



is measurable. A plurality is said to be that which is divisible potentially into parts which are not continuous; a magnitude, on the other hand, is that which is potentially divisible into parts which are continuous....Of these, a limited [*peperasmemon*] plurality is said to be a *arithmos*, a limited length a line, a limited width a surface, and a limited depth, a body.

**Metaphysics Iota (I)6, 1057a2-6:**

Plurality is as if it were [or "such as"; *hoion*] a genus of *arithmos*; for *arithmos* is a plurality measurable by the one. And in some sense [or "in a way"] the one and *arithmos* [or "a *arithmos*"] are opposed, not as contraries, but...as some relative things are; for the one in so far as it is a measure is opposed to *arithmos* in so far as *arithmos* is measurable.

**Metaphysics Nu (N)1, 1087b33-1088a15:**

The one signifies a measure, evidently. And in each case there is some different underlying subject [*hupokeimenon*, thing laid down], such as in the musical scale a quarter-tone; in magnitude a finger or a foot or some other such thing; and in rhythm a beat or a syllable....And this is also according to formula [or "definition" or "account": *logos*]; for the one signifies a measure of some plurality and the *arithmos* signifies a measured plurality [a plurality that has been measured] and a plurality of measures. Therefore it is also with good reason that the one is not a *arithmos*; for neither is a measure measures, but a measure is a principle [or "source", *arche*], and so is the one.

2] Based on Nicomachus' treatment of *arithmetike*, speculate on why the Greeks did not consider [a] fractions, [b] negative numbers, or [c] irrational numbers to be *arithmoi*, or group all these with *arithmoi* under as one category, similar to our concept of 'number'.

3] Do the terms 'arithmos' and 'number' connote the same 'category of understanding', and is the argument between ancients and moderns then an argument over the proper definition and extent of that category?

4] Consider the following definitions of numerical unit. Why do you think that, from the Greeks through at least the middle ages, 'one' was held to not be a number?

1] Pythagoreans: one is not a number since a measure is not the thing measured 1088a7

2] Thymarides: [Pythagorean 4th b.c.e.] unit as a 'limiting quantity' Iamblichus 11-12

3] . "an unities is no number but the beginning and original of number." Baker 1568

4] "...Ramus, and such that have written since his time, affirme not only that an unite or one, is a number, but also that every fraction or parte of an vnite, is a number..." Hylles 1592

5] "...unity alone out of all number, when it multiplies itself, produces nothing greater than itself...Unity, therefore, is non-dimensional and elementary." Nicomachus 238

6] [a] if from a number there is subtracted no number, the given number remains [b] if from 3 we take 1, 3 does not remain [c] therefore one is not 'no number.' Simon Stevin

7] Rabbi Ben Ezra *Sefer ha-Echad* (Book on Unity) (1140) first to entertain idea that one is a number

8] “Multiplicity is the genus of *arithmos*. Because of this *arithmos* and one are opposites.” Iamblichus [see also Aristotle *Metaphysics* 1056b 19ff]

9] “The one is the source of number” *Metaphysics* 1052b22ff.

10] “...unity is not an *arithmos* for neither is a measure measures, but a measure is a principle, and so is unity.” Aristotle, *Metaphysics* 1088a7

5] Is there any notable disagreement among the above definitions of unity?

6] Think back to Aristotle’s claim that a definition should aim at the ‘*ti esti*’ of something – the essence, or literally, the ‘what is it’. What would you, or any modern, say the ‘*ti esti*’ is of 3125 as opposed, say to the ‘*ti esti*’ of 6385)? What would Nicomachus say?

7] For Nicomachus what does it mean to ‘know’ a number?

8] One reason why 1 and 2 are not considered numbers by the Pythagoreans derives from their conception of a number as the sum of other numbers. In this conception, 1 and 2 can only be generators of number, not numbers themselves. The Pythagorean shape numbers in Nicomachus are a particular version of this conception of numbers as sums – in this case a number is a summation series. Why do you think the Pythagoreans might have thought that this quality of being a sum was definitive to the concept ‘number’?

#### ALTERNATE ARITHMOS QUESTIONS:

1] From all of the evidence of the material below, come up with a definition of *arithmos* that does not refer to what in particular is included or excluded (e.g. fractions, ‘irrational numbers’, one, etc.). It might help you to think (or to frame your answer) with the question of what distinguishes *arithmos* from our concept of number (but again, we are looking for the concept – the container – not the content of that container. There is something about the ‘containers’ *arithmos* and ‘number’ that justifies what they include and exclude – what is it?)

2] Beginning by thinking about the difference in the notions of proportion and ratio in Books V and VII, speculate on what the essential difference is for the Greeks between magnitude and multitude, and why it was so important for them to consider these essentially different kinds of entities. (Does Aristotle’s distinction between quantity and quality help at all?)

3] The Greeks have left us no rationale for why they only considered what we call positive integers to be *arithmoi*. To put it simply, this is because they did not have to contend with

anyone arguing the case for some category called ‘number’ which includes everything that we put in there. Try to tease out of this week’s readings any clues as to why the Greeks would be so unified in maintaining such a limited concept. (the answer to this will be implicated in the answer to #1, and you can combine the two answers if you’d like).

## **E. THE MODERN CONCEPTION OF NUMBER**

1] In his *Universal Arithmetick* (Newton’s name for algebra), Isaac Newton defines number thus: “By number we understand not so much a multitude of unities, as the abstracted ratio of any quantity, to another quantity of the same kind, which we take for unity. And this is threefold; integer, fracted, and surd: An integer, is what is measured by unity, a fraction, that which a submultiple part of unity measures, and a surd, to which unity is incommensurable” Is this a substantially different conception of number from that found in Nicomachus and/or Euclid?

2] Is Dedekind’s conception of number significantly different from that of Newton? If so, how is it different from that found in Nicomachus and/or Euclid

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