

## II. Logos and Arithmos

### A. Logos as Ratio

- 1] In the definitions in Book V and Book VII of Euclid's *Elements*, why is number defined but magnitude not?
- 2] Why do you think that for number, both part and parts are defined, in magnitude only 'part'?
- 3] Book V. defines a ration between magnitudes, but Book VII has no definition of ratio between arithmoi. Why?
- 4] Describing what a proportion is for magnitudes is a much more complicated affair than describing proportions for arithmoi. What does the definition of proportion for magnitudes mean, and why can it not be as simple as the definition of proportion for arithmoi?
- 5] Why does Euclid represent numbers with lines instead of points, as his Pythagorean predecessors do? What is the difference in the 'symbolic capacity' of the two means of representation?
- 6] Why do you think Diophantus precedes his known numbers (rather than his variables) with the symbol 'M' (an abbreviation of 'monad')?
- 7] Consider Euclid VII 37 "If a number be measured by a certain number, the number measured will have a part having the same name as the measuring number." What does this mean? What does it say about how Euclid considers fractions in relationship to the concept 'arithmos'?
- 8] Given the Greek understanding of fractions, why do you think they were only written as strings of unit fractions?
- 9] One reason why 1 and 2 are not considered numbers by the Pythagoreans derives from their conception of a number as the sum of other numbers. In this conception, 1 and 2 can only be generators of number, not numbers themselves. The Pythagorean shape numbers in Nicomachus are a particular version of this conception of numbers as sums – in this case a number is a summation series. Why do you think the Pythagoreans might have thought that this quality of being a sum was definitive to the concept 'number'?

## B. Magnitude and Multitude

1] It is said that the Pythagoreans were the first to consider geometry and arithmetike (number theory) as one subject, which they called 'mathematike' or 'learnable'. It was not until Aristotle that the term seemed to gain general usage as meaning specifically these kinds of study (including then the rest of the classic 'quadrivium', music and astronomy). It seems significant, and perhaps necessary, that in creating the axio-deductive system of theoretical geometry, the Pythagoreans convert geometry from a science of measurement to a science (perhaps) of shapes. In doing so they banish all numbering from geometry and create a disjunct between number and magnitude. Given this, why do you think the Pythagoreans thought that these subjects should be grouped together under one label?

2] Compare the definitions in Bk. V and Bk. VII of Euclid. Note the differences and explain them (most important, contrast and compare Def. V.5, 6 and Def. VII. 20).

3] Many of the propositions in Bk. V and Bk. VII are identical. Make note of anything that occurs in one book but not the other, and speculate on the cause or meaning of such distinctions.

4] What distinctions could one draw between magnitude and multitude if one considered only Euclid Bks. V. and VII?

5] Thinking about Aristotle's division of the genus 'quantity', and the Greek conception of *arithmos*, why do you think that magnitude and multitude are treated as entirely distinct subjects?

6] Euclid's Book VII takes four definitions to cover the same issues for magnitude that Book V. covers in one definition for multitude. What do you think this says about either the relative complexity of the two concepts, or their relative 'intuitiveness'?

7] Does the fact that most of the definitions about proportion come in the book on magnitude indicate that magnitude is the genus, of which multitude is a species (given that all of these definitions apply also to arithmoi)? Consult Aristotle on this.

8] In the definitions of Book VII there is an elaborate system for classifying arithmoi. Why is there not a similarly elaborate system of classification for magnitudes? (in our section on irrationals, you might consider whether or not Book X performs this function for magnitudes).

### C. Quality and Quantity

1] The first sentence in Ch. 6 of Aristotle's *Categories* divides quantity into the sub-categories 'discrete' and 'continuous'. This is the first thing that Aristotle has to say about quantity, thus one assumes that it is the most fundamental, if not most important statement one can make about quantity. If this is the beginning of a definition, what kind of definition is it?

2] These are the primary species of the genus 'quantity'. Why? Can you think of any other ways to subdivide the idea of 'quantity'?

3] The division of relative position vs. no relative position is the second way that Aristotle divides the genus quantity. The division does not coincide with the first division. Does it seem as important to you? As 'natural'?

4] Again, in Ch. 6, Aristotle says "That which is not a quantity can by no means, it would seem, be termed equal or unequal to anything else" Does this constitute a definition of quantity: if not an essential one, at least exclusive?

5] The Pythagoreans supposedly began the tradition that considered the four mathematical sciences (arithmetike, geometry, music, astronomy) to be related as parts of a greater science of 'quantity'. Does Aristotle's discussion of quantity support this contention?

6] The word 'Arithmos' does not cover the same ground as 'number.' We have a few phrases that come close to expressing what 'arithmos' expresses: counting numbers, positive integers, natural numbers. The Greeks did not have a word that covers exactly the same ground that our word number covers. Does '*poson*' [quantity, 'how

MIT OpenCourseWare  
<http://ocw.mit.edu>

ES.2H3 Ancient Philosophy and Mathematics  
Fall 2009

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.