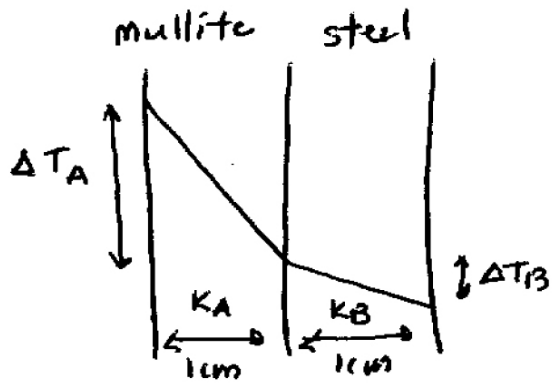
3.044 MATERIALS PROCESSING

LECTURE 3

We will often be comparing heat transfer steps/processes:

When can we neglect one and focus on the other?



Resistance:

$$10 > \frac{\frac{L_A}{k_A}}{\frac{L_B}{k_B}} > 0.1 \quad \Rightarrow \quad \begin{array}{ll} 10: & \text{"}B\text{" conducts fast, cannot sustain a gradient} \\ 0.1: & \text{"}A\text{" conducts fast, cannot sustain a gradient} \end{array}$$

Reduce Dimensionality:

$$\frac{\partial T}{\partial x} = \alpha \nabla^2 T : \quad T(t, x, y, z)$$

1. Steady State

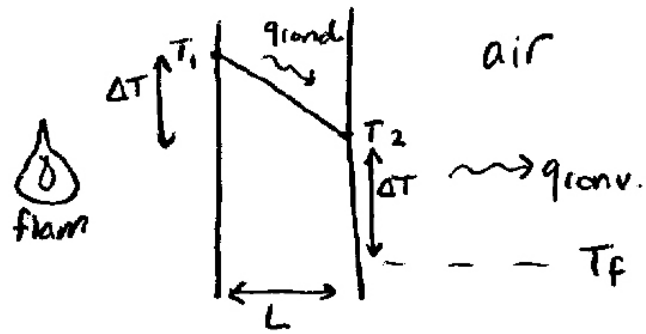
   $\frac{\partial T}{\partial t} = 0$

2. No Thermal Gradients

   $\nabla T = 0, \quad T = T(t)$ ONLY

   $\frac{\partial T}{\partial t} = ...$

Date: February 15th, 2012.



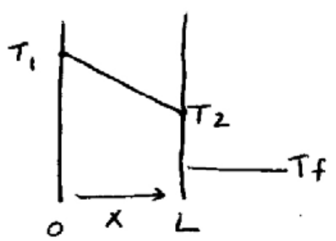
In general, for solid / "fluid" interfaces: $T_2 \neq T_f$

- constant T, B.C. is not appropriate
- fluid cannot always remove heat at the rate it is delivered

How is heat transferred / removed in the fluid?

- conduction: heat moves, atoms sit still
- convection: atoms flow away, carrying heat with them
    1. natural convection (T interacts w/ gravity)
    2. forced convection (mechanically driven flow)
- radiation: photons carries heat away

What are the proper B.C.?



1. $T_2 \neq T_f$
2. @ $x = L$, specify flux:

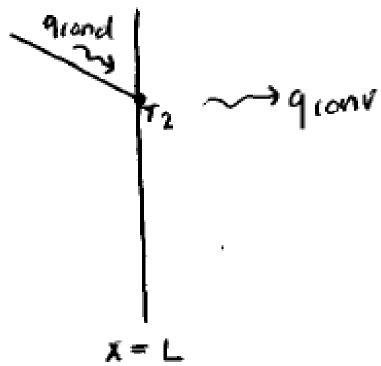
$$\overbrace{q}^{\text{heat}\left[\frac{\text{W}}{\text{m}^2}\right]} = \underbrace{h}_{\text{heat transfer coeff.}\left[\frac{\text{W}}{\text{m}^2\text{K}}\right]}(T_2 - T_f) \quad \Rightarrow$$ the hotter the material is with respect to the fluid, the faster heat will flow

$$\frac{\partial T}{\partial t} = 0 = \alpha \frac{\partial^2 T}{\partial x^2}$$

Step 1: Solve

$$\frac{T - T_1}{T_2 - T_1} = xL, \quad \text{where } T_2 \text{ is } \textbf{unknown}$$
$$\Theta = \chi$$

Step 2: B.C.



$$@\, x = L$$
$$q_{\text{cond}} = q_{\text{conv}}$$
$$-k\frac{\partial T}{\partial x} = h(T_2 - T_f)$$

Step 3: Solve for $\frac{\partial T}{\partial x}$

$$\frac{T - T_1}{T_2 - T_1} = \frac{x}{L}$$
$$T = T_1 + \frac{x}{L}(T_2 - T_f)$$
$$\frac{\partial T}{\partial x} = \frac{T_2 - T_1}{L}$$

Plug into: $-k\frac{\partial T}{\partial x} = h(T_2 - T_f)$

$$-k\frac{T_2 - T_1}{L} = h(T_2 - T_f)$$
$$\frac{kT_1}{L} + hT_f = \left(h + \frac{k}{L}\right)$$
$$T_2 = \frac{\frac{k}{L}T_f}{h + \frac{k}{L}}$$

Plug into: $T = T_1 + \frac{x}{L}(T_2 - T_f)$

$$T = T_1 + \frac{x}{L}\left[\frac{\frac{k}{L}T_1 + hT_f}{h + \frac{k}{L}} - T_1\right]$$

$$T - T_1 = \frac{x}{L}\left[\frac{h(T_f - T_1)}{h + \frac{k}{L}}\right]$$

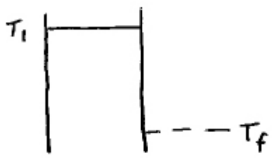
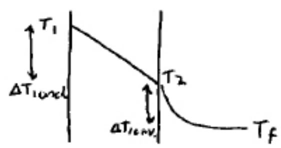
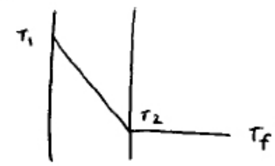
$$\frac{T - T_1}{T_f - T_1} = \frac{x}{L}\left[\frac{h\frac{L}{k}}{1 + h\frac{x}{L}}\right]$$

$$\boxed{\Theta = \chi\left[\frac{h\frac{L}{k}}{1 + h\frac{x}{L}}\right]}$$

$\frac{hL}{k} \Rightarrow \frac{h}{\frac{k}{l}} \Rightarrow \frac{\frac{L}{k}}{\frac{1}{h}}$ where $\frac{L}{k}$ is conductive resistance and $\frac{1}{h}$ is convective resistance

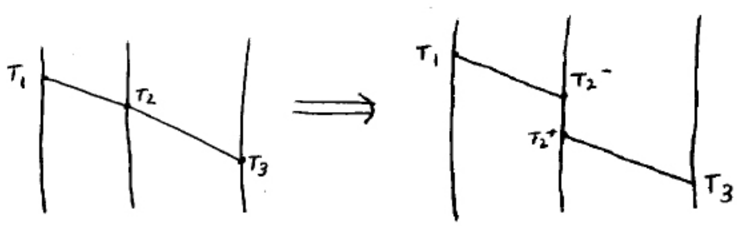
$\boxed{\text{Biot Number:} \frac{hL}{k}}$ dimensionless, ratio of resistances

Three Important Cases:

 B_i small	 $\boxed{B_i \approx 1}$	 B_i large
$T = T_1 = T_2$ slow/no convection "no gradients in solid" **convection is rate limiting** $\boxed{B_i \leq 0.1}$	transient solution conduction and convection equally important	$T_2 = T_f$ $\frac{T - T_1}{T_2 - T_1} = \frac{x}{L}$ rapid convection "fixed surface temp" **conduction is rate limiting** $\boxed{B_i \geq 10}$

Generalize:

$\boxed{1.}$ Imperfect interfaces:



$$q_{in} = q_{out}$$
$$= h(T_2^+ - T_2^-), \quad \text{where} \frac{1}{h} = \text{interface resistance}$$

$\boxed{2.}$ Geometry:

$$\frac{hL}{k} \rightarrow \text{What is L?}$$
$$L \approx \frac{\text{volume}}{\text{surface area}}, \text{ a characteristic dimension}$$

Examples:

1. plate heated on one side: $L =$ thickness
2. plate heated on both sides: $L =$ half thickness
3. cylinder: $L = \dfrac{\pi R^2 l}{2\pi R l} = \dfrac{R}{2}$
4. sphere (or other 3D shape): $L = \dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \dfrac{R}{3}$