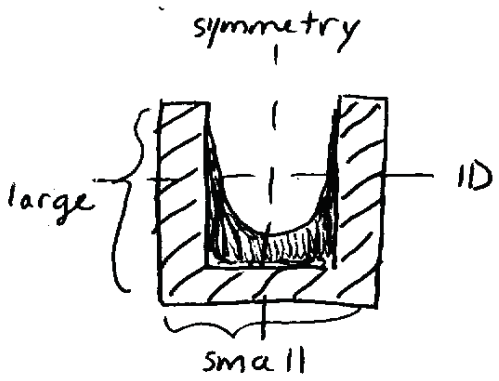
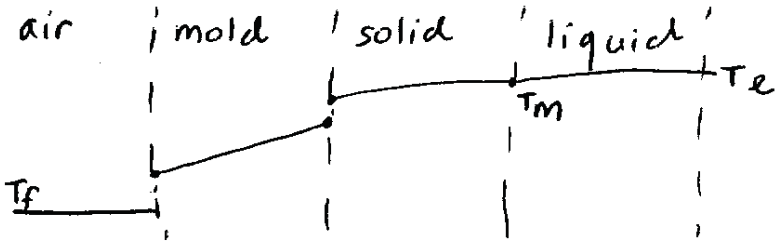
3.044 MATERIALS PROCESSING

LECTURE 9

Example 1: Casting into low conductivity molds





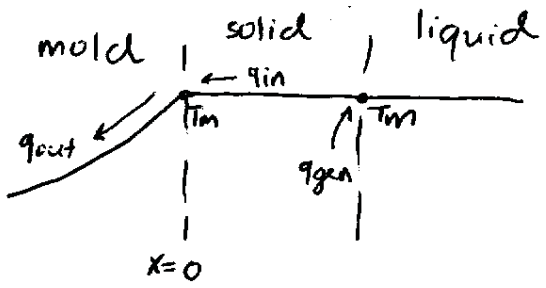
Greatly simplified if:

1. Mold is thick → can neglect air → semi-infinite geometry → erf

2. Mold controls heat loss → T_{mold} increases → k_{mold} decreases → **gradients are in mold**

3. Superheating is negligible → liquid poured at T_m

Date: April 2nd, 2012.



$$
\begin{aligned}
q_{\text{gen}} &= H_f \rho_s \frac{\Delta s}{\Delta t} A \\
q_{\text{out}} &= -k_m \frac{\partial T}{\partial x} \\
q_{\text{out}} &= q_{\text{gen}} \\
\frac{T(x,t) - T_m}{T_0 - T_m)} &= \text{erf}\left(\frac{-x}{2\sqrt{\alpha_m t}}\right) \\
T(x,t) &= (T_0 - T_m)\ \text{erf}\left(\frac{-x}{2\sqrt{\alpha_m t}}\right) + T_m \\
\left.\frac{\partial T}{\partial x}\right|_{x=0} &= (t_0 - T_m)\frac{1}{\sqrt{\pi \alpha_m t}} e^{\frac{-x^2}{4\alpha_m t}} \\
k_m \left.\frac{\partial T}{\partial x}\right|_{x=0} &= \frac{T_0 - T_m}{\sqrt{\pi \alpha_m t}} \\
\left.\frac{\partial T}{\partial x}\right|_{x=0} &= \sqrt{\frac{k_m \rho_m c_{p,m}}{\pi t (T_0 - T_m)}} \\
&= -\rho_s H_f \frac{ds}{dt} \\
\frac{ds}{dt} &= \frac{T_0 - T_m}{\rho_s H_f}\left(\frac{k_m \rho_m c_{p,m}}{\pi t}\right)^{\frac{1}{2}} \\
s &= \frac{2(T_0 - T_m)}{\rho_s H_f}\left(\frac{k_m \rho_m c_{p,m}}{t}\right)^{\frac{1}{2}} + A
\end{aligned}
$$

Boundary Condition: $@t = 0,\ s = 0$

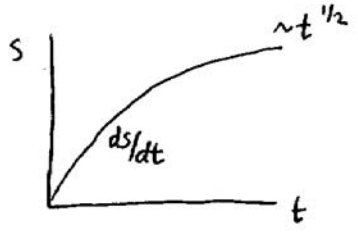
$$
s = \frac{2(T_0 - T_m)}{\rho_s H_f}\left(\frac{k_m \rho_m c_{p,m}}{t}\right)^{\frac{1}{2}}
$$

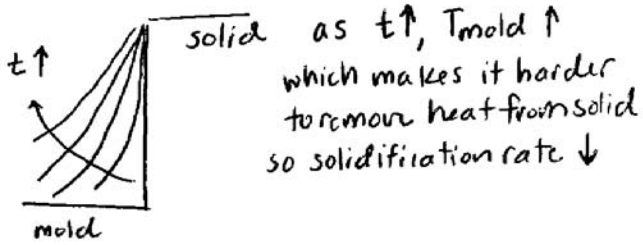
Boundary Condition: @$s = L$, $t = t_f$

$$\boxed{t_f \propto L^2}$$

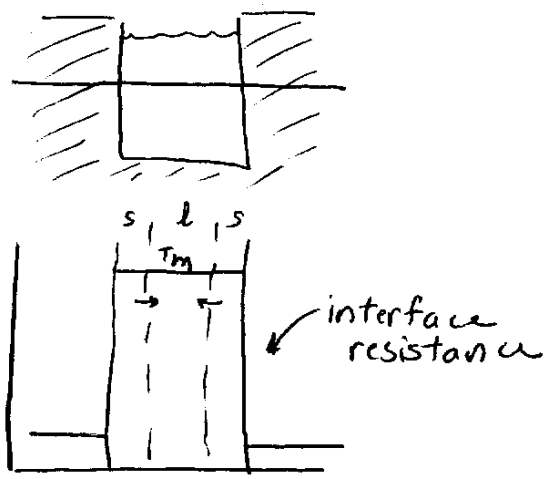
$$L \approx \frac{V}{A}$$

$$\boxed{t_f \propto \left(\frac{V}{A}\right)^2} \Rightarrow \textbf{Chvorinov's Rule}$$





Example 2: Thin castings / Cold molds / Highly conductive molds



$$\overbrace{-h(T_m - T_{mold})}^{out} = \overbrace{\rho_s H_f \frac{ds}{dt}}^{in}$$

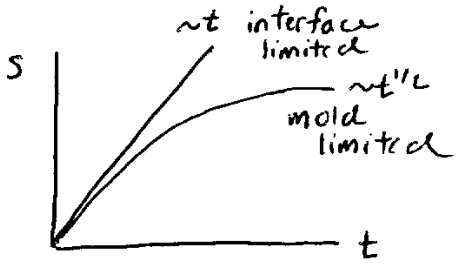
$$\int ds = \int \frac{-h(T_m - T_{mold})}{\rho_s H_f} dt$$

$$s = \frac{-h(T_m - T_{mold})}{\rho_s H_f} t + A$$

Boundary Condition: @$t = 0$, $s = 0$

$$A = 0$$

$$\boxed{s = \frac{-h(T_m - T_{mold})}{\rho_s H_f} t}$$



Typical Values:

$$\text{Al}: \ \rho_s = 2.7 \frac{\text{g}}{\text{cm}^3}, \qquad H_f = 10 \frac{\text{kJ}}{\text{mol}}, \qquad T_m = 660^\circ\text{C}$$

$$\text{Mold}: T_{mold} = 25^\circ\text{C}$$

$$\text{Time to Solidify}: t_f = 200 \frac{\text{s}}{\text{m}} \quad \Rightarrow 1\text{cm} \sim 1\text{s}, \ \Rightarrow \text{mm} \sim \text{sub-s}$$

3.044 Materials Processing

Spring 2013