24.118 – Paradox and Infinity

Problem Set 5: Infinity

How this problem set will be graded:

Unlike the previous problem set, this time you will be graded both on the basis of whether your answers are correct and on the basis of whether they are properly justified. There is no word limit.

As with all problem sets, you may consult published literature and the web. You must, however, credit all sources. Failure to do so constitutes plaigiarism and can have serious consequences. For advice about when and how to credit sources see:

http://web.mit.edu/academicintegrity/handbook/handbook.pdf

Problems:

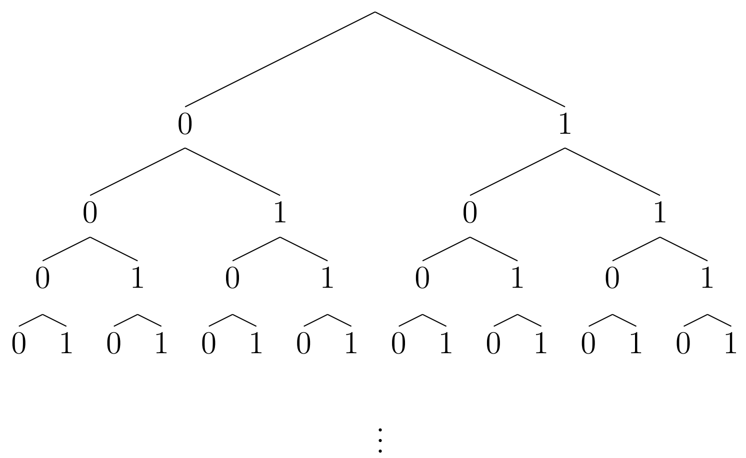
1. Show that there is a 1-1 correspondence between the whole numbers (...-3, -2, -1, 0, 1, 2, 3, ...) and the prime numbers.

2. Show that there is a 1-1 correspondence between the natural numbers and the set of all pairs $\langle n, m \rangle$ for n and m natural numbers.

3. Show that there is a 1-1 correspondence between [0,1) and [0,∞) (i.e. between the set of real numbers greater or equal to zero and smaller than 1, and the set of real numbers greater or equal to 0.)

4. Show that there can be no 1-1 correspondence between the natural numbers and the set of all functions from the natural numbers to the set $\{a, b, c, d, e\}$.

5. Consider the following infinite tree:



(When fully spelled out, the tree contains one row for each natural number. The zero-th row contains one node, the first row contains two nodes, the second row contains four nodes, and, in general, the nth row contains 2^n nodes.)

Is there a 1-1 correspondence between the *nodes* of this tree and the natural numbers? Is there a 1-1 correspondence between the *branches*[1] of this tree and the natural numbers? (Don't forget to justify your answers!)

[1] A branch is an infinite sequence of nodes which starts at the top of the tree and contains a node at every row, with each node connected to its successor by an edge. (Branches can be represented as infinite sequences of zeroes and ones.)

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Spring 2013