

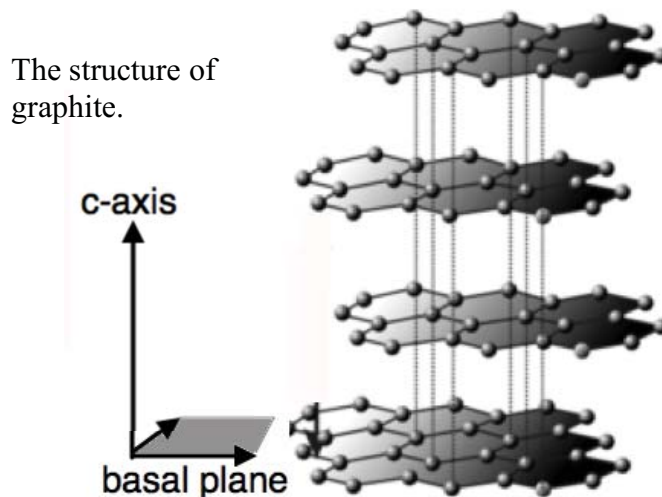
**Department of Materials Science and Engineering
Massachusetts Institute of Technology
3.044 Materials Processing – Spring 2013
Problem Set 1**

Problem 1

You are planning to investigate a new, weakly-doped graphite material as a potential superconductor. To see superconductive behavior, you must cool it to near absolute zero. To achieve this, you drop a single crystal cube of your graphite material into a liquid helium bath, held at 3K. Let us investigate the cooling of your sample.

Graphite is a hexagonal material. At your starting temperature of 300K, the thermal conductivity of graphite along any direction within the basal plane is 72 W/mK, and along the c-axis, it is 1.53 W/mK. At the final temperature of 3K, the thermal conductivity of graphite along any direction within the basal plane is 0.014 W/mK, and along the c-axis, it is 0.0056 W/mK (source for k data: G.A. Slack. “Anisotropic thermal conductivity of pyrolytic graphite.” *Phys. Rev.* **127**, 1962). The z-axis of your cube is aligned with the c-axis of the crystal.

- i. Write an expression for the thermal conductivity in each direction as a function of temperature. You may have to make some assumptions about k .
- ii. Derive the conduction equation in 3D for graphite.
- iii. Under what conditions is this a 3D conduction problem? 2D? 1D?
- iv. Solve for the temperature profile as a function of time, making any justifiable approximations.
- v. Can you explain why graphite is a better conductor within the basal plane than along its c-axis?



Problems 2-5

Over the next few problems we will analyze the temperature profile in a nuclear fuel rod. The rod is very long and is 20 cm in diameter, and the air around it is held at 25°C by a water cooling system.

Problem 2

Write down a “word equation” for conservation of heat in a small element of material of width Δr along the conduction direction. Include a term for heat generation. Then, introduce mathematical expressions for the terms in your word equation, assuming that \dot{q} is the amount of heat generated due to radioactive decay, per unit volume. Letting Δr go to a differentially small size, derive the one-dimensional heat conduction equation in the presence of heat generation.

Problem 3

Solve the equation you derived in Problem #2 for a fuel rod at steady state. Apply boundary conditions for heating in air at ambient temperature (T_0) with a convective boundary condition and a known heat transfer coefficient of h . (Hint: there is a symmetry b.c. for the rod, which may make the problem easier.)

Uranium undergoes a phase transition at 660C, and we want to avoid this because it would jeopardize the mechanical integrity of the rod. What is the maximum rate of uniform heat production due to radioactivity that the rod can tolerate?

Problem 4

Imagine that the heat transfer coefficient was reduced to 2 W/mK by placing the rods in a stagnant, inert atmosphere (to prevent oxidation). Now what can you say about the maximum allowable rate of heat production? Like many processing problems, there is an easier way that brute-force. For the same problem, calculate the Biot number and prove that conduction is fast in this situation. Then write down a simple expression that equates the heat generated in the rod to that lost by convection. Comment on the difference between your answer in question 3 and here.

Problem 5

Quantitatively determine the dominant heat transfer process in the following situations:

- i. Cooling a 1m wide, 0.4m thick billet of steel in a 1m-thick refractory (Al_2O_3 / CaO blend) mold, which is in air and open on top.
- ii. Blow-drying hair with a thin layer of water on the strand. Hair is ~ 80 microns in diameter, and the water layer is 40 microns thick.
- iii. Quenching 3cm-thick slabs of Cu in water, which has a thin layer (10 microns) of oxide on its surface (like we saw in the video on the first day of class).
- iv. Curing of a powder coating: 50 micron spheres of polyester resting on top of a 3mm-thick aluminum substrate in a furnace.

Problems 6-9

In this problem, you will learn an extremely powerful method for solving the heat equation, and de-mystify the “series solutions” in Poirier & Geiger. You will taken through the step-by-step process to solve the 1D heat equation with the boundary conditions $\Theta(0, \tau) = 0$ and $\Theta(1, \tau) = 0$; initial condition $\Theta(\xi, 0) = 1$ for $0 < \xi < 1$.

Problem 6

Assume that the solution takes the form of a Fourier series:

$$\Theta(\xi, \tau) = \sum_{k=0}^{\infty} A_k(\tau) \cos(k \xi) + B_k(\tau) \sin(k \xi)$$

Note that $A_k(\tau)$, $B_k(\tau)$, and the form of k are unknown. Also, this solution is periodic in ξ , which is okay, all we are concerned with is the range $[0, 1]$.

Write down the 1D heat equation. Also write down $\frac{\partial \theta}{\partial \tau}$, $\frac{\partial \theta}{\partial \xi}$, and $\frac{\partial^2 \theta}{\partial \xi^2}$.

Using your expressions for the derivatives of $\Theta(\xi, \tau)$, substitute them into the heat equation. Solve for $A(\tau)$ and $B(\tau)$ (do not apply the boundary conditions yet). You should include integration constants which we will call $A_{k,\tau=0}$ and $B_{k,\tau=0}$.

Problem 7

Apply the boundary condition $\Theta(0, \tau) = 0$ to find a restriction on $A_k(\tau)$ and/or $B_k(\tau)$. Then, apply the second boundary condition to place a restriction on k . Write your solution with the consequences of these two boundary conditions included.

Next, we will make use of the initial condition to find the integration constants $A_{k,\tau=0}$ and $B_{k,\tau=0}$. The initial condition, combined with the boundary conditions, and the periodic form of the solution, should suggest a very famous function that can be written in the same form as our

expression for $\Theta(\xi, \tau)$. You can use your Fourier skills for 18.03 or 3.016 to find this expression, or you can look it up in a textbook or on the web (but only from trustworthy sources, which you should cite). Now that you have found every unknown in the equation, write the solution, $\Theta(\xi, \tau)$.

Problem 8

Now that you have a complete solution to this problem, plot it. Instead of taking the series to infinity, plot it for several values of k_{\max} at $\tau = 0.001$. Be sure to include the case of only 1 term. How many terms does it take to converge?

Repeat the above plot, but at a later time of $\tau = 0.1$. Now how many terms are required for a good approximation? Now you should appreciate under which conditions the infinite sum can be simplified dramatically...

Problem 9

Plot the solution at $\tau = 0.001, 0.01, 0.1, 0.5,$ and 1 . Use as many terms as you judge are appropriate.

Many solutions to the conduction equation can be used in more than one situation. First, give 2 examples of real-world, engineering situations that this solution would apply to, and relate the symbols in the equation to real dimensions, temperatures, etc. Second, describe a different set of boundary and initial conditions in which this same equation can be used as the solution (hint: symmetry).

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