3.4E

4c

No, it does not follow. Sometimes it is the case that there is truth-value assignment that makes **P** true and one that makes **P** false, and a truth-value assignment that makes **Q** true and one that make **Q** false, but there is no one truth-value assignment such that both **P** and **Q** are true on that assignment.

An example: ‘A’ and ‘$\sim A$’ are both truth-functionally indeterminate, but {‘A’,‘$\sim A$’} is not truth-functionally consistent.

3.5E

With many of these questions, you can see what the answer is without constructing the truth-table. You need to construct the truth-table, nevertheless. It’s good for you.

I’ve used the same conventions as the last answers: main connectives singled out by vertical lines around their columns, numbers at the bottom to indicate the order of calculation.

1d

This argument is truth-functionally valid. Here’s the truth-table:

A	W	Y	$\sim$	$(Y$	$\equiv$	$A)$	$\sim$	Y	$\sim$	A	W	$\&$	$\sim$	W
T	T	T	F	T	T	T	F	T	F	T	T	F	F	T
T	T	F	T	F	F	T	T	F	F	T	T	F	F	T
T	F	T	F	T	T	T	F	T	F	T	F	F	T	F
T	F	F	T	F	F	T	T	F	F	T	F	F	T	F
F	T	T	T	T	F	F	F	T	T	F	T	F	F	T
F	T	F	F	F	T	F	T	F	T	F	T	F	F	T
F	F	T	T	T	F	F	F	T	T	F	F	F	T	F
F	F	F	F	F	T	F	T	F	T	F	F	F	T	F
			2	0	1	0	1	0	1	0	0	2	1	0

You may have the rows in a different order — that’s fine.

As you can see, there are no rows such that all of the premises get assigned T and the conclusion gets assigned F (as there are no rows such that all of the premises get assigned T at all). So the argument is truth-functionally valid.

2c

This argument is truth-functionally valid. Observe:

A	B	A	$\supset$	$\sim$	A	$(B$	$\supset$	$A)$	$\supset$	B	A	$\equiv$	$\sim$	B
T	T	T	F	F	T	T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	F	T	T	F	F	T	T	T	F
F	T	F	T	T	F	T	F	F	T	T	F	T	F	T
F	F	F	T	T	F	F	T	F	F	F	F	F	T	F
		0	2	1	0	0	1	0	2	0	0	2	1	0

There is one row that assigns both premises true: the $\langle F, T\rangle$ row (marked out by horizontal lines). As you can see, that row assigns T to the conclusion. So there are no rows such that all of the premises get assigned T and the conclusion gets assigned F. So the argument is truth-functionally valid.

2d

Ergh — one of the sentence letters is a 'T'. Don't confuse the sentence letter 'T' with the truth-value 'T' below.

This argument is truth-functionally invalid. Here is a shortened truth-table that shows that.

J	M	T	J	$\vee$	$[M$	$\supset$	$(T$	$\equiv$	$J)]$	$(M$	$\supset$	$J)$	$\&$	$(T$	$\supset$	$M)$	T	$\&$	$\sim$	M
T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	F	F	T
			0	3	0	2	0	1	0	0	1	0	2	0	1	0	0	2	1	0

4f

Let A = 'The butler murdered Devon'; B = 'The maid is lying'; C = 'The gardener murdered Devon'; D = 'The weapon was a slingshot'.

The argument:

$$\begin{array}{l} (A \supset B)\&(C \supset D) \\ (B \equiv \sim D)\&(\sim D \supset A) \\ \hline A \end{array}$$

This argument is not truth-functionally valid. Check it out:

A	B	C	D	$(A$	$\supset$	$B)$	$\&$	$(C$	$\supset$	$D)$	$(B$	$\equiv$	$\sim$	$D)$	$\&$	$(\sim$	D	$\supset$	$A)$	A
F	F	T	T	F	T	F	T	T	T	T	F	T	F	T	T	F	T	T	F	F
				0	1	0	2	0	1	0	0	2	1	0	3	1	0	2	0	

5c

No, it does not follow. Here is a counterexample. Let $\mathbf{P}$=$A \vee B$, $\mathbf{Q}$=A, $\mathbf{R}$=B. $\{`A \vee B'\}$ truth-funcitonally entails $A \vee B$, obviously. $\{`A \vee B'\}$ does not truth-functionally entail A (consider A false, B true), and it does not truth-functionally entail B (consider A true, B false).

3.6E

Throughout this section, when I say something like 'there is no truth-value assignment such that' blah, what I mean is 'there is no truth-value assignment such that, on that assignment' blah.

2b

$\Gamma \models \ulcorner \mathbf{P} \supset \mathbf{Q} \urcorner$ iff there is no truth-value assignment such that every member of Γ is true and $\ulcorner \mathbf{P} \supset \mathbf{Q} \urcorner$ is false. There is no truth-value assignment such that every member of Γ is true and $\ulcorner \mathbf{P} \supset \mathbf{Q} \urcorner$ is false iff there is no truth-value assignment such that every member of Γ is true, $\mathbf{P}$ is true and $\mathbf{Q}$ is false (by the definition of '$\supset$'). There is no truth-value assignment such that every member of Γ is true, $\mathbf{P}$ is true and $\mathbf{Q}$ is false iff there is no truth-value assignment such that every member of $\Gamma \cup \{\mathbf{P}\}$ is true and $\mathbf{Q}$ is false. There is no truth-value assignment such that every member of $\Gamma \cup \{\mathbf{P}\}$ is true and $\mathbf{Q}$ is false iff $\Gamma \cup \{\mathbf{P}\} \models \mathbf{Q}$. Therefore, $\Gamma \models \ulcorner \mathbf{P} \supset \mathbf{Q} \urcorner$ iff $\Gamma \cup \{\mathbf{P}\} \models \mathbf{Q}$.

Q.E.D.

3b

Suppose $\Gamma \models \mathbf{P}$ and $\Gamma \models \ulcorner \sim \mathbf{P} \urcorner$. Then

(1) there is no truth-value assignment such that every member of Γ is true and $\mathbf{P}$ is false, and

(2) there is no truth-value assignment such that every member of Γ is true and $\sim \mathbf{P}$ is false.

By (2) (and the definition of '$\sim$'), there is no truth-value assignment such that every member of Γ is true and $\mathbf{P}$ is true. From this and (1) it follows that there is no truth-value assignment such that every member of Γ is true and $\mathbf{P}$ is true and there is no truth-value assignment such that every member of Γ is true and $\mathbf{P}$ is false. But if there is any truth-value assignment such that every member of Γ is true, it is either such that every member of Γ is true and $\mathbf{P}$ is true or it is such that every member of Γ is true and $\mathbf{P}$ is false. So there is no truth-value assignment such that every member of Γ is true. So Γ is truth-functionally inconsistent.

So if $\Gamma \models \mathbf{P}$ and $\Gamma \models \ulcorner \sim \mathbf{P} \urcorner$, then Γ is truth-functionally inconsistent.

Q.E.D.

4a

Suppose $\{\mathbf{P}\} \models \mathbf{Q}$, and $\{\ulcorner \sim \mathbf{P} \urcorner\} \models \mathbf{R}$. Then

(1) there is no truth-value assignment such that $\mathbf{P}$ is true and $\mathbf{Q}$ is false, and

(2) there is no truth-value assignment such that $\sim \mathbf{P}$ is true and $\mathbf{R}$ is false.

By (2) (and the definition of '$\sim$'),

(3) there is no truth value assignment such that **P** is false and **R** is false.

Now, every truth-value assignment is either such that **Q** is true or such that **Q** is false. So, by (1),

(4) if a truth-value assignment is such that **P** is true, it is such that **Q** is true.

And every truth-value assignment is either such that **R** is true or is such that **R** is false. So, by (3)

(5) if a truth-value assignment is such that **P** is false, then it is such that **R** is true.

But every truth-value assignment is either such that **P** is true or such that **P** is false. So, by (4) and (5), every truth-value assignment is either such that **Q** is true or such that **R** is true. So (by the definition of '$\vee$'), every truth-value assignment is such that $\ulcorner \mathbf{Q} \vee \mathbf{R} \urcorner$ is true. So $\ulcorner \mathbf{Q} \vee \mathbf{R} \urcorner$ is truth-functionally true.

So if $\{\mathbf{P}\} \models \mathbf{Q}$, and $\{\ulcorner \sim \mathbf{P} \urcorner\} \models \mathbf{R}$, then $\ulcorner \mathbf{Q} \vee \mathbf{R} \urcorner$ is truth-functionally true.

Q.E.D.

4c

Suppose $\Gamma \models \mathbf{P}$ and $\Gamma' \models \mathbf{Q}$. Then

(1) there is no truth-value assignment such that every member of Γ is true and **P** is false, and

(2) there is no truth value assignment such that every member of Γ' is true and **Q** is false.

By (1), there is no truth-value assignment such that every member of $\Gamma \cup \Gamma'$ is true and **P** is false. And by (2), there is no truth-value assignment such that every member of $\Gamma \cup \Gamma'$ is true and **Q** is false. So (by the definition of '&') there is no truth-value assignment such that $\Gamma \cup \Gamma'$ is true and $\ulcorner \mathbf{P} \& \mathbf{Q} \urcorner$ is false. So $\Gamma \cup \Gamma' \models \ulcorner \mathbf{P} \& \mathbf{Q} \urcorner$.

So, if $\Gamma \models \mathbf{P}$ and $\Gamma' \models \mathbf{Q}$, then $\Gamma \cup \Gamma' \models \ulcorner \mathbf{P} \& \mathbf{Q} \urcorner$.

Q.E.D.