

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**

Departments of Electrical Engineering, Mechanical Engineering, and the Harvard-MIT Division  
of Health Sciences and Technology

6.022J/2.792J/BEH.371J/HST542J: Quantitative Physiology: Organ Transport Systems

**PROBLEM SET 8**

**SOLUTIONS**

April 15, 2004

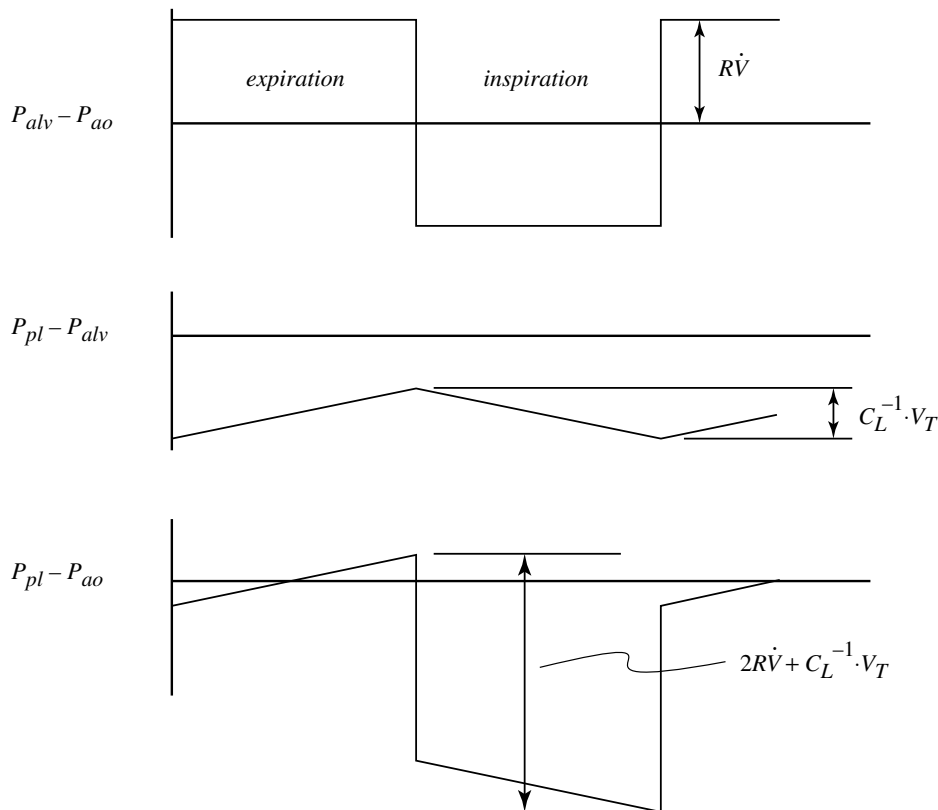
### Problem 1

A student with a mild case of asthma was feeling perfectly well when she left for class. On the way to class she breathes cold, dry air and develops a moderate asthma attack. She takes an inhaled medication (isoproterenol) that relieves the symptoms.

- A. Modeling the lung as a single resistor and a single capacitor attached in series, what are the peak-to-peak pleural pressure swings in each case below? Assume lung compliance  $C_L$  stays fixed at  $0.1 \text{ L}\cdot\text{cm H}_2\text{O}^{-1}$ , that the student breathes using a square-wave flow pattern with equal inspiratory and expiratory times, and that  $R_{insp}$  and  $R_{exp}$  are equal.

	$R_{aw}$ (cmH <sub>2</sub> O·L <sup>-1</sup> ·s)	$f$ (breaths/min)	$V_T$ (ml)
Normal	1	20	500
Asthma	20	40	250
Treated	5	25	400

The difference between pleural pressure ( $P_{pl}$ ) and atmospheric or airway opening pressure ( $P_{ao}$ ) can be thought of as the sum of  $(P_{pl} - P_{alv})$  and  $(P_{alv} - P_{ao})$ , where the former represents the effect of lung compliance and the latter, airway resistnace. These pressure differences for a square wave flow pattern are sketched below.



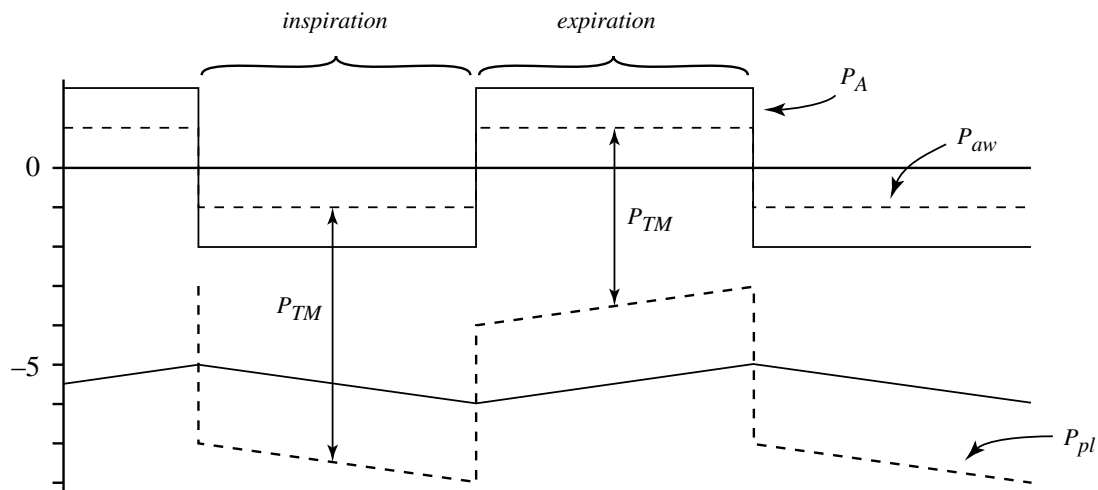
It can be seen that the maximum swing in  $P_{pl} - P_{ao}$  is equal to

$$2R_{aw}\dot{V} + \frac{V_T}{C_L}$$

The flow rate on inspiration is  $V_T/t_{insp}$  where  $t_{insp}$  is the inspiratory time per breathing cycle. The following can then be calculated:

$t_{insp} = t_{exp}$ (sec)	$\dot{V}$ (L·sec <sup>-1</sup> )	$\Delta (P_{pl} - P_{ao})_{max}$ (cm H <sub>2</sub> O)
1.5	1/3	5.7
0.75	1/3	15.8
1.2	1/3	7.3

- B. The situation described above has been idealized to keep the analysis simple. For example, in reality, inspiratory flow resistance during the attack will be significantly less than the resistance experienced during expiration. Why?



$$P_{TM} = P_{aw} - P_{pl}$$

$$\Delta 2V_{tf} R_{aw}$$

$P_{TM}$  is higher in inhalation and lower in exhalation.

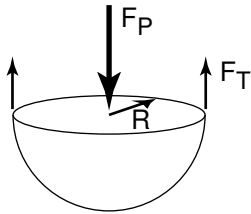
$$\therefore Area_{aw} = f(P_{TM})$$

in inhalation    Area  $\uparrow$   $\rightarrow$   $R_{aw}$   $\downarrow$   
in exhalation    Area  $\downarrow$   $\rightarrow$   $R_{aw}$   $\uparrow$

## Problem 2

- A. The alveolus may be modeled as a simple thin-walled sphere of radius  $R$ . Derive the relationship between the pressure inside the alveolus,  $P$ , and the tension,  $T$ , in the wall. Hint: the answer is  $P = 2T/R$ .

Consider a force balance between pressure and surface tension on half of the bubble:



$$F_T = \text{surface tension force} = 2\pi RT$$

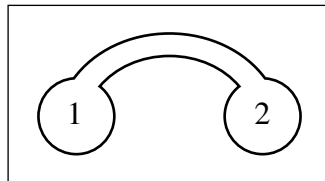
$$F_p = \text{pressure force} = \pi R^2 p$$

where  $p$  is the pressure difference between inside and outside of the bubble.

A force balance gives:

$$F_T = F_p; \quad p = \frac{2T}{R}$$

- B. Assume one has two bubbles of equal radius connected together by a tube. The bubbles contain a surfactant which causes their surface tension to change as their surface area,  $a$ , changes.



For each of the three surfactants listed below, the relationship between surface tension,  $T$ , and area,  $a$ , is given.

surfactant 1:  $T = ka \quad k = \text{constant}$

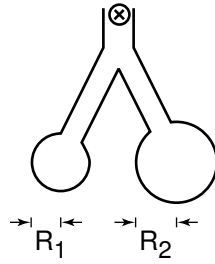
surfactant 2:  $T = k\sqrt{a} \quad k = \text{constant}$

surfactant 3:  $T = \frac{k}{\sqrt{a}} \quad k = \text{constant}$

Using Laplace's relationship, for each of the surfactants determine what will happen if the radius of one of the bubbles is increased. Why? Is the system stable?

*This system will be stable if, given a small perturbation, the bubbles return to their initial state. This happens when the pressures change in such a way that the larger bubble requires a larger pressure, i.e., that  $dp/dR > 0$ . When this condition is satisfied, if the system is*

perturbed so that  $R_1 > R_2$ , the pressures are such ( $p_1 > p_2$ ) that gas is driven from “1” to “2”, toward equilibrium.



Considering each of the three cases:

surfactant 1:

$$T = ka; \quad p = 2ka/R; \quad a = 4\pi R^2$$

$$p = 8k\pi R; \quad \frac{dp}{dR} = 8\pi k > 0$$

$\therefore$  stable,  $R_1 = R_2 = R_{equil}$ .

surfactant 2:

$$T = ka^{1/2}; \quad p = 4k\pi^{1/2}$$

$$\frac{dp}{dR} = 0$$

$\therefore$  stable, but the system will remain in the perturbed state. That is, all states are stable.

surfactant 3:

$$T = \frac{k}{a^{1/2}}; \quad p = \frac{k}{\pi^{1/2} R^2}$$

$$\frac{dp}{dR} = \frac{-2k}{R^3 \pi^{1/2}} < 0$$

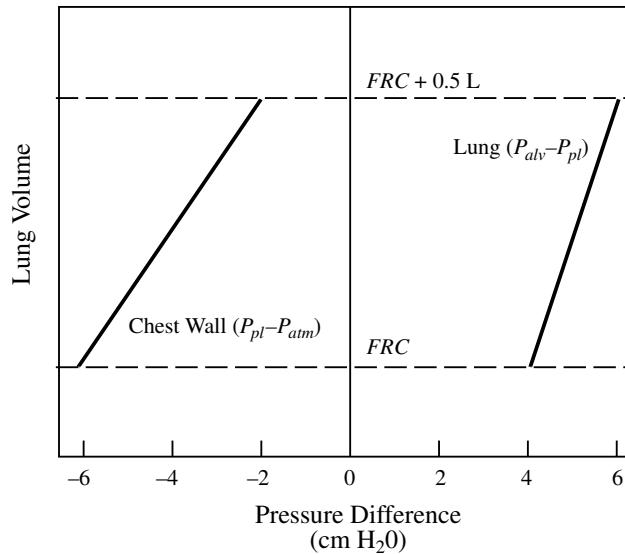
$\therefore$  The system is unstable; the smaller bubble will empty the larger one and ultimately collapse.

The presence of surfactant, since it reduces  $T$  when  $a$  decreases, will increase the tendency for stability.

### Problem 3

Consider a hypothetical patient without significant respiratory disease who enters the hospital for coronary bypass surgery. during anesthesia he will require mechanical ventilation which will be applied by a pump via an endotracheal tube. The lungs and chest wall (with respiratory muscles relaxed) of this patient have the characteristics illustrated in Figure 1.

Figure 1:



In all the following, for both normal and assisted ventilation, you may assume that:

- The patient's tidal volume remains constant at 0.5 L.
- His airways exhibit a flow resistance of  $2 \text{ cm H}_2\text{O} \cdot \text{L}^{-1} \cdot \text{s}$  with negligible inertance.
- His inspiratory and expiratory flow rates are both 0.5 L/s and are constant during each phase of breathing.
- His lung volume at end expiration is maintained constant at the level indicated as *FRC* in Figure 1.

A. What is the value of  $|P_{ao} - P_{alv}|$  (airway opening minus alveolar pressure) during inspiration and expiration at a constant rate of 0.5 L/s?

*The alveolar-airway opening pressure drop is just due to friction in this case (negligible inertance) and is given by:*

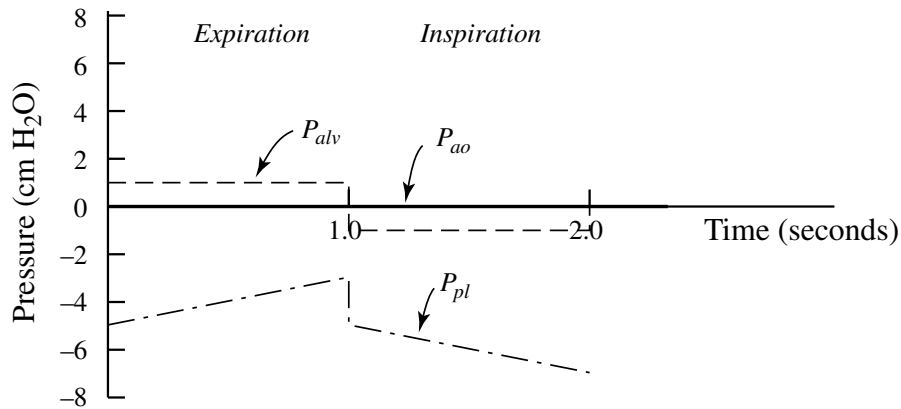
$$|P_{ao} - P_{alv}| = R \cdot \dot{V} = 1 \text{ cmH}_2\text{O}$$

B. What is the total change in  $P_{alv} - P_{pl}$  between end expiration and end inspiration?

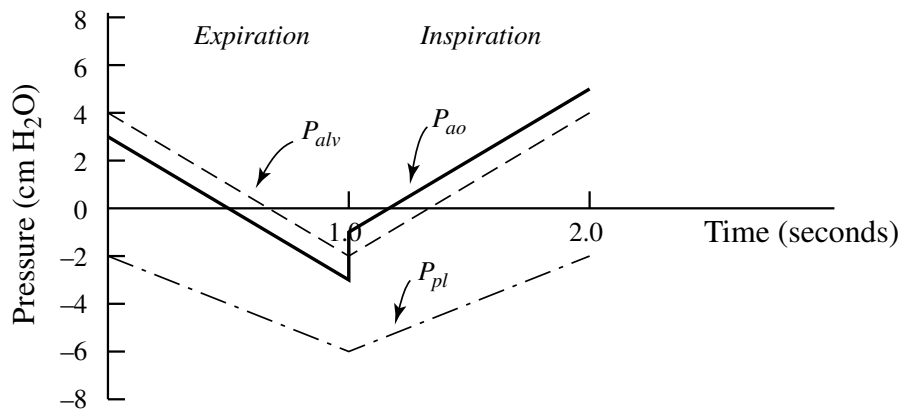
*The total change in alveolar minus pleural pressure can be read directly from Figure 1, and equals  $2 \text{ cm H}_2\text{O}$ .*

C. Sketch on the provided graphs the time-course during one complete breathing cycle of  $P_{ao}$ ,  $P_{pl}$ , and  $P_{alv}$  for each of the following cases (you may assume that Figure 1 still applies):

(i) During normal breathing before surgery.



(ii) During mechanical ventilation applied at the mouth just prior to surgery. (Assume that the respiratory muscles are completely inactive.)



(iii) At some point during the surgical procedure, during mechanical ventilation, when the patient's chest wall is open and his lungs are exposed to atmospheric pressure.

