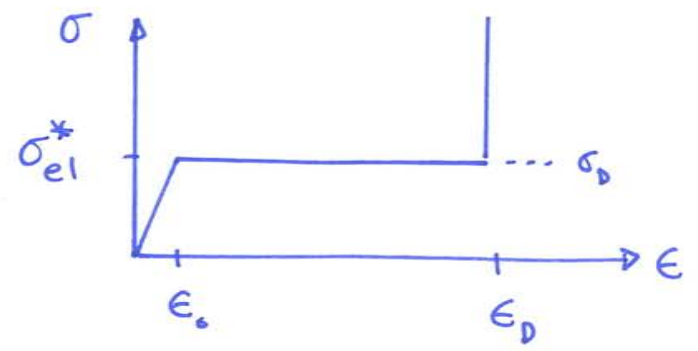
Elastomeric Foams
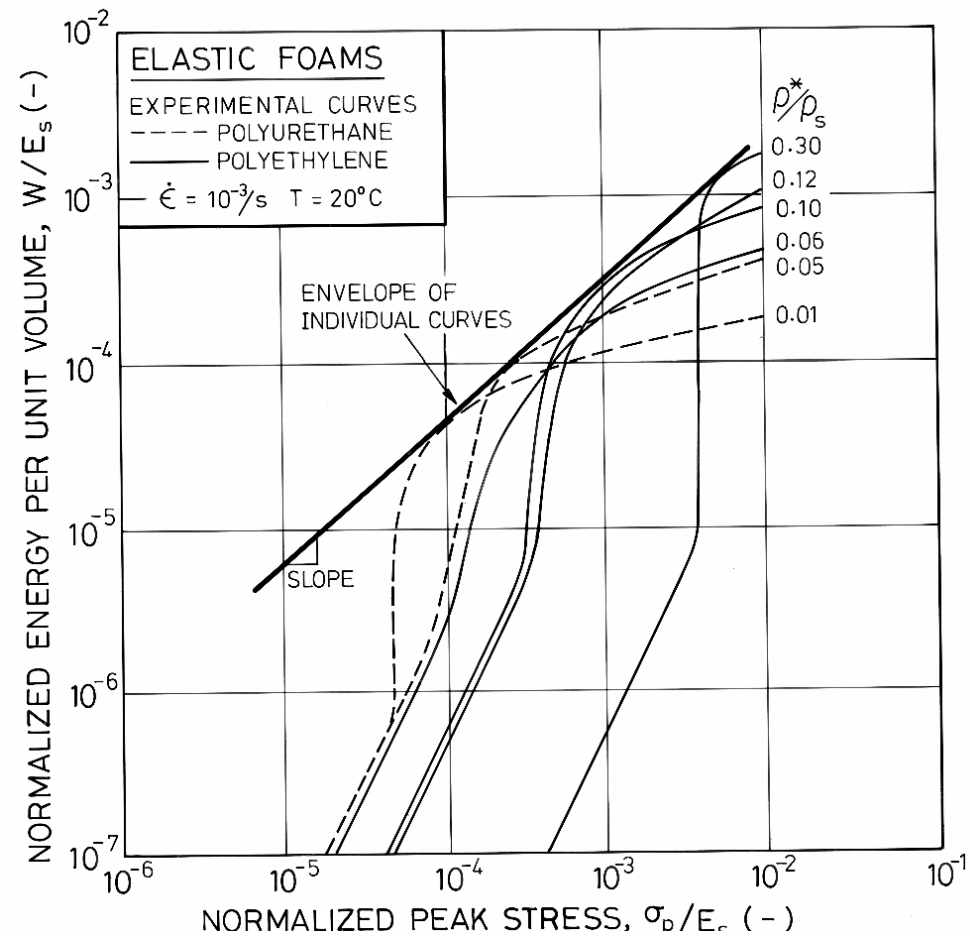
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Flexible Polyurethane

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Polymethacrylimid

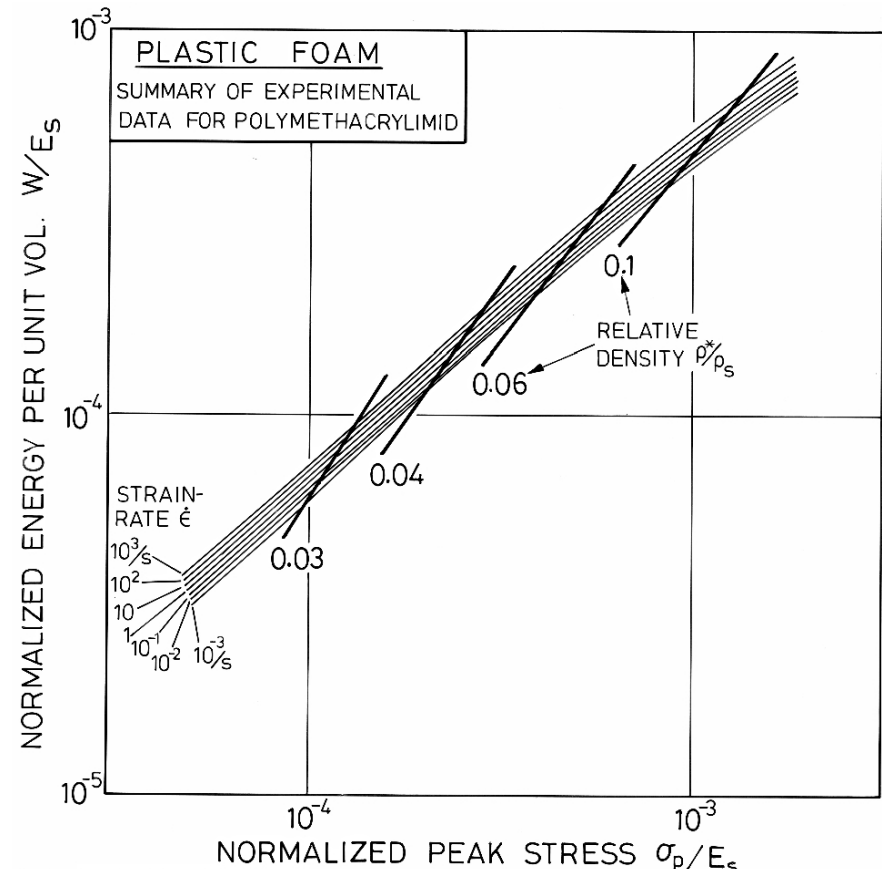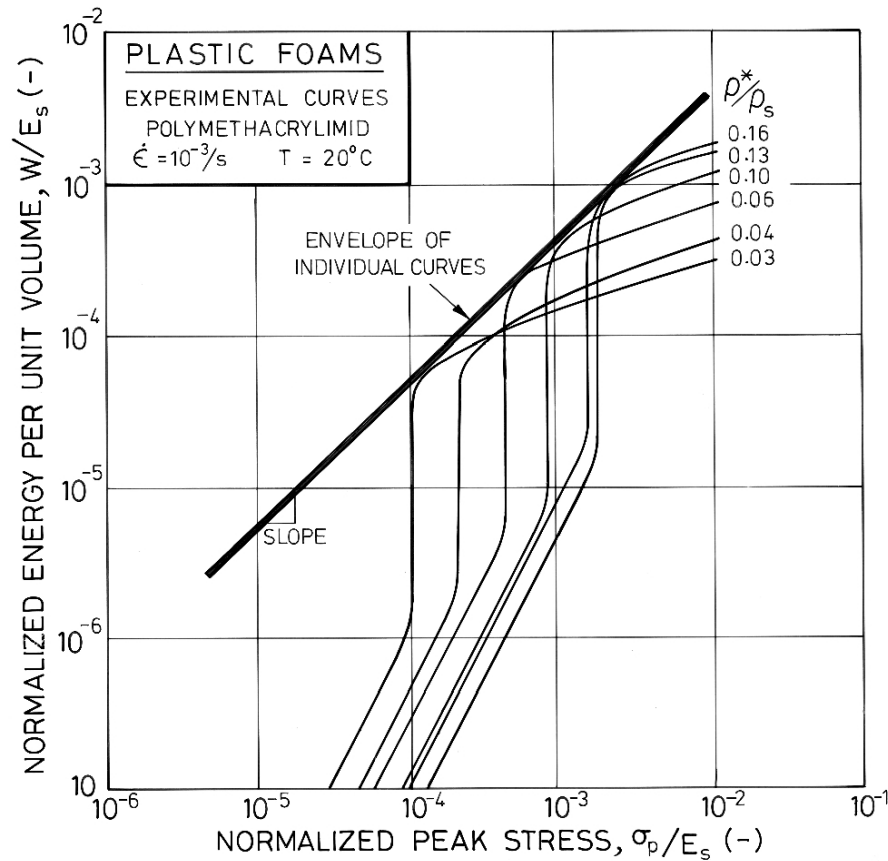

Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figures courtesy of Lorna Gibson and Cambridge University Press.

(a) linear elastic region $\epsilon < \epsilon_0$

$$W = \frac{1}{2}\frac{\sigma_p^2}{E^*} \qquad \frac{U}{E_s} = \frac{1}{2}\left(\frac{\sigma_p}{E_s}\right)^2 \frac{1}{(\rho^*/\rho_s)^2}$$

(b) stress plateau $\epsilon_0 < \epsilon < \epsilon_D$

$$dW = \sigma^*_{el}\, d\epsilon \qquad \frac{U}{E_s} = 0.05\,(\rho^*/\rho_s)^2\,(\epsilon - \epsilon_0)$$

- family of vertical lines on figure
- plateau ends at densification strain ϵ_D
- then W/E_s vs. σ_p/E_s becomes horizontal

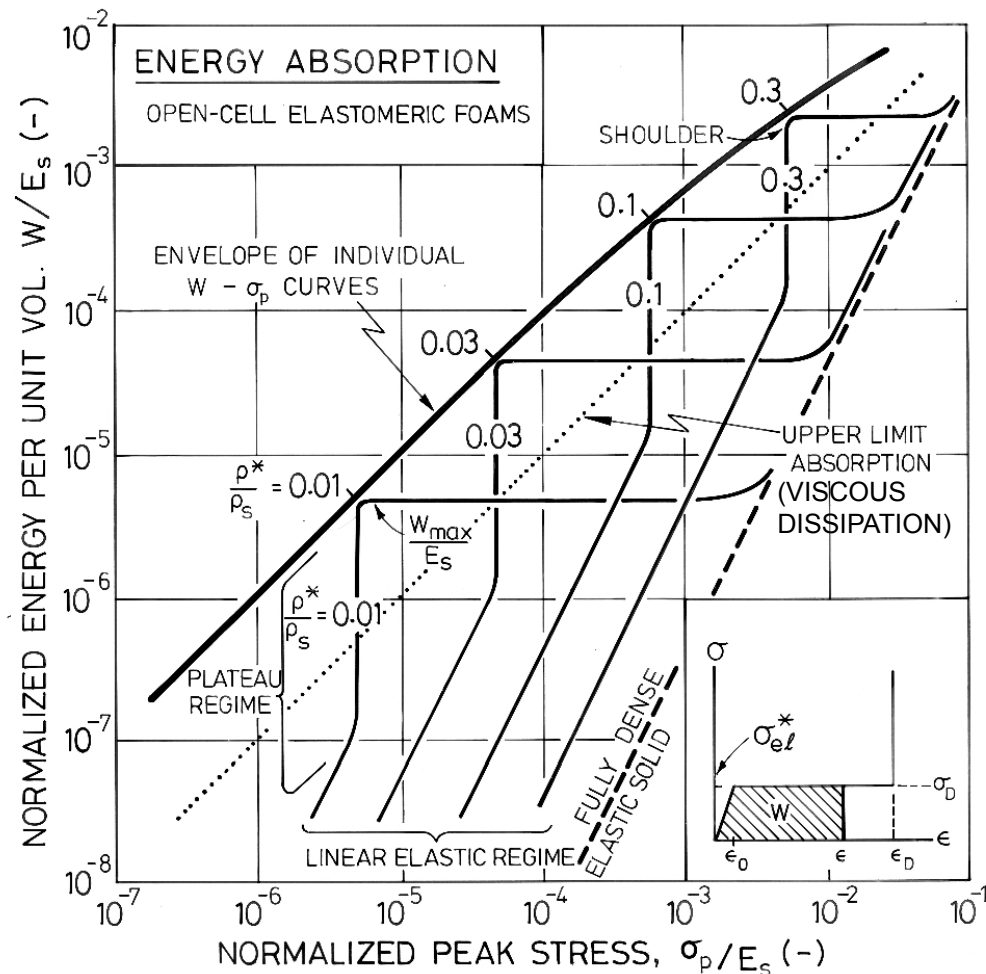(c) at end of stress plateau $\epsilon \sim \epsilon_D$

- maximum energy absorbed just before reach ϵ_D (shoulder point)

$$\frac{W_{max}}{E_s} = 0.05\,(\rho^*/\rho_s)^2\,(1 - 1.4\,\rho^*/\rho_s) \qquad (\text{assuming } \epsilon_0 \ll \epsilon_D \text{ + neglecting } \epsilon_0)$$

- optimum choice of foam is one with shoulder point that lies at $\sigma_p = \sigma_D$
- envelope of shoulder points, for optimum foams, at:

$$\sigma_p = \sigma_D = 0.05\, E_s\,(\rho^*/\rho_s)^2 \qquad \rho^*/\rho_s = \left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}$$

Open-cell Elastomeric Foams: Modelling

Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

substituting into eq'n for $\frac{W}{E_s}\big|_{max}$:

$$\frac{W_{max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 1.4\left(\frac{20\,\sigma_p}{E_s}\right)^{1/2}\right]$$

$$\frac{W_{max}}{E_s} = \frac{\sigma_p}{E_s}\left[1 - 6.26\left(\frac{\sigma_p}{E_s}\right)^{1/2}\right]$$

· line of slope 1 at low stresses, falling to $7/8$ at high σ


(d) densification

· when foam fully densified + compressed to a solid, then energy
absorption curve joins that for the fully dense elastomer

$$\frac{W}{E_s} = \frac{1}{2}\frac{\sigma_p^2}{E_s}$$

Note:
· model curves have same shape as expts.
· model shows W/E_s depends on σ_p/E_s + ρ^*/ρ_s only – one diagram for all elastomer foams
· for a given W/E_s, σ_p/E_s for the foam less than that of the fully
dense solid, by a factor of 10^{-3} to 10^{-1}

Closed-cell Elastomeric Foams: Modelling
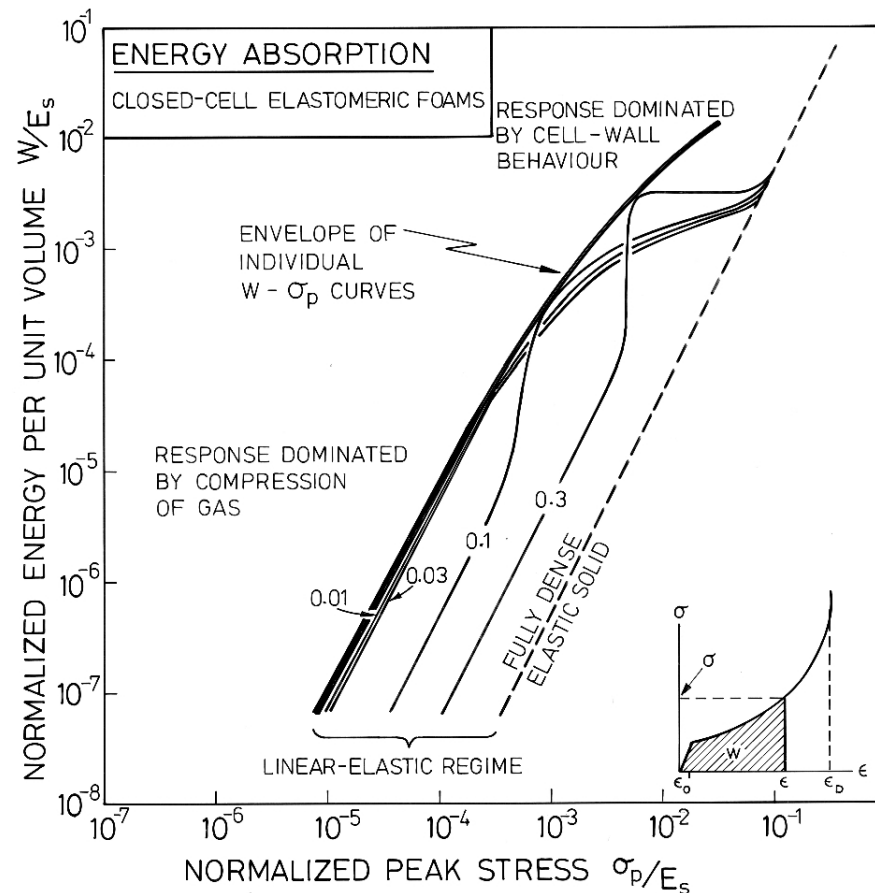
Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Modelling: open-cell foams that yield

- analysis similar to elastomeric foams, with σ^*_{pl} replacing σ^*_{el}
- note that some closed cell foams that yield, face contribution to E^* σ^*_{pl} negligible
- neglect fluid contribution
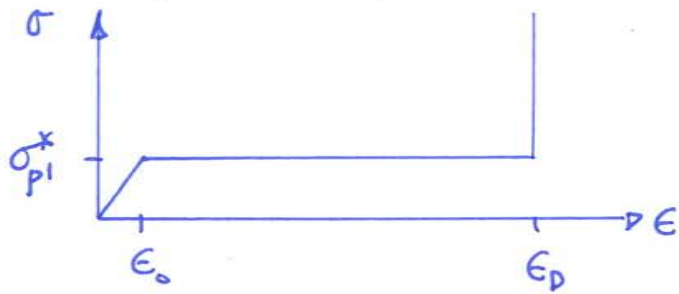
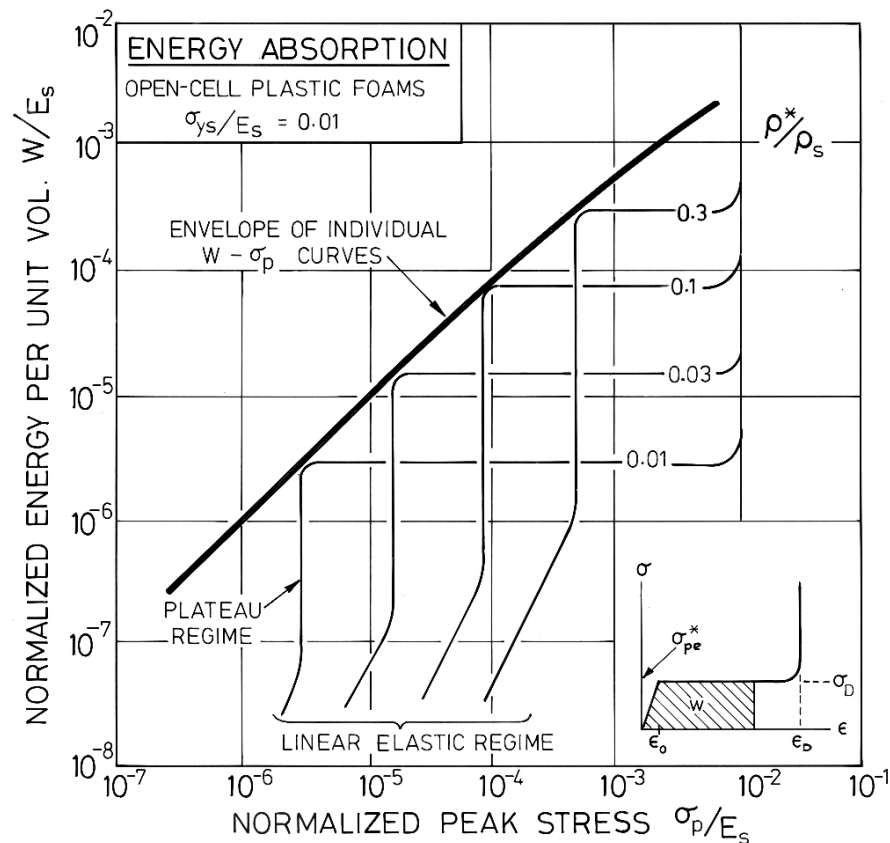$$\sigma^*_{pl} = 0.3\, \sigma_{ys} \left(\rho^*/\rho_s\right)^{3/2}$$

(a) linear elastic regime: same as for elastomeric foam:
$$\frac{U}{E_s} = \frac{1}{2}\left(\frac{\sigma_{pl}}{E_s}\right)^2 \frac{1}{(\rho^*/\rho_s)^2}$$

(b) stress plateau:
$$\frac{W}{E_s} = 0.3\, \frac{\sigma_{ys}}{E_s}\left(\frac{\rho^*}{\rho_s}\right)^{3/2}(\epsilon - \epsilon_o)$$

(c) end of stress plateau:
$$\frac{W_{max}}{E_s} \approx 0.3\, \frac{\sigma_{ys}}{E_s}\left(\frac{\rho^*}{\rho_s}\right)^{3/2}(1 - 1.4\,\rho^*/\rho_s)$$

- optimum choice of foam - absorbs maximum energy without σ_p rising sharply at ϵ_D

Plastic Foams: Modelling

Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

- Curve of optimum energy absorption (heavy line on figure) is envelope that touches $W - \sigma_p$ curve at shoulder points

- for σ_p, $\qquad \rho^*/\rho_s = \left(\dfrac{3.3\,\sigma_p}{\sigma_{ys}}\right)^{2/3}$

- substituting in W_{max}/E_s eqn:

$$\frac{W_{max}}{E_s} = \frac{\sigma_D}{E_s}\left\{ 1 - 3.1\left(\frac{\sigma_D}{\sigma_{ys}}\right)^{2/3} \right\}$$

- model curves explain general features of experimental curves

- modelling - curves less general than for elastomers

    - this curve for a particular value of $\sigma_{ys}/E_s = 1/100$ (typical value for polymers)

Design + selection of foams for impact protection

· typically know object to be protected + some details about it

mass, m

contact area, A

max drop height, h

(or energy to be absorbed, u

Max allowable acceleration, a

(eg. head injury ~ 100g)

peak stress allowable, σ_p

· variables: foam material, density, thickness

Example 1

Given: mass, $m = 0.5$ kg

contact area, $A = 0.01$ m²

drop height, $h = 1$ m

max deceleration, $a = 10g$

foam: flexible polyurethane $E_s = 50$ MPa

Find: optimum foam density

" " thickness

Example 1:
Find Foam Density and Thickness

Table 8.2 Example I: selection of foams

Specification of the problem

Mass of the package object, $m = 0.5\,\text{kg}$

Area of contact between foam and object, $A = 0.01\,\text{m}^2$

Velocity of package on impact (drop height $h = 1\,\text{m}$), $v = 4.5\,\text{m/s}$

Energy to be absorbed, $U = mv^2/2 = 5\,\text{J}$

Maximum allowable package force (based on deceleration of $10g$), $F = ma = 50\,\text{N}$

Maximum allowable peak stress, $\sigma_\text{p} = F/A = 5\,\text{kN/m}^2$

Solid modulus in foam (flexible polyurethane), $E_\text{s} = 50\,\text{MN/m}^2$

Maximum allowable normalized peak stress, $\sigma_\text{p}/E_\text{s} = 10^{-4}$

Iterative procedure

	$t_1 \gg t$	$t_1 \ll t$
1st Iteration		
Initial choice of t_1	1 m	0.001 m
Resulting strain-rate, $\dot{\epsilon} = v/t_1$	$4.5\,\text{s}^{-1}$	$4.5 \times 10^3\,\text{s}^{-1}$
Resulting (W/E_s) at $\sigma_\text{p}/E_\text{s} = 10^{-4}$	5.25×10^{-5}	7.4×10^{-5}
Energy absorbed per unit volume, W	$2620\,\text{J/m}^3$	$3700\,\text{J/m}^3$
2nd Iteration		
Revised t_2 (from $U = WAt$)	0.19 m	0.14 m
Revised $\dot{\epsilon} = v/t_2$	$24\,\text{s}^{-1}$	$32\,\text{s}^{-1}$
Revised (W/E_s)	6.6×10^{-5}	6.7×10^{-5}
Revised W	$3300\,\text{J/m}^3$	$3350\,\text{J/m}^3$
3rd Iteration		
Revised t_3 (from $U = WAt$)	0.15 m	0.15 m
Optimum density, ρ^*/ρ_s (Fig. 8.8)	A little below 0.01	
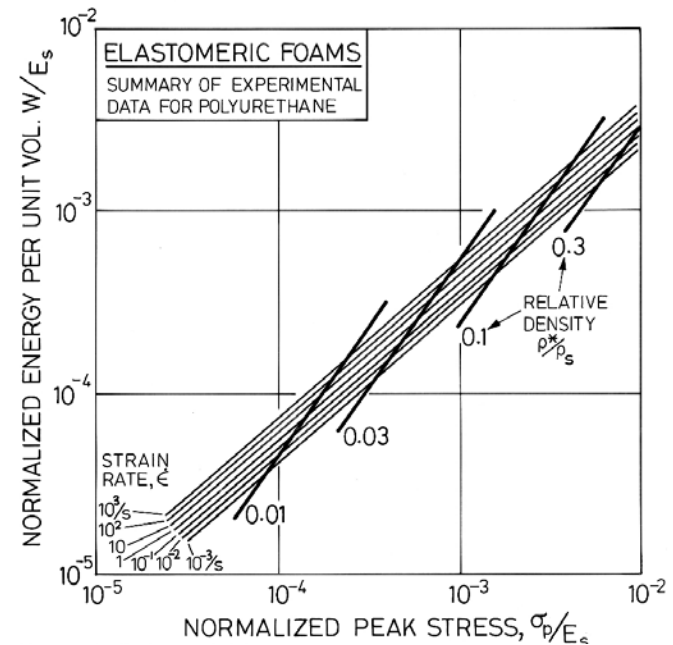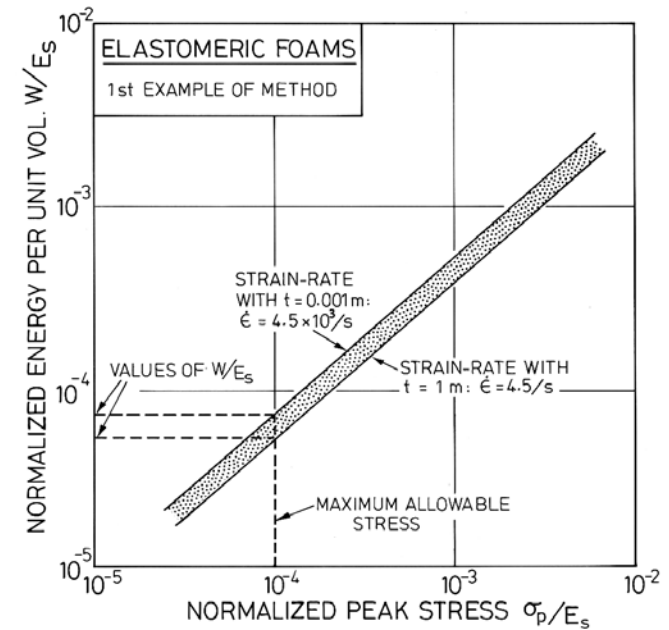
- energy to be absorbed, $u = mgh = (0.5 kg)(10 m/s^2)(1 m) = 5 J$

- max. allowable force on package $= F = ma = (0.5 kg)(10 g) = 50 N$

- peak stress, $\sigma_p = F/A = 50 N / 0.01 m^2 = 5 kN/m^2$

- normalized peak stress, $\sigma_p / E_s = 5 kPa / 50 MPa = 10^{-4}$

- draw vertical line on energy absorption diagram @ $\sigma_p / E_s = 10^{-4}$

- need to know $\dot{\epsilon} \approx v/t$ velocity $v = \sqrt{2gh} = 4.5 m/s$

- iterative approach - choose arbitrary thickness, t

- e.g. $t_1 = 1 m$

  $\dot{\epsilon} = 4.5/s$

  $W/E_s = 5.25 \times 10^{-5}$

  $W = 2620 J/m^3$

($u = WAt$) $t_2 = \dfrac{WA}{u} = 0.19 m$

  $\dot{\epsilon}_2 = 24/s$

  $W/E_s = 6.6 \times 10^{-5}$

  $W = 3300 J/m^3$

$t_1 = 0.001 m$

  $\dot{\epsilon} = 4.5 \times 10^3 /s$

  $W/E_s = 7.4 \times 10^{-5}$

  $W = 3700 J/m^3$

  $t_2 = 0.14 m$

  $\dot{\epsilon}_2 = 32/s$

  $W/E_s = 6.7 \times 10^{-5}$

  $W = 3350 J/m^3$

Third iteration: $t_3 = 0.15 m$
  (both W).
optimum density (Fig).
  $\rho^*/\rho_s \sim 0.01$.

Note: t converges quickly
even from very different
initial guesses for t

Example 2

Given $m = 2.5 kg$

$A = 0.025 m^2$

$t = 20 mm$

$h = 1 m$

$a = 100 g$

Find foam material
foam density

Calculate W, σ_p, $\dot{\epsilon}$

$$W = \frac{mgh}{At} = \frac{(2.5 kg)(10 m/s^2)(1 m)}{0.025 m^2 (0.02 m)} = 5 \times 10^{-4} J/m^3$$

$$\sigma_p = \frac{F_{max}}{A} = \frac{ma}{A} = \frac{(2.5 kg)(100)(10 m/s^2)}{0.025 m^2} = 10^5 N/m^2$$

$$\dot{\epsilon} = \frac{v}{t} = \frac{\sqrt{2gh}}{t} = \frac{\sqrt{2(10 m/s^2)(1 m)}}{0.02 m} = \frac{4.5 m/s}{0.02 m} = 225/s$$

Select arbitrary value of $E_s = 100 MPa$

Plot $W/E_s = 5 \times 10^{-4}$ point A

$\sigma_p / E_s = 10^{-3}$

Example 2:
Find Foam Material and Density

Table 8.3 Example 2: selection of foams

Specification of the problem

Mass of the package object, $m = 2.5\,\text{kg}$

Area of contact between foam and object, $A = 0.025\,\text{m}^2$

Thickness of foam, $t = 20\,\text{mm}$

Drop height, $h = 1\,\text{m}$

Velocity of impact $v = (2gh)^{1/2} = 4.5\,\text{m/s}$

Strain-rate $\dot{\epsilon} = v/t = 225/s$

Energy to be absorbed $U = mgh = 25\,\text{J}$

Energy to be absorbed per unit volume of foam $W = U/At = 5 \times 10^4\,\text{J/m}^3$

Maximum allowable force (based on declaration of $100g$) = 2500 N

Maximum allowable peak stress $\sigma_p = F/A = 10^5\,\text{N/m}^2$

Trial design point A, using $E_s = 100\,\text{MN/m}^2$

Normalized energy $W/E_s = 5 \times 10^{-4}$

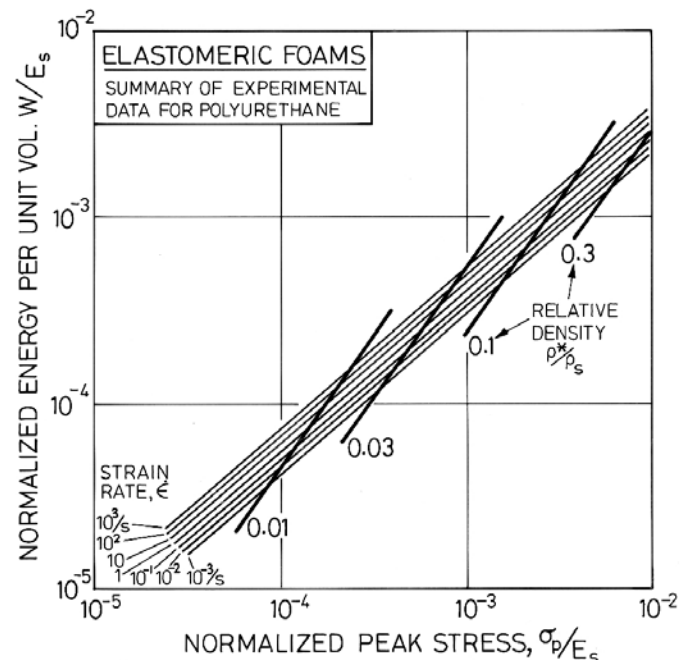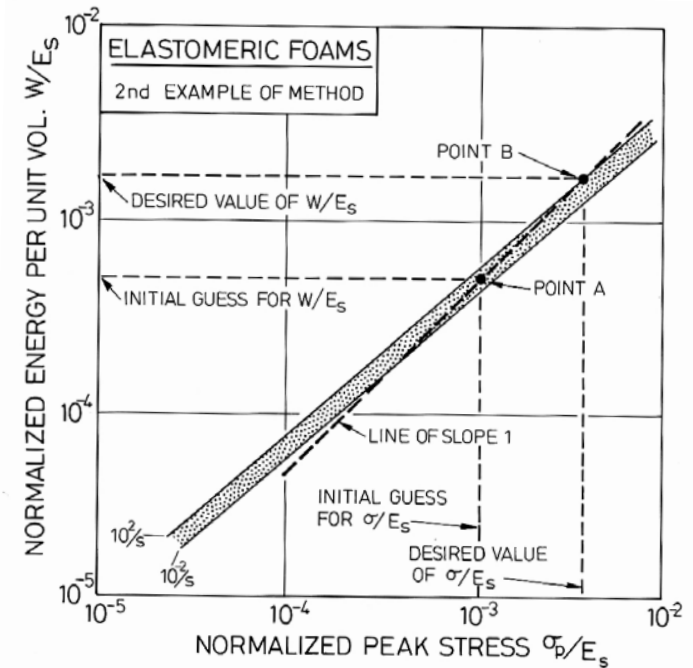Normalized peak stress $\sigma_p/E_s = 10^{-3}$

Final design point B, read from diagram

Normalized energy $W/E_s = 1.8 \times 10^{-3}$

Normalized stress $\sigma_p/E_s = 3.7 \times 10^{-3}$

Resulting derived value of $E_s = 28\,\text{MN/m}^2$

Desired foam density ≈ 0.1

- Construct a line of slope 1 through this point (broken line)

- moving along this line simply changes E_s

- select the point where the broken line intersects the appropriate $\dot\varepsilon \sim 10^2/s$
  (point B)

- read off values of $W/E_s = 1.8 \times 10^{-3}$

$$\sigma_{pl}/\bar{E}_s = 37 \times 10^{-3}$$

- resulting value of $E_s = 28\,MPa \Rightarrow$ low modulus, flexible polyurethane

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- replotting on more detailed figure: $\rho^*/\rho_s = \cancel{0.05}\ 0.1$

- If point A above all energy contours + line of slope 1 does not intersect them, specification cannot be achieved, A or t has to increase

- if point A below all contours, then A & t larger than need to be — can be reduced

Case study: design of car head rest

- head rest should absorb kinetic energy of head while keeping force less than that which would cause injury.

- example in book:

  mass of head $= 2.5\,kg$

  Max. deceleration $= a = 50g = 500\,m/s^2$

  area of contact, $A = 0.01\,m^2$

  thickness of padding $t = 0.17\,m$

Max. allowable force $F = ma = 1250\,N$

"   "   stress $\sigma_p = F/A = 125\,kN/m^2$

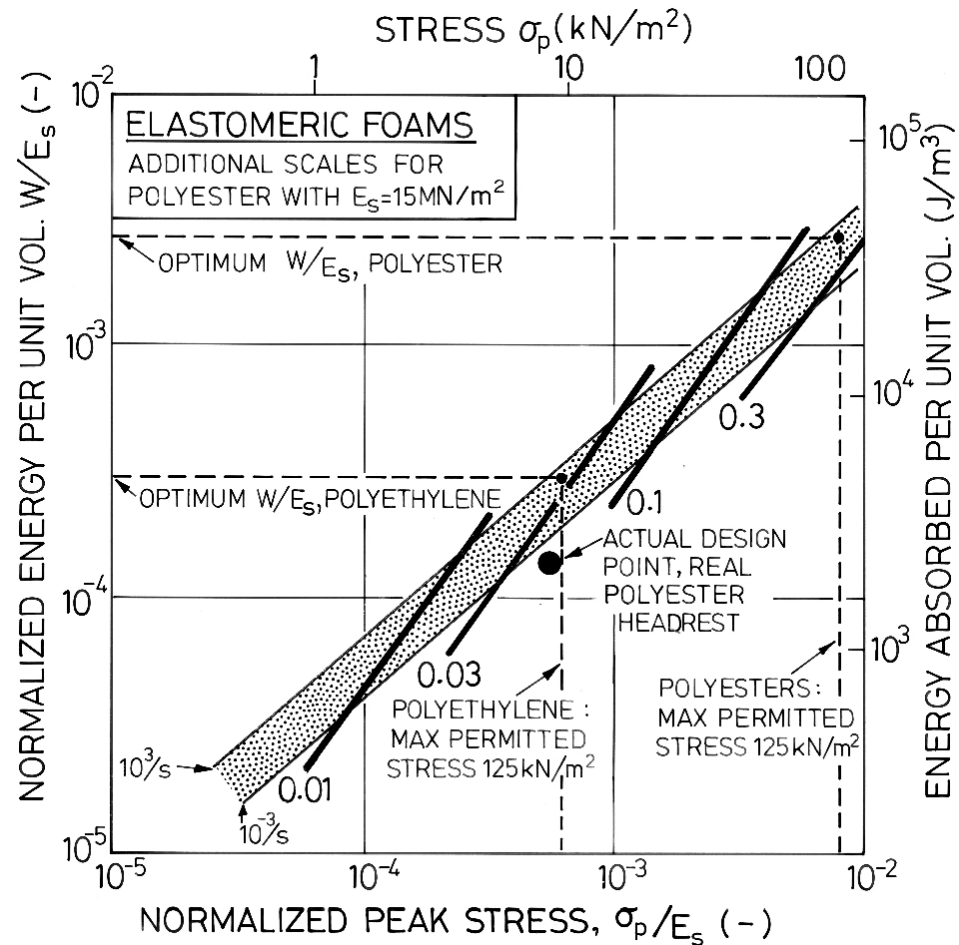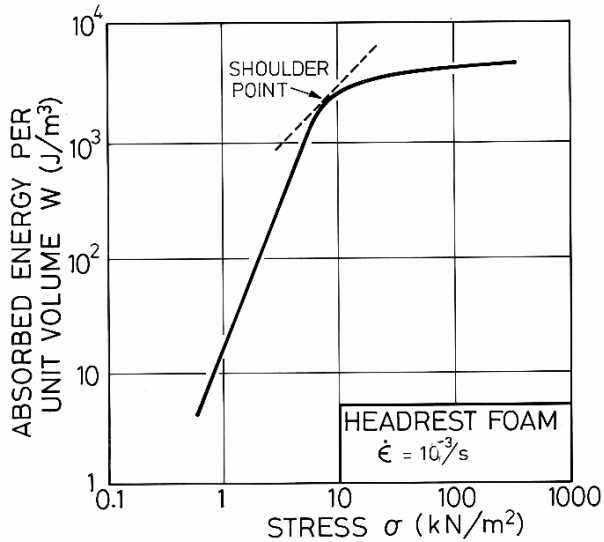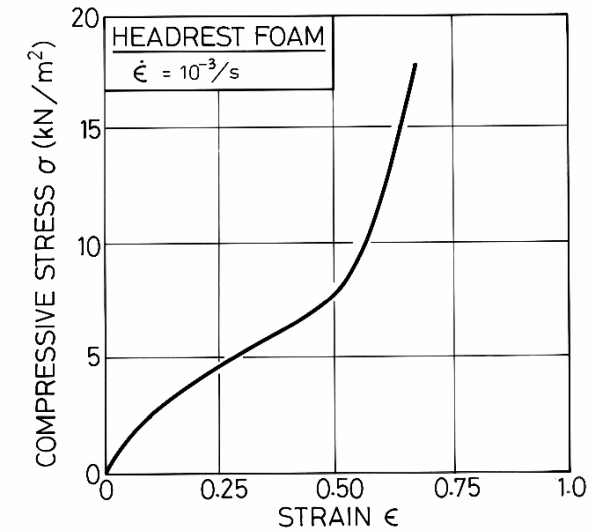energy to be absorbed /vol, $W = \dfrac{\frac{1}{2}mv^2}{At} = 735\,v^2\ J/m^3$

peak strain rate $\dot{\varepsilon} = v/t \quad [s^{-1}]$

current material – flexible polyester foam $\quad \rho^*/\rho_s = 0.06$

from plot: for $\sigma_p = 125\,kN/m^2 \quad W = 5.\times 10^3\ J/m^3$

maximum collision velocity $= v = \sqrt{\dfrac{W}{735}} = \sqrt{\dfrac{5\times 10^3}{735}} = 2.6\,m/s = 5.8\,mph$

Car Head Rest Design

Alternative design #1

- Consider en. abs. diag. for elastomeriz foams
- add scales for polyester (using $E_s = 15$ MPa)
- for $\sigma_p = 125$ kN/m² could use polyester foam $\rho^*/\rho_s = 0.2$

  then $W/E_s = 2.6 \times 10^{-3}$ & $v = 7.3$ m/s = 16 mph

Alternative design #2

- use different material e.g. low density open cell polyethylene $E_s = 200$ MPa

---

- $\sigma_p / E_s = \dfrac{0.125}{200} = 6.3 \times 10^{-4}$

- at $\dot{\varepsilon} = v/t \approx 100/s$ (estimated)

  $W/E_s = 3.2 \times 10^{-4}$ (from fig).

  $W = (3.2 \times 10^{-4})(200 \text{ MPa}) = 6.4 \times 10^4$ J/m³

  $v = \sqrt{\dfrac{W}{735}} = \sqrt{\dfrac{6.4 \times 10^4}{735}} = 9.3$ m/s = 21 mph

- reading from figure: $\rho^*/\rho_s = 0.03$

Case study: foams for bicycle helmets

US: 600-700 bicycle deaths/yr   (US Nat. Hwy Traffic Safety Admin
       >90%   not wearing a helmet   Bicycle Helmet Safety Inst.)
       ~50,000   cyclists injured (2009)

- helmets consist of solid outer shell + foam liner (eg. expanded PS)

- liner thickness typically 20mm

- wish to absorb as much energy as possible while keeping peak acc'n less

  than that to cause head injury

- foam liner

  - redistributes load over larger area, reducing stress on head

  - peak stress on head limited by plateau stress of foam (as long as don't reach
                                                              densification)

  - max. tolerable acc'n = 300g (if for a few milliseconds)

  - mass of head ≈ 3kg

  $$F_{max} = ma = (3kg)(300)(10m/s^2) = 9kN$$

  - as foam crushes, it distributes load over area ~ A ~ 0.01 m² (may be high)

  $$\sigma_p = \frac{9kN}{0.01m^2} = 0.9 \text{ MPa}$$

figure → EPS $\rho^* = 0.05$ Mg/m³

absorbs $W = 0.8$ MJ/m³

- diagram allows easy identification of possible candidate materials

- more complete analysis can then be done

- energy absorbed $u = 0.8 \times 10^6 \frac{J}{m^3} \times 0.01\,m^2 \times 0.02\,m = 160\,J$ ($u = WAt$)

- $\frac{1}{2}mv^2 = u$; $v_{max} = \sqrt{\frac{2u}{m}} = \sqrt{\frac{2\,(160\,J)}{3\,kg}\,kg\,\frac{m^2}{s^2}} = 10\,m/s \approx 22\,mph.$

Case Study:
Foams for Bicycle Helmets
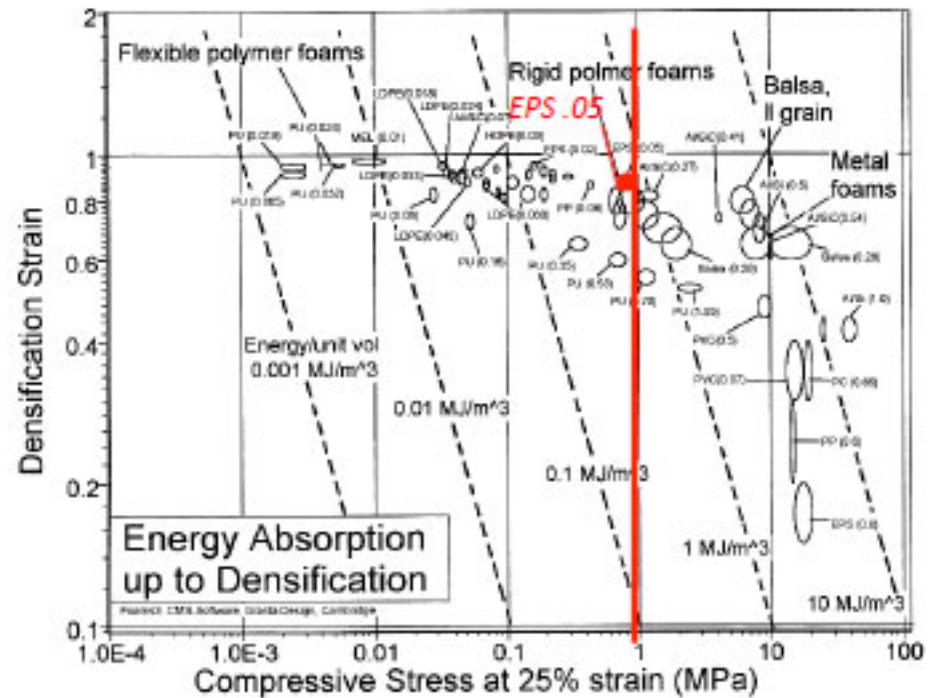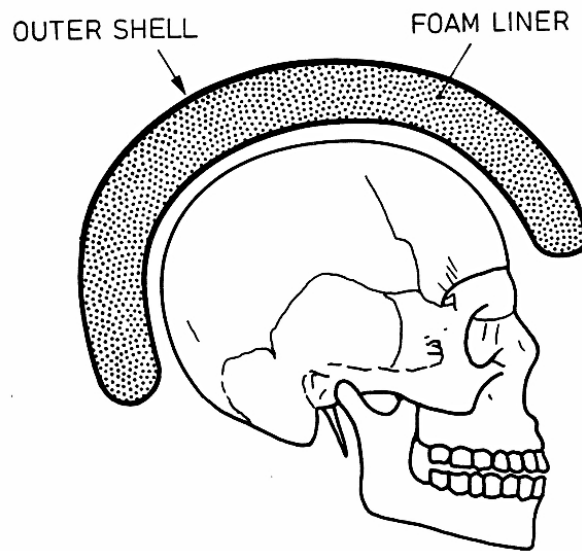
3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2014