24.241 — Logic I, Fall 2009
Final Exam

You may not use any notes, handouts, or other material during the exam. All cell phones must be turned off. Please read all instructions carefully. Good luck with the exam and with the rest of your time at MIT!

Part One: English (1 pt. each)

1. Fill in the blank: An argument (in English) is deductively valid iff . . . .

2. Fill in the blank: English sentences **P** and **Q** are logically equivalent iff . . . .

3. True / False: For any argument A (in English), if the premises and conclusion of A are all true, then A is deductively sound.

4. True / False: For any set Γ of English sentences, if no member of Γ is logically true, then some argument whose premises and conclusion are members of Γ is deductively invalid.

5. True / False: For any argument A (in English), if A's conclusion is logically equivalent to one of A's premises, then A is deductively valid.

Specify whether each of the following arguments is deductively valid or invalid.

6.
    Flounders don't snore.
    If flounders snore, then they annoy sharks.
    ―――――――――――――――――――――
    Flounders don't annoy sharks.

7.
    Light travels faster than any spaceship can travel.
    Every spaceship can travel faster than any cheetah can run.
    ―――――――――――――――――――――
    No cheetah can run faster than some spaceship can travel.

8.
    Everyone loves anyone who loves someone.
    John loves Mary.
    ―――――――――――――――――――――
    Everyone loves everyone.

9.
    6+3=63.
    ―――――――――――――――――
    Some dogs bark.

Part Two: Sentential Logic

10. Symbolize each of the following English sentences **in SL**, revealing as much of their logical form as possible. Be sure to indicate which sentences your sentence letters abbreviate. If you think the English sentence is ambiguous among multiple symbolizations, provide them all. (1 pt. each)

   (a) Flounders snore if all fish snore.

   (b) Flounders don't snore or sleep.

   (c) Only if it rains will the concert be cancelled.

   (d) John will turn into a frog if and only if either he doesn't receive an antidote or the antidote doesn't work.

   (e) Neither rain nor snow will stop the postal service.

   (f) Mary and Bill are tall, but their children aren't.

   (g) Mary and Bill are a married couple.

   (h) At least one of John, Mary, and Bill is a lobbyist, but if Mary is, then John and Bill are too.

11. Complete the following truth-table. (3 pts.)

A	B	C	$\sim ((A \supset B) \lor \sim (A\& \sim B)) \supset (C\& \sim C)$

12. Construct truth-tables to establish each of the following claims. Be sure to indicate which parts of the truth-table demonstrate the point. (3pts. each)

(a) $\sim A \supset (A \supset B)$ is truth-functionally true.

(b) $\{A \equiv (B \lor C), \sim A\& \sim C\}$ is truth-functionally consistent.

13. Which of the following sorts of claims, if true, can be proved **in SD**? (3 pts. total)

    (a) That a certain set of sentences is truth-functionally consistent.

    (b) That a certain set of sentences is truth-functionally inconsistent.

    (c) That a certain set of sentences truth-functionally entails a certain other sentence.

    (d) That a certain set of sentences does not truth-functionally entail a certain other sentence.

    (e) That a certain sentence is truth-functionally true.

    (f) That a certain sentence is not truth-functionally true.

    (g) That a certain sentence is truth-functionally false.

    (h) That a certain sentence is not truth-functionally false.

    (i) That a certain sentence is truth-functionally indeterminate.

    (j) That two particular sentences are truth-functionally equivalent.

    (k) That two particular sentences are not truth-functionally equivalent.

    (l) That a certain argument is truth-functionally valid.

    (m) That a certain argument is not truth-functionally valid.

14. Construct derivations **in SD** to prove each of the following:

    (a) $\{D\&E, D \equiv C\} \vdash (A\& \sim A) \vee C$ (3 pts.)

(b) $\{P \vee Q, \sim Q, P \supset Q\} \vdash R \& \sim R$ (3 pts.)

(c) $\{A \supset B, C \supset D, A \vee C\} \vdash B \vee D$ (3 pts.)

(d) $\{A \supset B\} \vdash \sim A \lor B$ (4 pts.)

15. Fill in the blank: A set of connectives is truth-functionally complete iff ...   (2pts.)

16. What is the minimum number of connectives (not necessarily the familiar connectives of SL) required to form a truth-functionally complete set? (1 pt.)

17. Provide examples of (a) a truth-functionally complete set of connectives from SL and (b) a set of two or more connectives from SL that is not truth-functionally complete. (2 pts.)

18. Let Γ_n be the set of assumptions open at line n of a derivation in SD, and $\mathbf{P_n}$ be the sentence occurring on line n. Prove that if, for all lines up to and including k, $\Gamma_k \vDash \mathbf{P_k}$, then if line k+1 is justified by $\vee$I, then $\Gamma_{k+1} \vDash \mathbf{P_{k+1}}$.   (4 pts.)

19. Prove that for every set Γ^* of SL sentences that is maximally consistent in SD, $\Gamma^* \vdash \mathbf{P}$ iff $\mathbf{P} \in \Gamma^*$.   (3 pts.)

20. Prove that for every set Γ^* of SL sentences that is maximally consistent in SD, if $\mathbf{P} \notin \Gamma^*$ then $\sim \mathbf{P} \in \Gamma^*$. You may appeal to the result above.   (3 pts.)

21. Prove that for every set Γ* of SL sentences that is maximally consistent in SD, **Q** ∨ **R** ∈ Γ* iff **Q** ∈ Γ* or **R** ∈ Γ*. You may appeal to any result above.   (3 pts.)

Part Three: Predicate Logic

22. Provide an argument in English whose symbolization in SL in invalid in SD but whose symbolization in PL in valid in PD. Provide both symbolizations.   (1 pt.)

23. Fill in the blank: A sentence of PL is true on interpretation I iff ...   (1 pt.)

24. Fill in the blank: A sentence (∃**x**)**P** of PL is satisfied by variable assignment d on interpretation I iff ...   (1 pt.)

25. Symbolize the following English sentences **in PL**, revealing as much of their logical form as possible. If you think the English sentence is ambiguous among multiple symbolizations, provide them all. In the space below, you may provide a single symbolization key to cover all the sentences. (1 pt. each)

(a) Some students understand TLB.

(b) Professors who bore someone are common.

(c) All professors like some of their students.

(d) Every student is passed by some professor who understand him or her.

(e) Any professor who is bored by everything bores all his or her students.

(f) Professors pass all and only those of their students they are understood by.

(g) No professor who understands TLB bores any of his or her students.

(h) If a student understand TLB, some professor passes him or her.

26. Whenever any truth-functional property listed in question (13) holds of certain sentences or among certain sentences and sets of sentences of SL, truth-tables provide a method for proving that that property holds. When we turn to PL, do we have an equally powerful method? Which quantificational properties, when they obtain, can be proved in PD to obtain? (3 pts.)

27. One of the restrictions on the use of ∃E is that the instantiating constant must not occur in an undischarged assumption. Explain, using an example, why this restriction is necessary. (3 pts.)

28. Provide derivations to prove each of the following claims. You may use any rules of PD as well as the rules =I and =E.

    (a) $\{(\forall x)Fx\} \vdash (\exists x)Fx$ (3 pts.)

(b) $\{(\exists x)(Fx \supset Ga)\} \vdash (\forall x)Fx \supset Ga$ (3 pts.)

(c) $\{(\forall x)(Fx \lor (\exists y)Gxy)\} \vdash (\forall x)(\exists y)(Fx \lor Gxy)$ (3 pts.)

(d) $\{Hb \lor (\forall z) \sim Hz, (\exists y)Hy\} \vdash Hb$ (4 pts.)

(e) $\emptyset \vdash \sim a = b \supset \sim b = a$ (3 pts.)

29. Provide a sentence of PL or PLE that is true on some interpretation, and that is true only on interpretations whose UD contains infinitely many things.   (2 pts.)

30. Provide a sentence of PLE that is true on some interpretation, and is not true on any interpretation whose UD contains infinitely many things.   (2 pts.)

31. The *Löwenheim-Skolem Theorem* tells us:

    If a set Γ of sentences of PL is quantificationally consistent, then there is an interpretation with the set of positive integers as the UD on which every member of Γ is true.

    Prove this result. You may appeal to any of our meta-logical results from class, including our proofs of the soundness and completeness of PD.   (3 pts.)

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