3.012 Practice Problems for Recitation 3 (10.04.05)
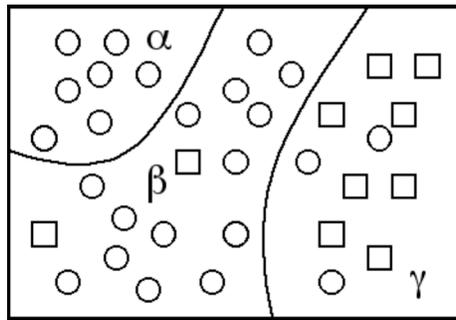
Part I. Chemical work in a multi-component system.

Consider a three-phase, two-component system. Phase α is composed entirely of A molecules, phase β is rich in A and dilute in B molecules, and phase γ is B-rich and A-poor.

a) Draw a picture of this system.

... where A is represented by circles, and B by squares.

b) What are all the relevant chemical work terms?

$$ dw = \mu_A^\alpha dn_A^\alpha + \mu_A^\beta dn_A^\beta + \mu_A^\gamma dn_A^\gamma + \mu_B^\beta dn_B^\beta + \mu_B^\gamma dn_B^\gamma $$

c) Do you remember how to think about work terms in general, as two parts?

Force times a differential displacement: $F\ dx$.

Here, F is μ, and intensive property, while x is n (moles), an extensive property. In place of n, intensive properties such as concentration or mole fraction are sometimes used, by making appropriate conversions.

Part II. Heat and temperature changes.

Consider problem 10 from problem set 2. What happens if you only allow 46 kJ of heat into the system? Recall:

Manganese: n = 2 moles
$\overline{C}_p^\alpha$ = 21.6 + 0.0159 T J / mol K
$\overline{C}_p^\beta$ = 34.9 + 0.0028 T J / mol K
$\overline{H}^{\alpha \to \beta}$ = 2010 J/mol; T_{tr} = 993 K

a) What is the final temperature of the sample, and what phase(s) are present in what proportions?

The general method should more or less be covered in your pset 2 solutions (in which 100+ kJ of heat resulted in a full change to the δ phase). In class, I showed how to do the full $C_p\, dT$ integration (integrating A + BT).

For going from 298-993 K (step 1), the energy used was 44, 270 J, leaving 1730 J still available for the phase transition (step 2). This is less than the 4020 J required to convert 2 moles from α to β. Thus, we use a proportion to see how many moles are converted, assuming the change is linear:

Ratio: 1730/4020 x 100 = 43% β-phase, or 0.86 moles; 57% is still α-phase.

Final T is 993 K.

b) What is the total entropy change for this process?

Again, a lot of this should be covered in the pset 2 solutions. In class, I showed fully how to do the $(C_p/T)\, dT$ integration (integrating (A/T + B) dT).

The entropy for step 1 is the usual entropy integral: $\Delta S = \int n \dfrac{C_p}{T} dT = 74.1$ J/K.

For a phase transition (step 2), normally $\Delta S = \dfrac{\Delta H_{tr}}{T_{tr}}$.

Here, since we did not fully convert the material, the entropy change is two moles times 1730 / 993, *not* times 4020/993.

If you need further assistance, please see me at office hours.