Question 1

Here is the truth-table. Same conventions as usual.

A	C	$\sim$	$(C$	$\vee$	$A)$	$\sim$	$(C$	$\equiv$	$\sim$	$A)$
T	T	F	T	T	T	T	T	F	F	T
T	F	F	F	T	T	F	F	T	F	T
F	T	F	T	T	F	F	T	T	T	F
F	F	T	F	F	F	T	F	F	T	F
		2	0	1	0	3	0	2	1	0

As you can see, there are no lines in which the premise is true and the conclusion false — the only line in which the premise is true is the last one, and on that line the conclusion is true too. So this argument is truth-functionally valid.

Question 2

All of these are proved by providing derivations in SD. I do so below.

Part (a)

1	$A \supset B$	A
2	| $\sim B$	A/$\supset$I
3	| | $A \& D$	A/$\sim$I
4	| | A	3, &E
5	| | B	1,4 $\supset$ E
6	| | $\sim B$	2 R
7	| $\sim (A \& D)$	3-6 $\sim$I
8	$\sim B \supset \sim (A \& D)$	2-7 $\supset$I

Part (b)

1	$(A \supset B) \supset \sim B$	A
2	B	A/$\sim$I
3	A	A/$\supset$I
4	B	2 R
5	$A \supset B$	3-4, $\supset$I
6	$\sim B$	1,5 $\supset$E
7	B	2 R
8	$\sim B$	2-8 $\sim$I

Part (c)

1	$F \supset (G \vee H)$	A
2	$\sim (\sim F \vee H)$	A
3	$\sim G$	A
4	H	A/$\sim$I
5	$\sim F \vee H$	4 $\vee$I
6	$\sim (\sim F \vee H)$	2 R
7	$\sim H$	4-6 $\sim$I

Question 3 (5.3E 12(b))

Let ‘A’ = ‘The recipe calls for flavouring’; ‘B’ = ‘The recipe calls for eggs’; ‘C’ = ‘The recipe is a recipe for tapioca.’

The relevant set of sentences is $\{$‘$(\sim A \vee \sim B) \supset \sim C$’, ‘$B \supset (C \& \sim A)$’, ‘$B$’$\}$.

The below derivation shows that this set is inconsistent in SD.

1	$(\sim A \vee \sim B) \supset \sim C$	A
2	$B \supset (C \& \sim A)$	A
3	B	A
4	$C \& \sim A$	2,3 $\supset$E
5	$\sim A$	4 &E
6	$\sim A \vee \sim B$	5 $\vee$I
7	$\sim C$	1,6 $\supset$E
8	C	4 &E

Question 4 (5.3E 13(a))

This derivation rule allows one to derive something false from something true. To see this: let $\mathbf{P} = A$, $\mathbf{Q} = B$, and consider the truth-value assignment that assigns false to A and true to B. Under that assignment, '$A \vee B$' is true, but 'B' is false.

That's bad — the only derivation rules we want are ones that guarantee that the sentence you derive is true, given that the sentences you started with are true.

Question 5 (5.3E 13(e))

Because, in SD, you can derive any conclusion from the negation of theorem.

Why is that? Suppose we have a derivation in SD with the negation of some theorem $\mathbf{P}$ as an assumption on line i. As $\mathbf{P}$ is a theorem, we can derive it in this derivation; say we do so on line j. Then, starting at line $k > max(i, j)$, we can construct a sub-derivation of the following form:

k	$\sim \mathbf{Q}$	A/$\sim$E
$k+1$	$\mathbf{P}$	j R
$k+2$	$\sim \mathbf{P}$	i R
$k+3$	$\mathbf{Q}$	k–$k+1$ $\sim$E

One can do this for any value of $\mathbf{Q}$. So, one can derive any sentence from the negation of a theorem, in SD.

Question 6

I like this one.

1	$\sim((A \supset B) \vee (B \supset A))$	A/$\sim$E
2	A	A/$\sim$I
3	B	A/$\supset$I
4	A	2 R
5	$B \supset A$	3-4 $\supset$I
6	$(A \supset B) \vee (B \supset A)$	5 $\vee$I
7	$\sim((A \supset B) \vee (B \supset A))$	1 R
8	$\sim A$	2-7 $\sim$I
9	A	A/$\supset$I
10	$\sim B$	A/$\sim$E
11	A	9 R
12	$\sim A$	8 R
13	B	10-12 $\sim$E
14	$A \supset B$	9-13 $\supset$I
15	$(A \supset B) \vee (B \supset A)$	14 $\vee$I
16	$\sim((A \supset B) \vee (B \supset A))$	1 R
17	$(A \supset B) \vee (B \supset A)$	1-16 $\sim$E

Question 7

Consider an SD derivation and sentences $\mathbf{P}, \mathbf{Q}$ of SL such that $\ulcorner \mathbf{P} \vee \mathbf{Q} \urcorner$ appears on line i of the derivation, to the right of m scope-lines, and $\ulcorner \sim \mathbf{P} \urcorner$ appears on line j of the derivation, to the right of n scope lines. Starting at row $k > max(i, j)$, and to the right of $max(m, n) - 1$ scope lines, one can construct a sub-derivation of the following form:

k	$\quad$ $\mathbf{P}$	A/$\vee$E
$k+1$	$\qquad$ $\sim \mathbf{Q}$	A$\sim$E
$k+2$	$\qquad$ $\mathbf{P}$	k R
$k+3$	$\qquad$ $\sim \mathbf{P}$	j R
$k+4$	$\quad$ $\mathbf{Q}$	$k+1$-$k+3$ $\sim$E
$k+5$	$\quad$ $\mathbf{Q}$	A/$\vee$E
$k+6$	$\quad$ $\mathbf{Q}$	k+5 R
$k+7$	$\mathbf{Q}$	i, $k-k+4$, $k+5-k+6$ $\vee$E

Using this construction, one can always derive $\mathbf{Q}$, from $\ulcorner \mathbf{P} \vee \mathbf{Q} \urcorner$ and $\ulcorner \sim \mathbf{P} \urcorner$, in SD. So anything we can derive in SD* we can derive in SD.