Transportation Management

Vehicle Routing

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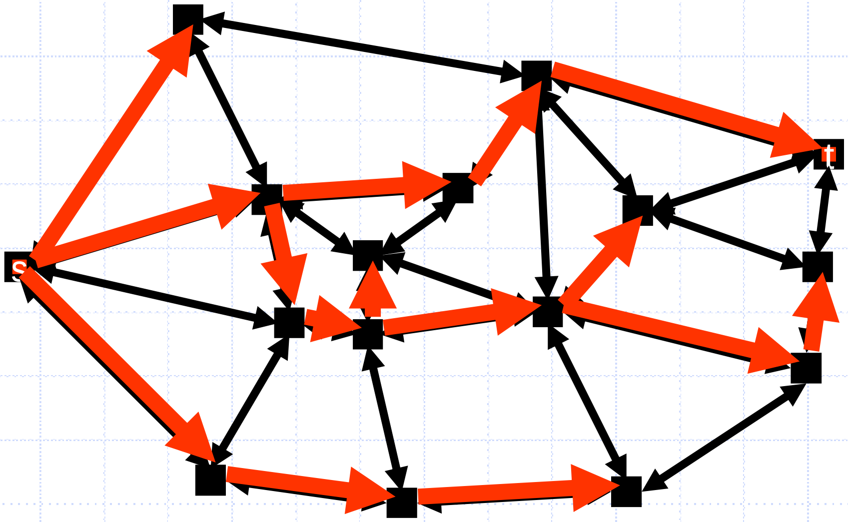
Local Routing

- Large Number of Network Problems – we will look at four
  - Shortest Path Problem
    - Given: One origin, one destination
    - Find: Shortest path from single origin to single destination
  - Transportation Problem
    - Given: Many origins, many destinations, constrained supply
    - Find: Flow from origins to destinations
  - Traveling Salesman Problem
    - Given: One origin, many destinations, sequential stops, one vehicle
    - Find: Shortest path connecting each stop once and only once
  - Vehicle routing Problem
    - Given: One origin, many destinations, many capacitated vehicles
    - Find: Lowest cost tours of vehicles to destinations

Shortest Path Problem

- Find the shortest path in a network between two nodes – or from one node to all others
- Result is used as base for other analysis
- Connects physical to operational network
- Issues
  - What route in practice is used? Shortest? Fastest? Un-restricted?
  - Frequency of updating the network
  - Using time versus distance (triangle inequality)
  - Impact of real-time changes in congestion
  - Speed of calculating versus look-up

Shortest Path



Shortest Path Matrix

i\j	1	2	3	4	. . .	n
1		d_{12}	d_{13}	d_{14}		d_{1n}
2			d_{23}	d_{24}		d_{2n}
3				d_{34}		d_{3n}
4						d_{4n}
. . .						
n						

- Network
  - Arc/Link & Nodes
  - Cost is on nodes, c_{ij}
- Think of a string model
- Basic SP Algorithm (s to t)
  1. Start at origin node, s=i
  2. Label each adjacent nodes, j, $L'_j=L_i+c_{ij}$ iff $L'_j<L_j$
  3. Pick node with lowest label, set it to i, go to step 2
  4. Stop when you hit node t
- Building Shortest Path Tree
- Many, many variations on this algorithm,
  - Label Setting
  - Label Correcting

Transportation Problem

- Find minimum cost routes for between multiple origins and destinations
- Flow is fungible – same products
  - Cost on arcs, c_{ij},
  - Flow on arcs, x_{ij}
- Many solution approaches
  - Balanced problem – Supply=Demand
  - Unbalanced –
  - Transhipment Problem – neutral nodes

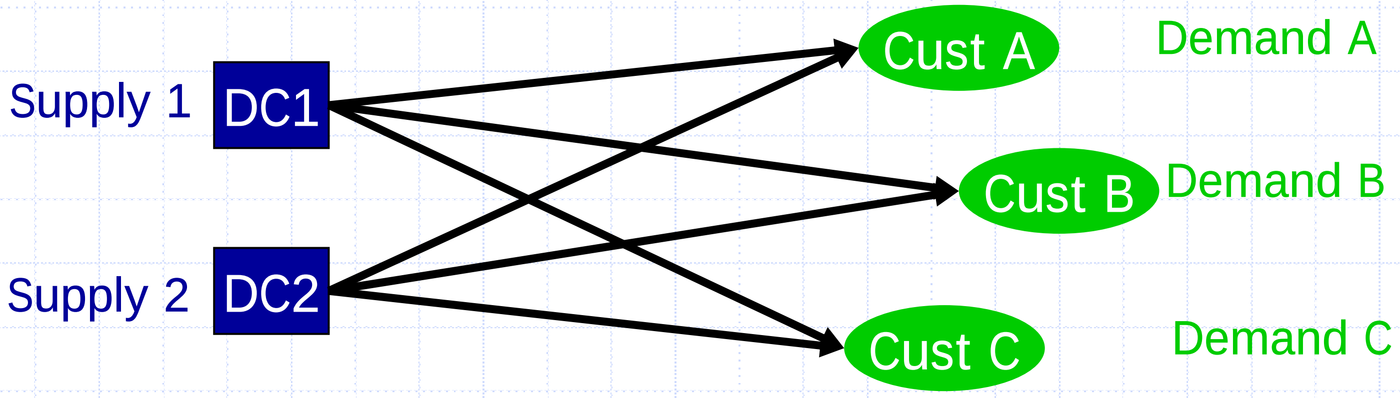
$$Min \quad \sum_{ij \in N} c_{ij} x_{ij}$$

$$s.t.$$

$$\sum_{j=1}^{n} x_{ij} = Supply_i \quad \forall i$$

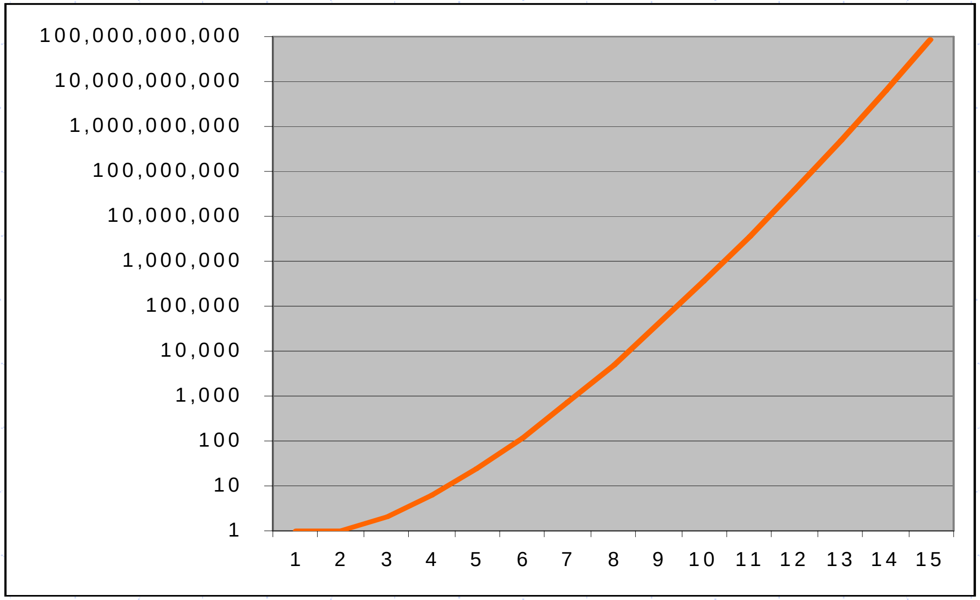
$$\sum_{i=1}^{n} x_{ij} = Demand_j \quad \forall j$$

$$x_{ij} \geq 0 \quad \forall ij$$



Traveling Salesman Problem

- Starting from an origin, find the minimum distance required to visit each destination once and only once and return to origin.
- *m*-TSP: best tour for *m* salesmen
- Very old problem ~1832
  - For history, see: http://www.tsp.gatech.edu/index.html



Number of tours with n cities

TSP Solution Approaches

- Heuristics
  - Construction
    - Nearest neighbor
    - Greedy (complete graph, pick shortest edge until Hamiltonian path)
    - Sweep (example of Cluster-First, Route-Second)
    - Space filling curve (example of Route-First, Cluster-Second)
    - Insertion (nearest, cheapest)
    - Savings (Clarke-Wright)
  - Local Improvement
    - 2-opt
    - 3-opt
  - Meta-heuristics
    - Tabu Search
    - Ant System
    - Simulated Annealing
    - Genetic Algorithms
    - Constraint Programming

Traveling Salesman Problem

- Nearest Neighbor Heuristic
  - Start at any node and connect tour to closest adjacent node
  - In practice 20% above optimal
- Insertion Heuristic
  - Form some sub tour (convex hull) and add in the nearest/furthest/cheapest/random node one at a time
  - In practice 19% / 9% / 16% / 11% above optimal
- 2-Opt Heuristic
  - Method of improving a solution
  - Select two edges (a,b) and (c,d) where total tour distance decreases the most if reformed as (a,c) and (b,d).

Vehicle Routing Problem

- Find minimum cost tours from single origin to multiple destinations using multiple vehicles
- Who needs to solve the problem?
  - Shippers – retailers, distributors, manufacturers
  - Carriers – LTL, package
  - Service companies – repair, waste, utility, postal, snow removal
- Types of problems
  - Commercial delivery (retailers, distributors, manufacturers)
  - Commercial pickup (retailers, distributors, manufacturers)
  - Mixed pickup & delivery (LTL and package carriers)
  - Residential appointment (online grocery, medical gases, repair)
  - Residential sweep (postal, waste, utility, snow removal)

Initial Routes

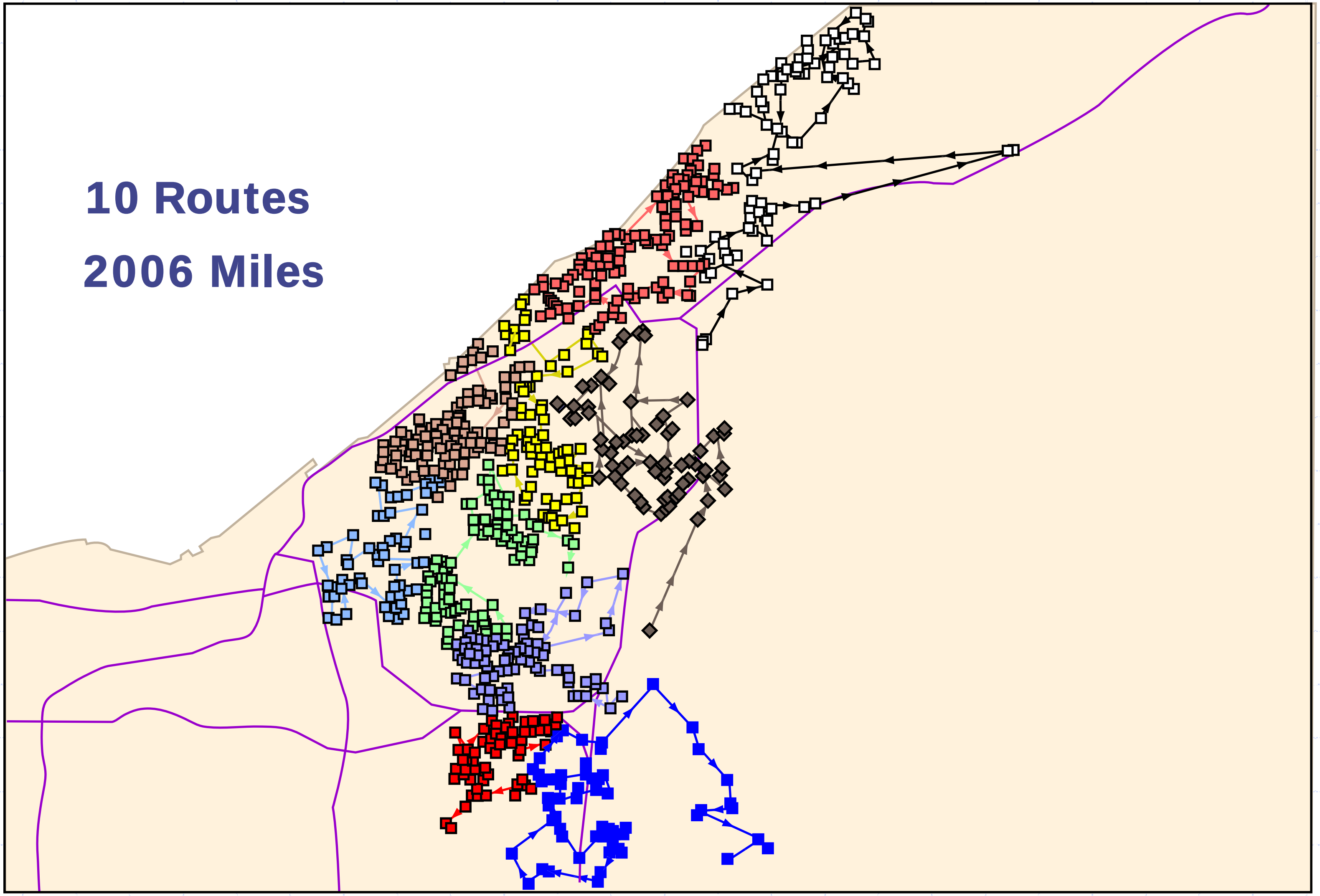
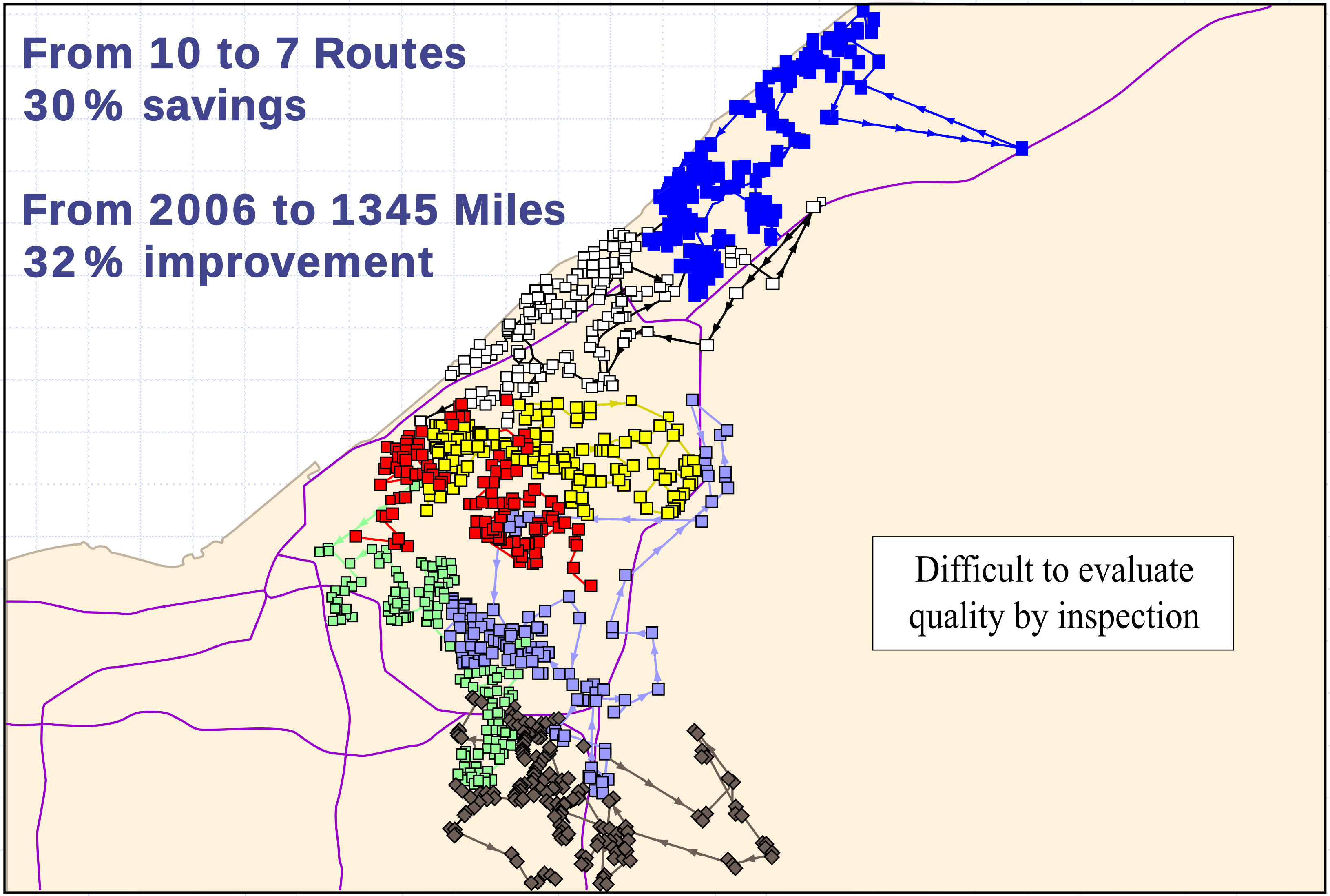


Figure by MIT OCW.

Adapted from Goentzel 2004

Optimized Routes



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VRP is NP-Hard

Difficult to evaluate quality by enumeration

Combinatorial Growth

- 3 stops
- 5 stops
- 10 stops

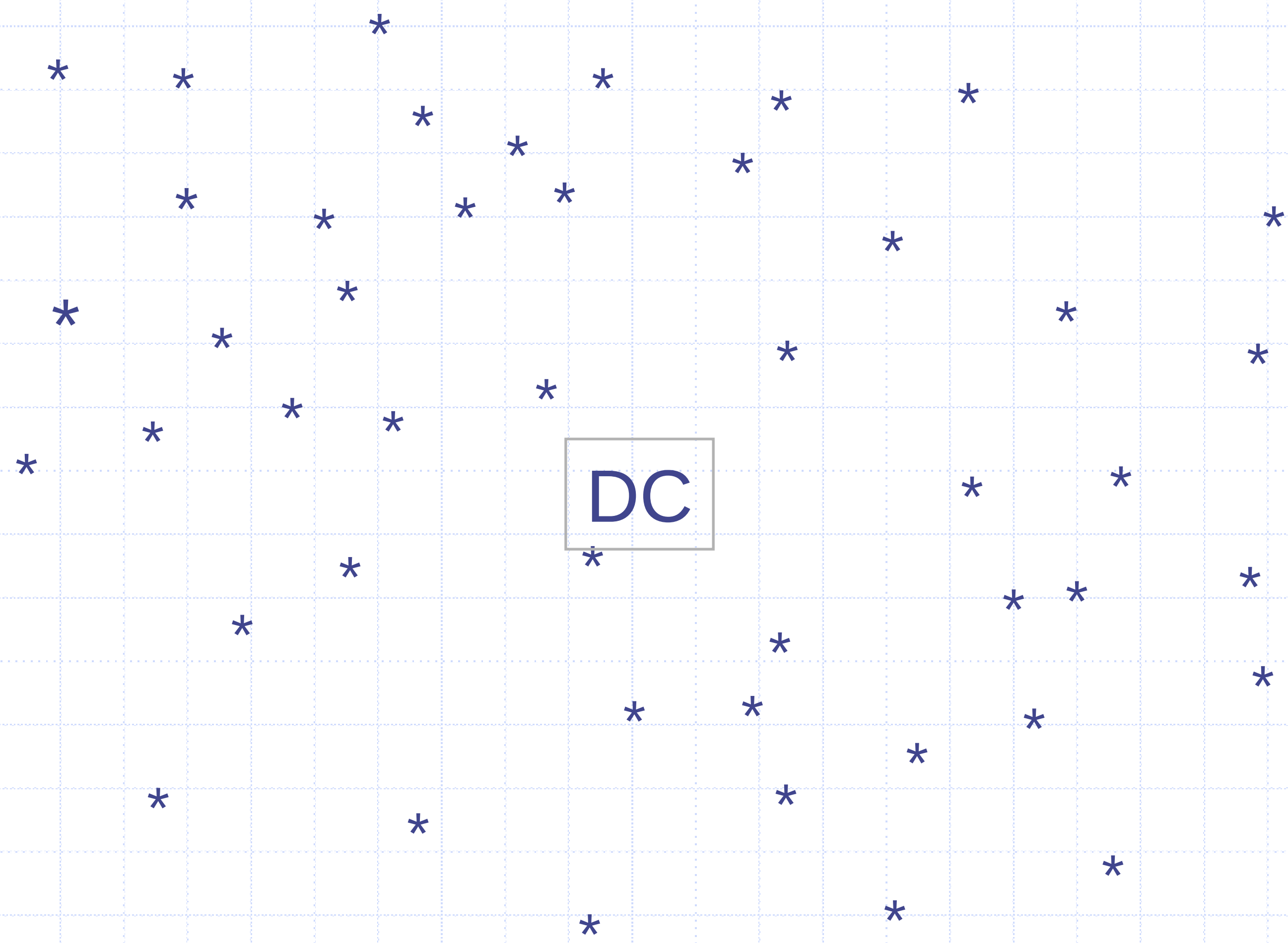
Customers total	Customers on the route	Ways to select customers for the route	Ways to select and sequence the route	Hours of work to evaluate one per second
10	3	120	720	0.20
20	3	1,140	6,840	1.9
30	3	4,060	24,360	6.8
40	3	9,880	59,280	16
50	3	19,600	117,600	33
60	3	34,220	205,320	57
70	3	54,740	328,440	91
80	3	82,160	492,960	137
90	3	117,480	704,880	196
100	3	161,700	970,200	270
				Days of work to evaluate one per second
10	5	252	30,240	0.35
20	5	15,504	1,860,480	22
30	5	142,506	17,100,720	198
40	5	658,008	78,960,960	914
50	5	2,118,760	254,251,200	2,943
60	5	5,461,512	655,381,440	7,585
70	5	12,103,014	1,452,361,680	16,810
80	5	24,040,016	2,884,801,920	33,389
90	5	43,949,268	5,273,912,160	61,041
100	5	75,287,520	9,034,502,400	104,566
total	on the route			**Years** of work to evaluate one per second
10	10	1	3,628,800	0.12
20	10	184,756	670,442,572,800	21,260
30	10	30,045,015	109,027,350,432,000	3,457,235
40	10	847,660,528	3,075,990,524,006,400	97,539,020
50	10	10,272,278,170	37,276,043,023,296,000	1,182,015,570
60	10	75,394,027,566	273,589,847,231,501,000	8,675,477,145
70	10	396,704,524,216	1,439,561,377,475,020,000	45,648,191,828
80	10	1,646,492,110,120	5,974,790,569,203,460,000	189,459,366,096
90	10	5,720,645,481,903	20,759,078,324,729,600,000	658,266,055,452
100	10	17,310,309,456,440	62,815,650,955,529,500,000	1,991,871,225,125

Adapted from Goentzel 2004

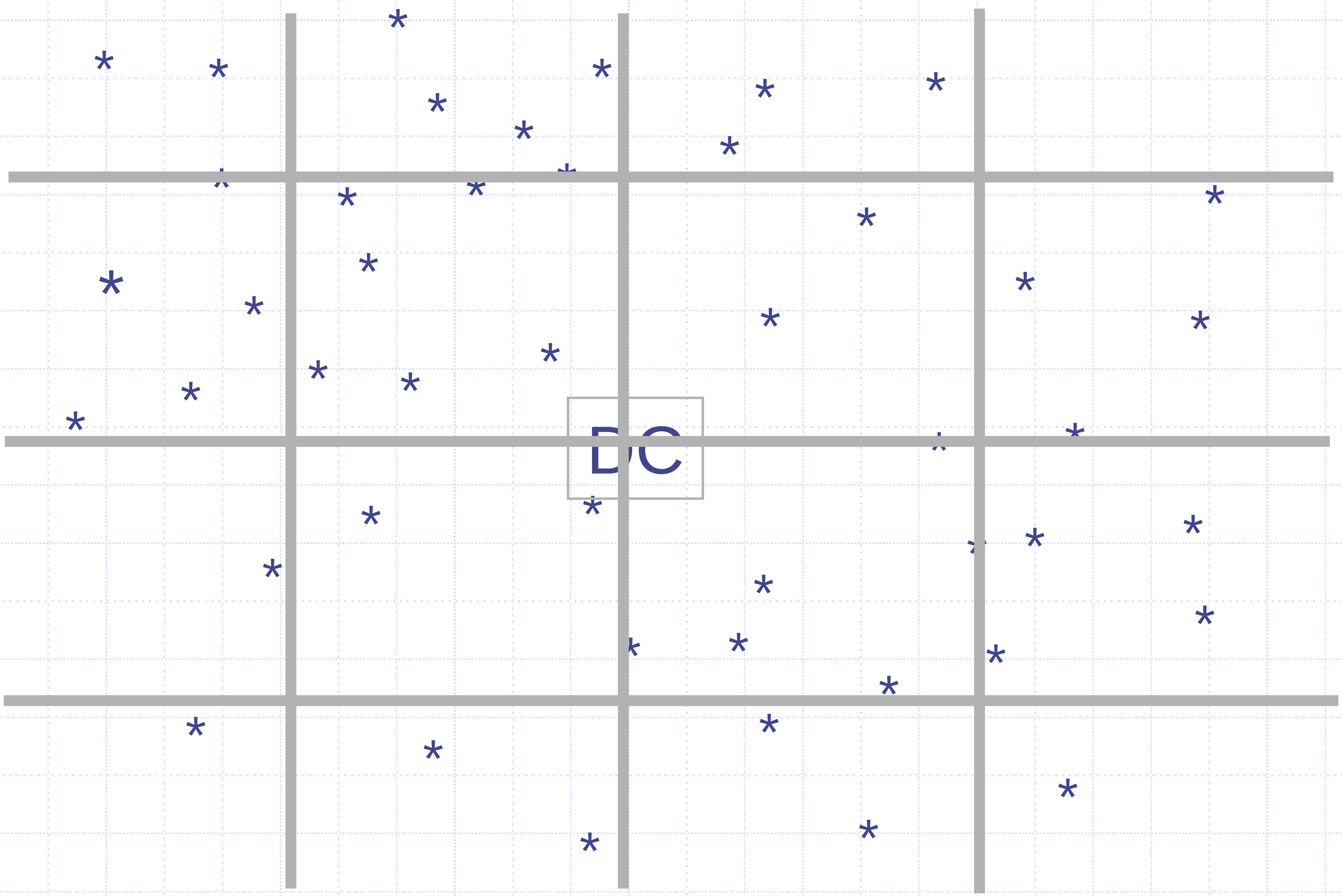
Vehicle Routing Problems

- General Approaches
  - Heuristics
    - Route first Cluster second
      - Space filling curve
      - Any earlier heuristic can be used
    - Cluster first Route second
      - Sweep Algorithm
      - Savings (Clarke-Wright)
  - Optimal
    - MILP – Column Generation

Heuristic Approach – Cluster & Sweep



Heuristic Approach – Cluster & Sweep



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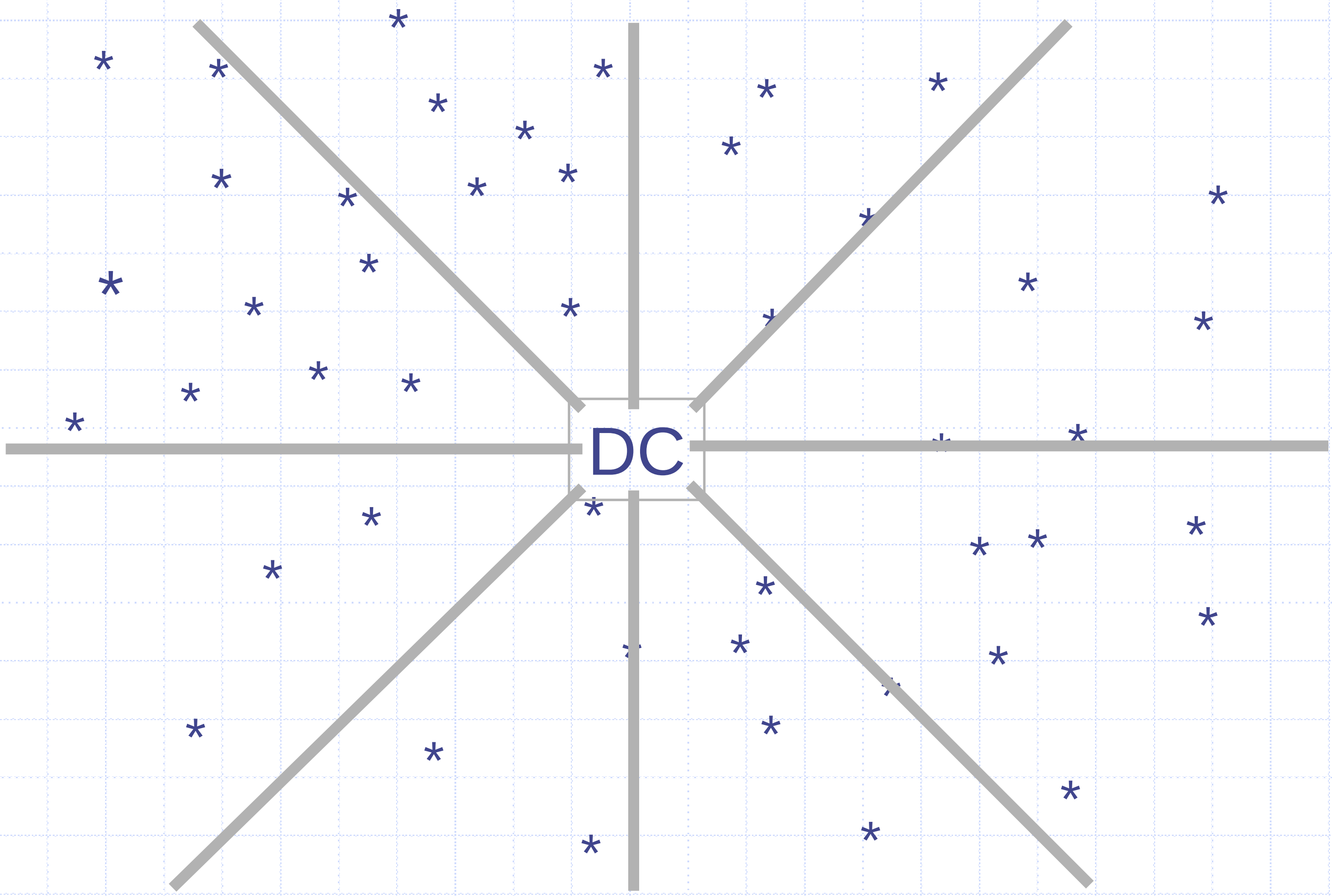

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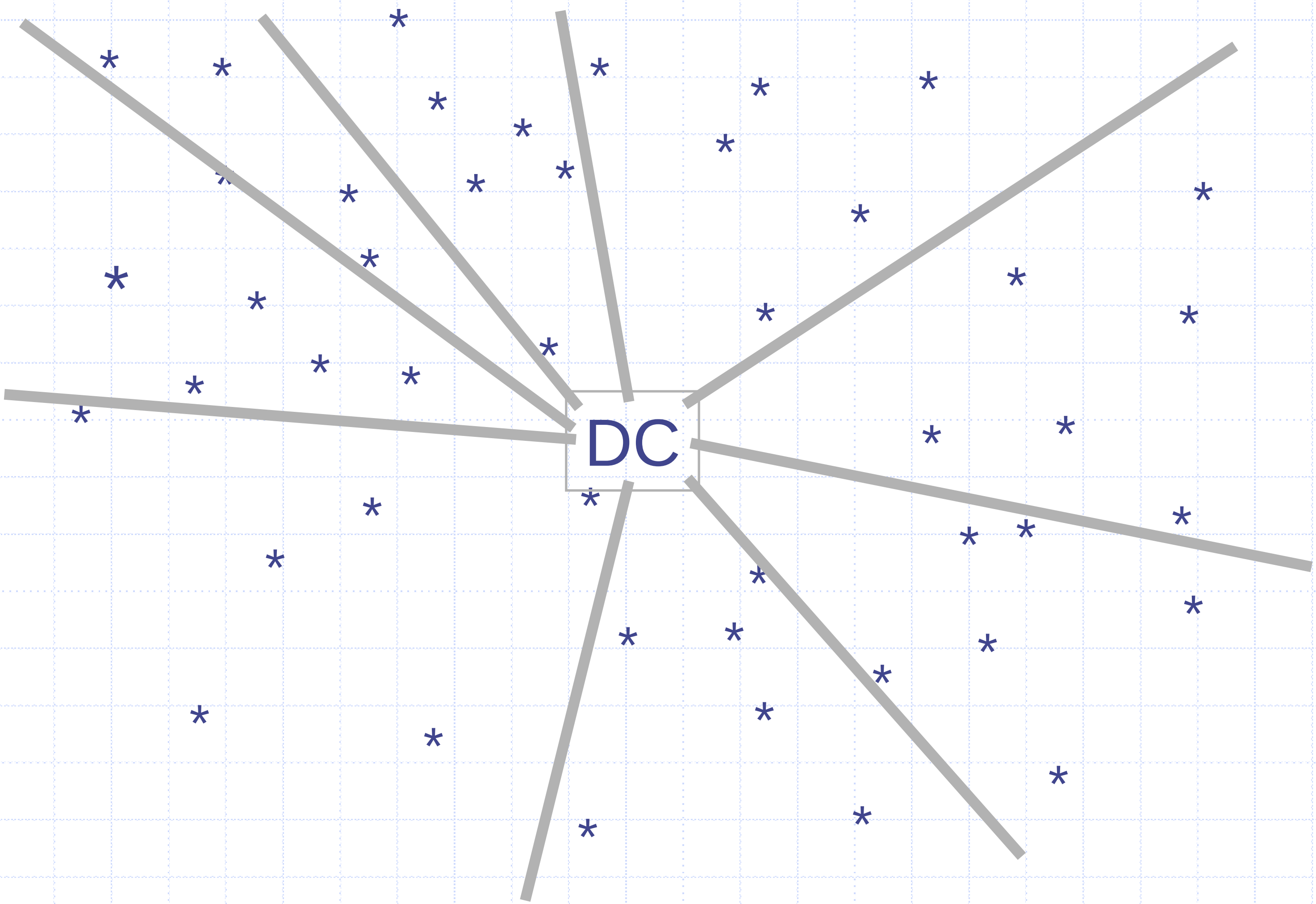
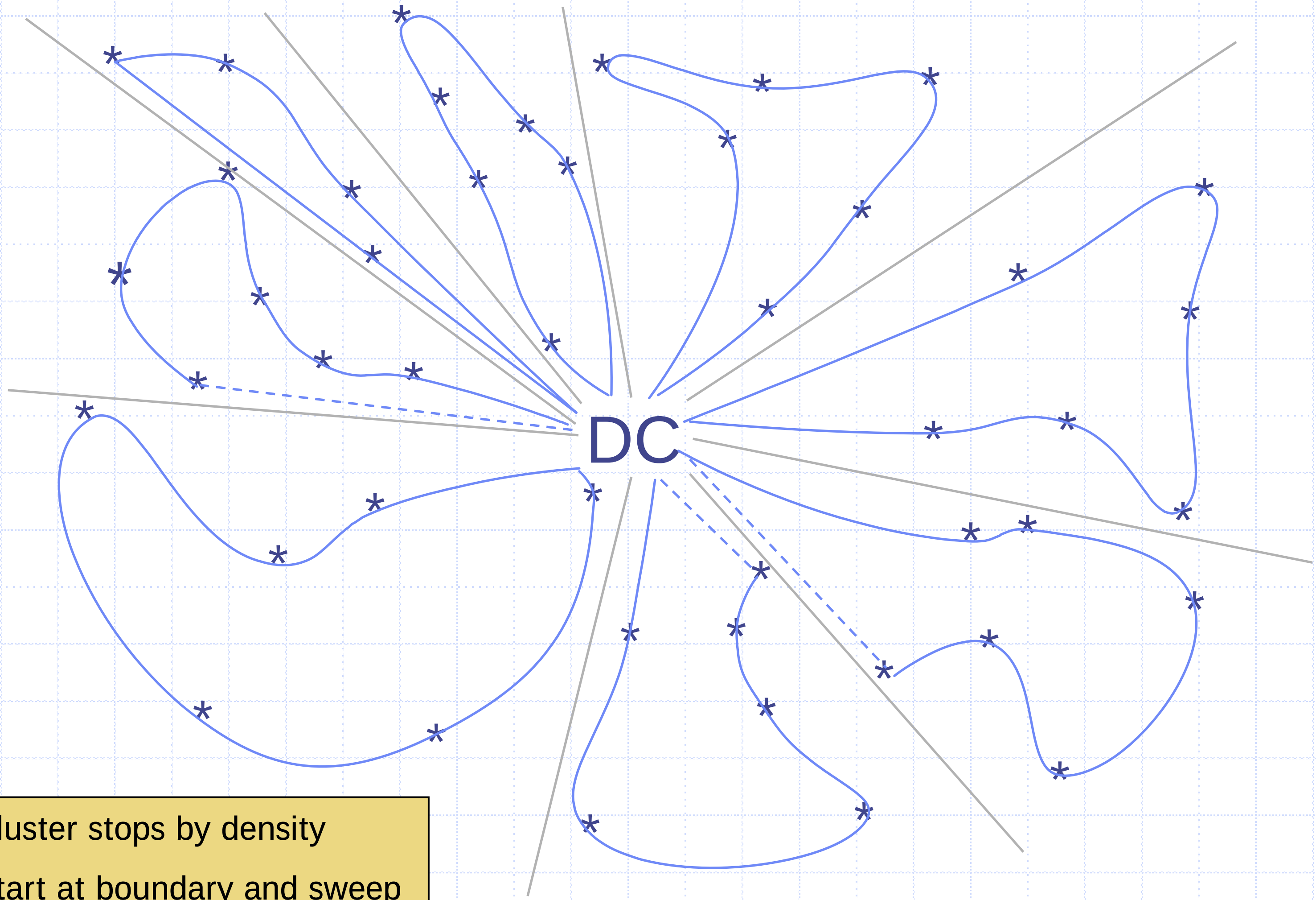


Heuristic Approach – Cluster & Sweep



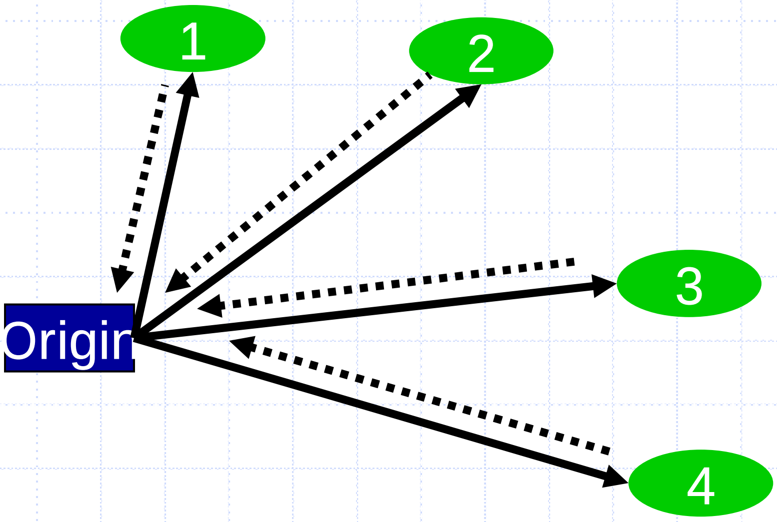
1. Cluster stops by density
2. Start at boundary and sweep CW adding stops until $=V_{MAX}$

Savings Algorithm

- Clark-Wright Algorithm
  - Serve each node directly
  - Identify savings for combining two nodes on same tour
  - Add nodes together if savings >0
    - $2c_{0i} + 2c_{0j} > c_{0i} + c_{ij} + c_{j0}$
    - Savings = $c_{0i} + c_{j0} - c_{ij}$

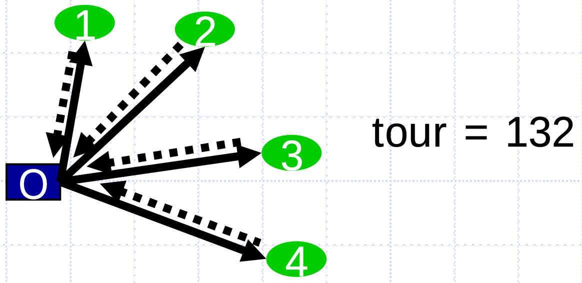
Shortest Path Matrix

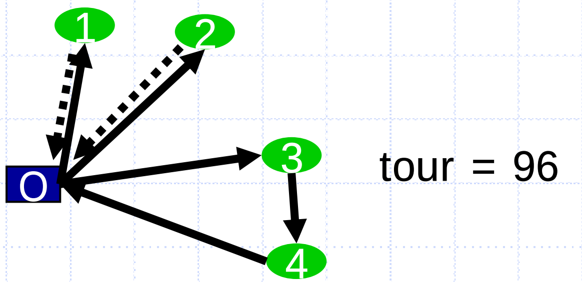
i\j	0	1	2	3	4
0		10	15	19	22
1			8	23	35
2				12	21
3					5

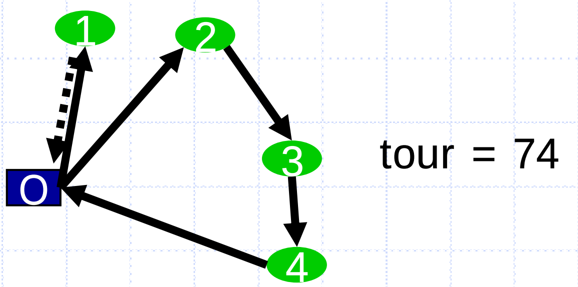


Savings Algorithm

- Suppose Max Capacity = 3
- Savings = $c_{0i} + c_{j0} - c_{ij}$
  - S(1,2) = 10 + 15 – 8 = 17
  - S(1,3) = 10 + 19 – 23 = 6
  - S(1,4) = 10 + 22 – 35 = -3
  - S(2,3) = 15 + 19 – 12 = 22
  - S(2,4) = 15 + 22 – 21 = 16
  - S(3,4) = 19 + 22 – 5 = 36







Shortest Path Matrix

i\j	0	1	2	3	4
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Benefits of this approach?

Optimal Approach – MILP w/CG

	Route 1	Route 2	Route 3	....	.....	Route M	
	C1	C2	C3	....	....	Cm	
Stop A	1	0	0	1	0	1	1
Stop B	1	1	0	0	1	0	1
Stop C	1	1	1	1	0	0	1
Stop D	0	1	1	0	1	1	1
Stop E	0	0	1	1	0	0	1
Stop F	0	0	0	0	1	0	1
Stop G	0	0	0	0	0	1	1
...	0	0	0	0	0	0	1
..	0	0	0	0	0	0	1
Stop N	0	0	0	0	0	0	1

Each Row is a stop

Each Column is a generated vehicle route and its cost

Each matrix coefficient, a_{ij}, is {0,1}, identifying the stops on the j'th route

Define Y_j, {0,1}, "1" if the route is used ; else "0"

Minimize: $\sum_j C_j Y_j$

Subject to:

$$\sum_{j=1}^{J} a_{ij} Y_j \geq D_i \text{ ; for all I}$$

$$Y_j = \{0,1\} \text{ , for all J}$$

Same Example

- Each tour is a column
  - How are tours generated?
  - Could each column be a solution?
  - How could this be enhanced?

Shortest Path Matrix

i\j	0	1	2	3	4
0		10	15	19	22
1			8	23	35
2				12	21
3					5

Total Dist
132

Dec Var	1	1	1	1	0	0	0	0	0	0	0	0	0	0			
	Route 1	Route 2	Route 3	Route 4	Route 5	Route 6	Route 7	Route 8	Route 9	Route 10	Route 11	Route 12	Route 13	Route 14	Sum		RHS
Stop 1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1	>=	1
Stop 2	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1	>=	1
Stop 3	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1	>=	1
Stop 4	0	0	0	1	0	0	1	0	1	1	0	1	1	1	1	>=	1
Capacity	1	1	1	1	2	2	2	2	2	2	3	3	3	3			
Distance	20	30	38	44	33	52	67	46	58	46	59	61	60	54			

Regardless of Approach

- Rules of Thumb
  - Good routes are "rounded", not "star shaped"
  - Good routes don't cross themselves or others
  - Good sectors are "pie shaped", not "checker board"
  - Good solutions "look like a daisy“
- Good Practice Tips
  - Always use a Preview-Analyze-Review methodology
  - Periodically visit the internal logic within the TMS
  - Never discount the salty expert who has been doing this longer than you’ve been alive
  - Identify all special conditions (customer A must be delivered to first) and then validate or reject them

Other Extensions to VRP

- More dimensions/elements
  - Sourcing
    - Multiple depot
    - Dynamic sourcing (depot varies)
  - Order
    - Multiple dimensions (e.g. cube, weight)
    - Mixed pickup and delivery
    - Time window
    - “Vendor Managed Inventory”
  - Plan
    - Fixed / Static / Master
    - Variable / Dynamic / Daily
    - Zone
    - Real-time dispatch
  - Resource
    - Backhaul
    - Continuous moves

- Academic problems
  - Multiple Depot VRP (MDVRP)
  - Multi-commodity VRP
  - Vehicle Routing Problem with Pick-up and Delivering (VRPPD)
  - VRP with time windows (VRPTW)
  - Inventory Routing Problem (IRP)
  - Stochastic VRP (SVRP) – minimize expected costs for satisfying realized demand/customers
  - Dynamic VRP – redirect trucks during the execution of their route to accommodate new orders
  - Vehicle Routing Problem with Backhauls (VRPB)

Adapted from Goentzel 2004

Fixed vs. Dynamic Route Plans

- Fixed/static routes
  - Routes repeat on a cycle
    - Daily, weekly, whenever there is sufficient demand
  - Routes are changed when customer base changes
    - Quarterly, annually
  - Routes are based on "forecast" demand
  - Routes are designed for "heavy days" related to truck capacity and driver hours
  - Primary advantages
    - Driver familiarity
    - Ease of execution
  - Primary disadvantages
    - Inefficiency caused by variability
    - Difficulty of efficient customer day assignment

- Variable/dynamic routes
  - Routes change continually
    - Typically every day
  - Routes based on "actual" shipment requirements
  - Routes are designed for vehicle and driver constraints
  - Primary advantages
    - Utilization of trucks and drivers
    - Flexibility in customer ordering
  - Primary disadvantages
    - Difficulty of determining optimum routes
    - Difficulty of maintaining route planning process
    - Execution may not match plan

Adapted from Goentzel 2004

Real-World Issues

- The real world does not behave according to uniform assumptions
  - Dock configuration
  - Dock hours
  - Trailer types
  - Moveable bulkheads (bulk liquids, grocery reefers)
  - Truck types
  - Truck-trailer combos: doubles & triples (pups)
  - Compatibility: order-vehicle, order-order, vehicle-site
  - Preferred customers (big box)
  - Driver preferences (seniority, local knowledge)
  - Driver skills (service technician)
  - Rush hour traffic
  - Real-time dispatching (deployed vehicles)
  - Refueling
  - Maintenance

Adapted from Goentzel 2004

Element Interactions

- Truck & Trailer
  - Trailers the tractor can handle – length, pups, specialized (e.g. car hauler)
- Vehicle & Customer
  - Must be able to visit the customer (loading dock, cornering, parking)
- Vehicle & Order
  - Products may not be deliverable on certain resources -- HazMat, loading/handling equipment (tanks, racks), capabilities (refrigeration), physical dimensions, etc.
- Vehicle & Driver
  - Not licensed for the truck, not able to load/unload trailer
- Order & Order
  - Products may not mix (lumber & light bulbs, bottled water & dehydrated food, etc.)

Adapted from Goentzel 2004

Manual Planning

- Plan using paper, pencil, and experience
- Advantages
  - Cheap and easy
- Challenges
  - Cannot generate multiple solutions
  - Difficult to evaluate result
  - Decentralized

Image of drawn-on map removed due to copyright restrictions.

Adapted from Goentzel 2004 **Map-on-the-wall**

Interactive GIS

- Plan using human intuition, guided by simple heuristics
- Advantages
  - Evaluation is easier (distance, time, cost calculations, and visual)
- Challenges
  - Time consuming (and typically there is limited time for planning)
  - Requires "super-users"
    - Need technical aptitude
    - Requires regular training
  - Typically decentralized

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Adapted from Goentzel 2004

Map-on-the-PC

Automated Heuristics

- Plan using construction, local improvement, & other heuristics
- Advantages
  - Provides solutions relatively quickly
- Challenges
  - Solution quality hard to predict
    - Heuristics that work well for one problem may work poorly for another
    - Solution quality from heuristics can change drastically when the data changes
    - Hard to know when to settle on a solution
  - Complexity
    - Not as good if there are complex constraints or shipments vary in size
    - Need sophisticated expert to improve or tune
  - Typically users stick with the same approach and manually edit plans

Adapted from Goentzel 2004

Optimization

- Column generation and set covering IP
- Advantages
  - Determines best solution among the options considered
- Challenges
  - Quality depends on quality of options created (column generation)
  - Requires significant computing power (parallel computing is advantageous)
  - Requires regular maintenance by domain and technology experts

Adapted from Goentzel 2004

Questions?